Physical seismic modelling of shear-wave singularities on a sphere of orthorhombic phenolic: A research note

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INTRODUCTION

A great deal of theoretical development and numerical modelling of seismic anisotropic wave propagation has been carried out, particularly within the last decade. Only within the past few years have a half dozen or so institutions begun scaled laboratory experiments, or physical modelling, to determine how well the various numerical schemes predict the actual physical results. A particular numerical modelling algorithm might be inadequate for several possible reasons, for instance: the basic theoretical assumptions on which the model rests may be in partial error or incomplete; the algorithm itself might involve computational approximations, perhaps to make the problem numerically tractable, that introduce significant error; or the routine might be derived on an idealized basis that is somewhat at variance with the physical reality (e.g. an elastic model representing an anelastic reality). There is therefore considerable interest on the part of seismic anisotropists to compare the results of numerical and physical modelling for which "the same medium" is used in the modelling procedure, that is, using input parameter values (e.g. stiffnesses, velocities) for a numerical "experiment" that are identical to those of the particular physical medium used in the corresponding laboratory experiment.

One such area of interest, at present, is in the behaviour of shear waves near special directions of propagation known as singular directions or *singularities*, which occur at places where the two shear-wave phase-velocity surfaces meet (i.e. touch or intersect). Near the commonest kind of singularity, the *point* singularity, a shear wave might exhibit anomalous behaviour, such as rapid variation in polarization or amplitude, similar to what might be observed near cusps on the group-velocity surface, even when the anisotropy is not sufficiently strong to cause cusps (Crampin and Yedlin, 1981; Crampin, 1991). Point singularities have now been recognized by Bush and Crampin (1987) and Bush (1990) in VSP data from the Paris Basin, and such observations may become increasingly important in exploration seismology, not least because point singularities may well occur along nearly vertical raypaths in sedimentary basins. If it were possible to determine the directions of such singularities, it could place tight constraints upon the nature of the internal anisotropy of the rockmass (Crampin, 1991).

Our objectives in this physical modelling work are, in the short term, to look for singular directions in an orthorhombic modelling medium, to examine the variations in polarization and amplitude near such directions, and to compare our physical results with numerical results obtained by collaborators; and in the long term, to elaborate how one might use singularity-related observations in exploration and/or development, and to develop the necessary processing code required to this end.

SHEAR-WAVE SINGULARITIES

The concept of singularities in shear-wave slowness and phase-velocity surfaces for anisotropic propagation media has been known for many years. Duff (1960), for example showed that the two shear-wave sheets come into contact at least twice, and usually much more frequently, in directions (of slowness or phase velocity) known variously as singular directions, singular points, or simply singularities. There are three types of singularity: *kiss* singularities, *line* singularities and *point* singularities, in all of which the slowness and phase-velocity surfaces are analytically continuous. Kiss singularities are points where the two surfaces touch tangentially but do not intersect (Figure 1a). They may occur in any anisotropic symmetry system and, in the case of hexagonal or transverse-isotropy symmetry, there is always one at the cylindrical symmetry axis. Line singularities (Figure 1b), which only occur for transverse-isotropy symmetry, are no more than simple intersections of the two surfaces in a circle centred on the axis of cylindrical symmetry (Crampin and Kirkwood, 1981; Crampin and Yedlin, 1981; Crampin, 1991). As Crampin (1991) states, kiss and line singularities are not expected to cause major disturbances to shear wavetrains; nor are any associated anomalies in polarizations or amplitudes likely to be observable in seismic data.

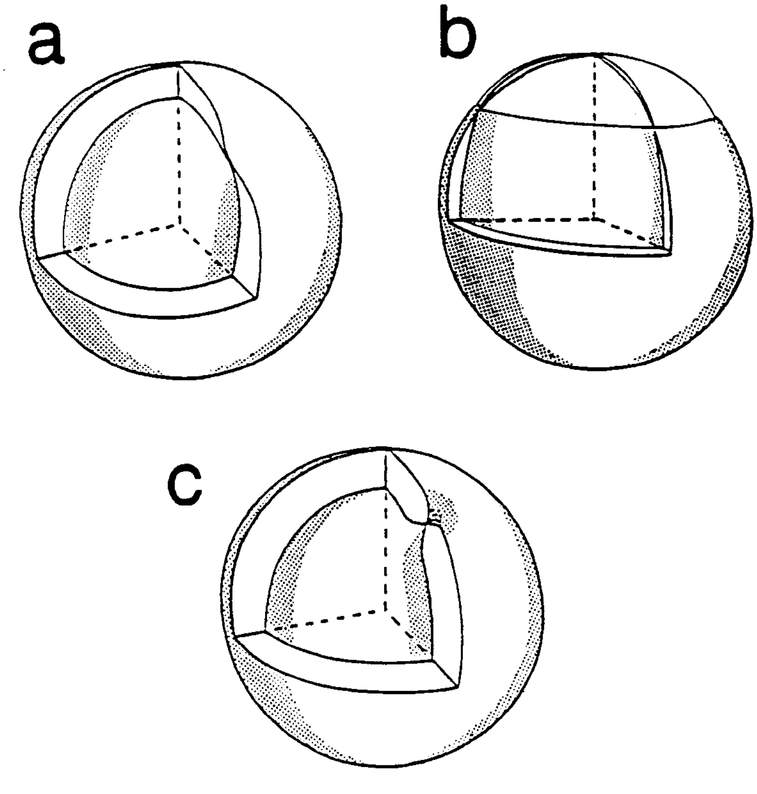

FIG. 1. Sketches of the three kinds of singularity: (a) kiss singularity; (b) line singularity; and (c) point singularity (after Crampin, 1991).

Point singularities are points where the surfaces not only touch but also cross each other, and in such a manner that they are continuous through the vertices of cones on the inner and outer velocity sheets (Figure 1c). They are sometimes known in the crystallographic literature as conical points. These singularities do not occur in transverse isotropy, but necessarily occur in all other symmetry systems. Point singularities may significantly disturb the behaviour of shear waves on neighbouring rays. Related to the fact that the curvature of the phase-velocity surface near such a point is very great, and that continuous paths on the surface passing through the singular point cross from the inner to the outer sheet (and vice versa), is the complicated behaviour of shear-wave polarizations, which can vary by up to 180° over neighbouring rays in the vicinity of the singularity.

In order to simulate the wave propagation in the Paris Basin, Bush and Crampin (1987) and Bush (1990) assumed a model of uniform rock with a combination of periodic thin horizontal layering and parallel vertical fluid-filled cracks. Periodic thin layering (PTL) alone leads to transverse isotropy (having a vertical symmetry axis), while parallel vertical fluid inclusions alone, an example of extensive-dilatancy anisotropy (EDA), produces azimuthal anisotropy (having a horizontal symmetry axis), each of these a special case of hexagonal or transverse-isotropy symmetry. The combination, however, gives rise to orthorhombic symmetry and, contrary to the case for either PTL or EDA alone, the necessary existence of at least some point singularities. Thus, for the purpose of physically modelling propagation phenomena near point singularities – relevant to the simulation of basins with both PTL and EDA – orthorhombic materials are certainly appropriate, whereas transversely isotropic materials are quite inadequate.

EXPERIMENTAL PROCEDURE AND INITIAL OBSERVATIONS

Our previous physical modelling experiments using the industrial laminate Phenolic CE were always carried out on rectangular prisms of the material (e.g. cubes and slabs) (Cheadle et al., 1991; Brown et al., 1991). In these studies, stiffnesses of each sample were determined and these showed small but significant differences. Some of these differences were probably due to the nonuniformity of different samples but some was probably also due to the fact that, relative to zero-offset or axial raypaths, oblique raypaths were necessarily of increasing length as offset increased, possibly introducing uncertainties into the comparison of traveltimes (and velocities) as a result of possible source/receiver array effects, anelasticity, etc. (Brown et al., 1991). This is one reason why we wish to shoot and record over a sphere of material, for which all raypaths will not only be of equal length but also will impinge normally on all source and receiver transducers. We will therefore determine the stiffnesses of the sphere independently – for all of the above reasons.

We have machined a phenolic sphere and initiated experiments on it using the set-up shown in Figure 2, together with the source/receiver transducers and the data acquisition described previously (Cheadle et al., 1991). The procedure is to acquire, in general, nine traces at each point over the sphere. The nine traces correspond to the nine combinations of source and receiver polarizations, using as the three component directions: vertical (normal to the surface), north-south (tangential) and east-west (tangential). This will, in effect, yield a vector transfer function (or impulse response) for each point or propagation direction, allowing one to simulate seismic traces for any source polarization (Igel and Crampin, 1990).

To date, we have acquired data along seven directions, which we specify by their direction cosines, the 1–direction being the fastest and the 3–direction the slowest directions for *P*–wave propagation. The seven initial directions are: (1, 0, 0), (0, 1, 0), (0, 0, 1), $(0, \sqrt{2}/2, \sqrt{2}/2)$, $(\sqrt{2}/2, 0, \sqrt{2}/2)$, $(\sqrt{2}/2, \sqrt{2}/2, 0)$ and $(\sqrt{3}/3, \sqrt{3}/3, \sqrt{3}/3)$. At this writing, the traces from these shots have not been processed or analyzed but we intend to discuss them at the 1991 CREWES sponsor meeting.

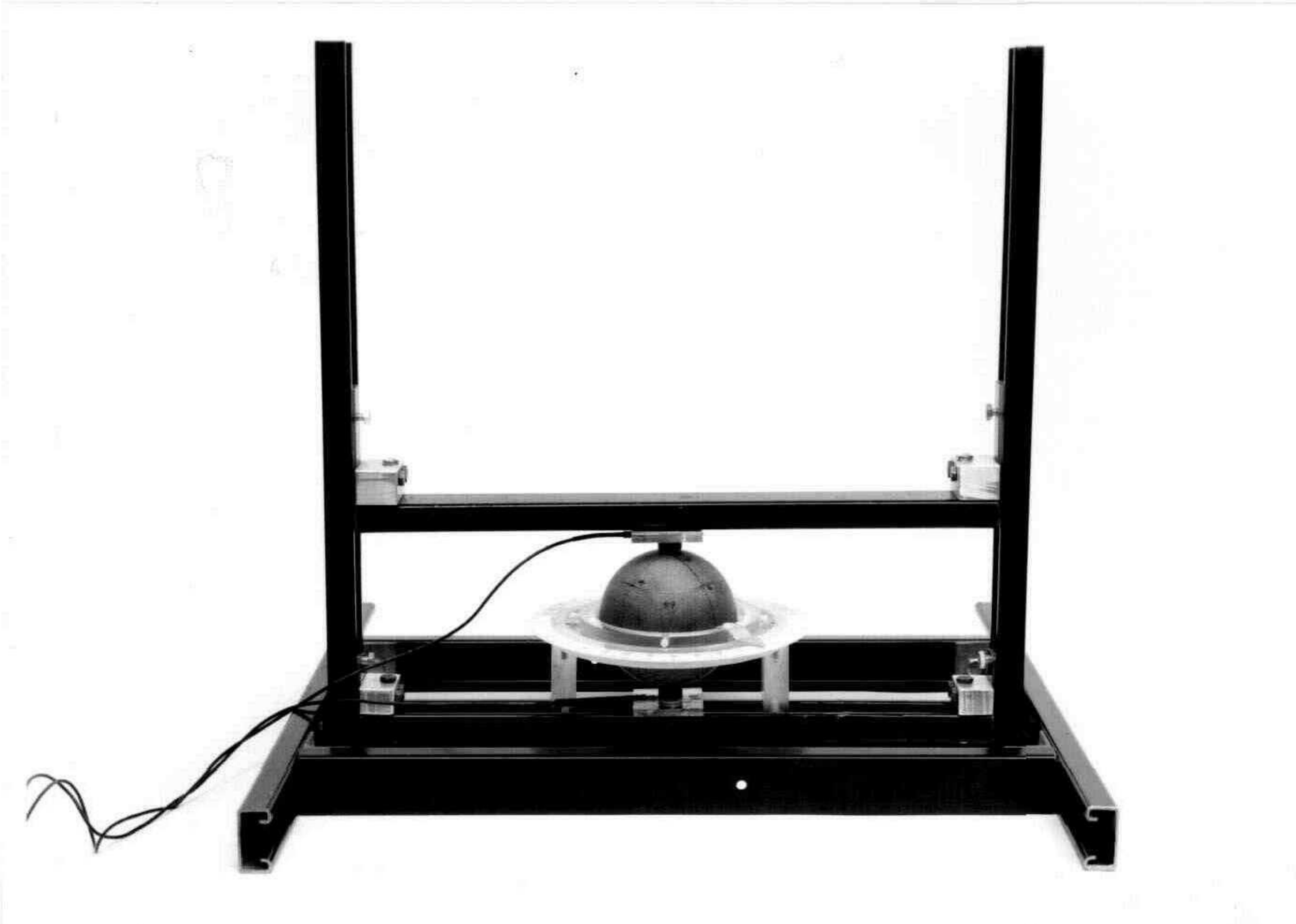

FIG. 2. Photograph of the laboratory set-up used in shooting and recording on the phenolic sphere. Transducers are in contact with the sphere at its top and bottom.

FUTURE WORK

The stiffnesses, which can be determined from the first six of the seven initial shots, will be used as input to the ANISEIS numerical modelling software of the Edinburgh Anisotropy Project, the output of which will be compared with the physical modelling results of the CREWES Project. We have some partial results from ANISEIS which, at present, are based on the stiffnesses determined for a 10–cm cube of Phenolic

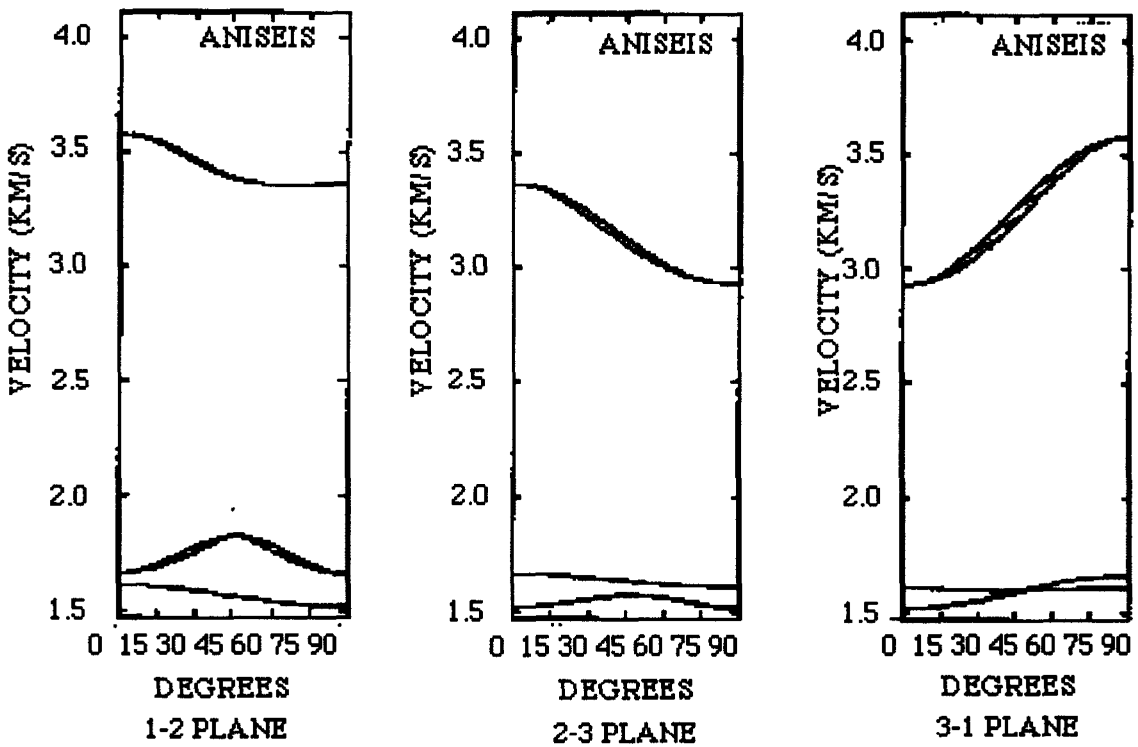

FIG. 3. Variation of qP, $qS1$, and $qS2$ phase velocities (solid curves) and group velocities (dashed curves) for a cube of phenolic CE, computed by ANISEIS (courtesy of S. Crampin) for the stiffnesses given by Cheadle et al. (1991). A shear-wave point singularity is seen at about 45° in the 3-1 plane.

CE (Cheadle et al., 1991). These are shown in Figure 3, where a point singularity can be seen. However, singularities are quite sensitive to variations in stiffnesses, so we await numerical results from the sphere before proceeding with comparisons.

Practical problems will undoubtedly arise in this work. For example, how can we compare recorded amplitudes generated by different source transducers (P versus S) which may have different piezoelectric responses and probably couple differently to the material surface? In fact, we can use the principles of seismic reciprocity, in particular negative reciprocity (Knopoff and Gangi, 1959; Brown et al., 1991) to equalize e.g. a *P–SH* trace (*P* source, *SH* receiver) and an *SH–P* trace, and so on.

We also still have the problem of relatively large transducers (compared to dominant wavelengths and specimen size) which mean that any one trace is a sort of composite record of a rather thick pencil of rays. For traces generated and recorded on a sphere, this is not as great a problem as for a cube or slab because the transducer faces are everywhere normal to the raypaths for the sphere. For continued work, however, we are looking into methods to reduce the effective transducer sizes, such as: miniature transducers, deconvolution of recorded traces (incorporating knowledge of the transducer directivity functions or radiation patterns), or more advanced technology based on interferometry or pulsed laser methods (e.g. Castagnede et al., 1991).

ACKNOWLEDGEMENT

We thank Stuart Crampin of the Edinburgh Anisotropy Project (British Geological Survey) for his suggestion and encouragement to carry out this research, as well as for generating the ANISEIS output (Figure 3).

REFERENCES

Brown, R.J., Lawton, D.C., and Cheadle, S.P., 1991, Scaled physical modelling of anisotropic wave propagation: multioffset profiles over an orthorhombic medium: Geophys. J. Int., **107**, in press.

Bush, I., 1990, Modelling shear-wave anisotropy in the Paris Basin: Ph.D. thesis, Univ. of Edinburgh.

Bush, I., and Crampin, S., 1987, Observations of EDA and PTL anisotropy in shear-wave VSPs: 57th Ann. Int. Mtg., Soc. Expl. Geophys., Expanded Abstracts, 646–649.

Castagnede, B., Kim, K.Y., Sachse, W., and Thompson, M.O., 1991, Determination of the elastic constants of anisotropic materials using laser-generated signals: J. Appl. Phys., **70**, 150–157..

Cheadle, S.P., Brown, R.J., and Brown, R.J., 1991, Orthorhombic anisotropy: A physical seismic modelling study: Geophysics, **56**, in press.

Crampin, S., 1991, Effects of point singularities on shear-wave propagation in sedimentary basins: Geophys. J. Int., **107**, in press.

Crampin, S., and Kirkwood, S.C., 1981, Velocity variations in systems of anisotropic symmetry: J. Geophys., **49**, 35–42.

Crampin, S., and Yedlin, M., 1981, Shear-wave singularities of wave propagation in anisotropic media: J. Geophys., **49**, 43–46.

Duff, G.F.D., 1960, The Cauchy problem for elastic waves in an anisotropic medium: Phil. Trans. R. Soc., **A252**, 249–273.

Igel, H., and Crampin, S., 1990, Extracting shear-wave polarizations from different source orientations: Synthetic modelling: J. Geophys. Res., **95**, 11,283–11,292.

Knopoff, L., and Gangi, A.F., 1959, Seismic reciprocity: Geophysics, **24**, 681–691.