A simple algorithm for band-limited impedance inversion

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ABSTRACT

This paper describes a seismic inversion method which has been cast as a MATLAB algorithm called BLIMP (Band Limited IMPedance). It can be used to invert seismic data using a well log, or some known impedance function, to provide low frequency which, using common acquisition methods, is required by the inversion process.

Vertical component data from the Blackfoot broad-band 2D 3C seismic survey were used as inversion examples. A comparison of the inversion results from the 10 Hz and 2 Hz data sets showed that the recorded low frequencies in the 2 Hz data were reliable. BLIMP was found to be simple to use, robust, and provided accurate impedance estimates

INTRODUCTION

To approximate the impedance of the subsurface using seismic data, it is necessary to account the band-limited nature of seismic data, especially at low-frequencies (Lines and Treitel, 1984). Waters (1978) described an inversion scheme which he dubbed ‘SAIL’ (Seismic Approximate Impedance Log). This method is simple approach to deriving impedance values from seismic data. An *impedance estimate*, from a well-log or stacking velocities, is first combined in the frequency domain with integrated seismic data. The result is inverse Fourier transformed to provide an impedance trace. Detailed impedance values are thus provided by the integrated seismic data, and the low-frequency trend is provided by the well-log.

Waters’ (1978) method can be summarized as follows:

1) Obtain an impedance estimate. If using a well log convert it from depth to time and tie it to the seismic section.

2) Integrate then exponentiate each trace of the seismic section.

3) Fourier transform the impedance estimate and the integrated traces.

4) Scale the spectra of the integrated traces to that of the impedance estimate.

5) High-cut filter the impedance estimate and add it to the low-cut filtered spectra of the integrated seismic traces.

6) Inverse Fourier transform the combined spectra. The filter cut-off should be dictated by the lowest reliable seismic frequencies.

The above process, with some modifications, has been cast as a MATLAB function ‘BLIMP’ (Band Limited IMPedance) and requires a minimum of user input.

In this report BLIMP is demonstrated by first developing a relationship between seismic impedance and the seismic trace. Then, using seismic data from the Blackfoot broad-band 2D 3C seismic survey (Miller et al., 1995), a single 10 Hz trace is inverted and compared to a log-impedance which ties the line. This result is then compared to the inversion of the equivalent 2 Hz trace. The entire sections, for the 10 Hz and the 2 Hz data are then inverted and examined

METHOD

The BLIMP method begins with specifying the relationship between the seismic trace and seismic impedance. Thus, define the normal incidence reflection coefficient as:

$$r_j = \frac{I_{j+1} - I_j}{I_{j+1} + I_j}, \tag{1}$$

where I_j is seismic impedance, and r_j is seismic reflectivity.

Solve equation (1) for I_{j+1}

$$I_{j+1} = I_j\left(1 + \frac{2r_j}{1-r_j}\right) = I_j\left(\frac{1+r_j}{1-r_j}\right),\ I_n = I_1\left(\frac{1+r_1}{1-r_1}\right)\left(\frac{1+r_2}{1-r_2}\right)\cdots\left(\frac{1+r_{n-1}}{1-r_{n-1}}\right)$$

$$\Rightarrow I_{j+1} = I_1 \prod_{k=1}^{j}\left(\frac{1+r_k}{1-r_k}\right). \tag{2}$$

Divide (2) by I_1 and take the logarithm,

$$\ln\left(\frac{I_{j+1}}{I_1}\right) = \sum_{k=1}^{j} \ln\left(\frac{1+r_k}{1-r_k}\right) \approx 2\sum_{k=1}^{j} r_k. \tag{3}$$

The last step follows from an approximation for ln which is valid for small r. Solve (3) for I_{j+1}:

$$I_{j+1} = I_1 \exp\left(2\sum_{k=1}^{j} r_k\right) \tag{4}$$

Model the seismic trace as scaled reflectivity : $S_k = \frac{2r_k}{\gamma}$, then (4) becomes:

$$I_{j+1} = I_1 \exp\left(\gamma\sum_{k=1}^{j} S_k\right). \tag{5}$$

Equation (5) thus integrates the seismic trace and then exponentiates the result to provide an impedance trace. BLIMP uses equation (5) to invert the seismic trace in the same manner as Waters (1978), but with some modifications related to the pre-conditioning of the required impedance estimate and the scaling of the seismic trace.

The following is a step by step description of the BLIMP method:

1) Compute the linear trend of the impedance estimate and subtract it (this reduces edge effects during subsequent frequency domain operations).

2) Compute the Fourier spectra of (1).

3) Apply a band-limited integration filter to each seismic trace and exponentiate the result.

4) Compute the Fourier spectra of (3).

5) Determine a scalar to match the mean power of (4) and (2) over the seismic signal band.

6) Multiply the spectra of (4) by the scalar from (5).

7) Low-pass filter (2) and add to (6).

8) Inverse Fourier transform (7).

9) Add the low-frequency trend from (1) to (7)

The required filters in steps (3), (5) and (7) are designed using the same user specified Gaussian rolloff at high and low frequencies.

INVERSION EXAMPLES

The data of Figure 1 was obtained from the Blackfoot Broad-Band 2D 3C seismic survey. A well-log 14-09-23-23W4, which tied the line (Figure 2), was obtained, and used to provide the required low-frequency impedance estimate.

To illustrate the need to compensate for missing low frequency, a recursive inversion was computed using the 10 Hz trace at the well location (Figure 1a). This was done using the following equation:

$$I_{j+1} = I_j \frac{\left(1 + r_j\right)}{\left(1 - r_j\right)} \tag{8}$$

The resulting recursive impedance is compared to the well log impedance (Figure 3). Note how, for low frequency (0 - 10 Hz), the recursive inversion has failed to give an accurate impedance. However, for mid to high frequency (10 - 90 Hz), the log and the inversion estimate have much the same character. This result implies that, for mid to high frequency, seismic data may resolve impedance but requires a low frequency trend to complete the estimate.

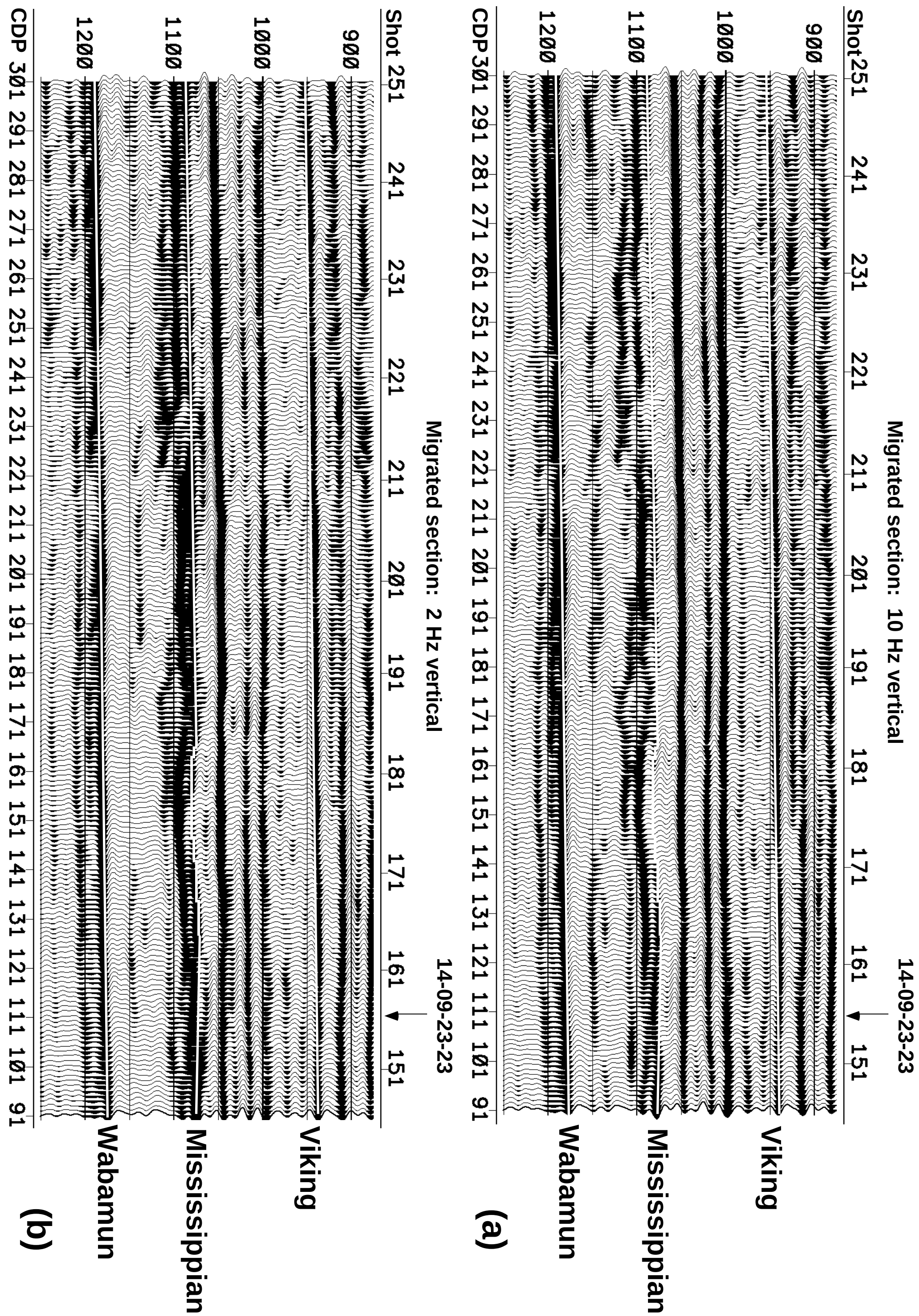

Fig 1. Migrated sections. The 10 Hz (a) and 2 Hz (b) data are shown with AGC applied. Inversions were computed without AGC. The trace used in Figures 5 and 6 was taken from the 10 Hz section at 14-09-23-23. The trace used in figure 7 came from 14-09-23-23 on the 2 Hz section. A possible sand-channel is interpreted at the Mississippian level, between shotpoints 171 and 181.

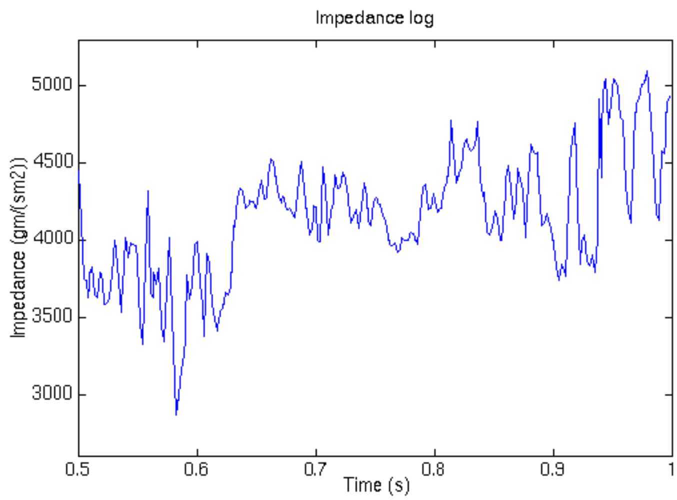

Fig 2. Impedance log from well 14-09-23-23W4

The 10 Hz trace was then inverted using BLIMP. Figures 4 and 5 are step-by-step illustrations of the inversion process. Figures 4 (a), (b) and (c) show how the log-impedance is conditioned by removing a linear impedance trend. Figures 4 (d), (e) and (f) show frequency spectra of the conditioned log, a 10 HZ high-cut filter, and the application of the filter. Figures 5 a, b and c show the spectra of the filtered log combined with the scaled spectrum of the integrated rc series. Figures 5 (d), (e) and (f) show the inverse Fourier transform of the result and the restoration of the linear trend to form the final impedance (Figure 5 (f)). Figure 6 shows a comparison of the log, recursive and band-limited inversion results.

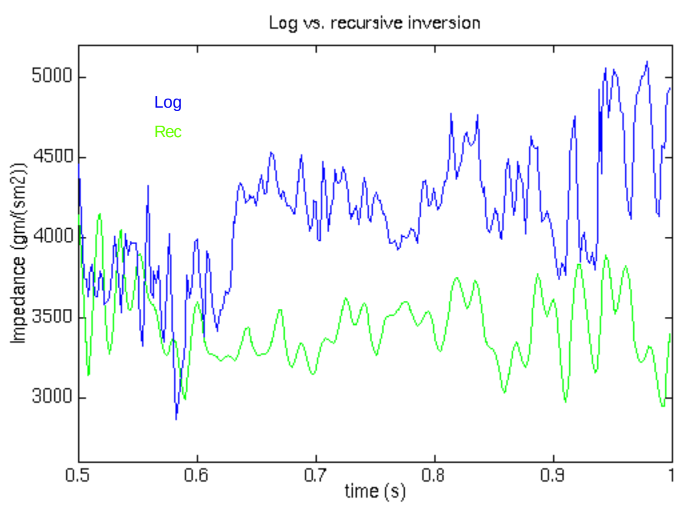

Fig 3. Comparison between impedance derived from the seismic trace by recursive inversion (equation 8) and the true log impedance. Note good correlation of values in a mid- to high-frequency sense (10 - 90 Hz), and poor low-frequency (0 - 10 Hz) correlation due to seismic band-limiting.

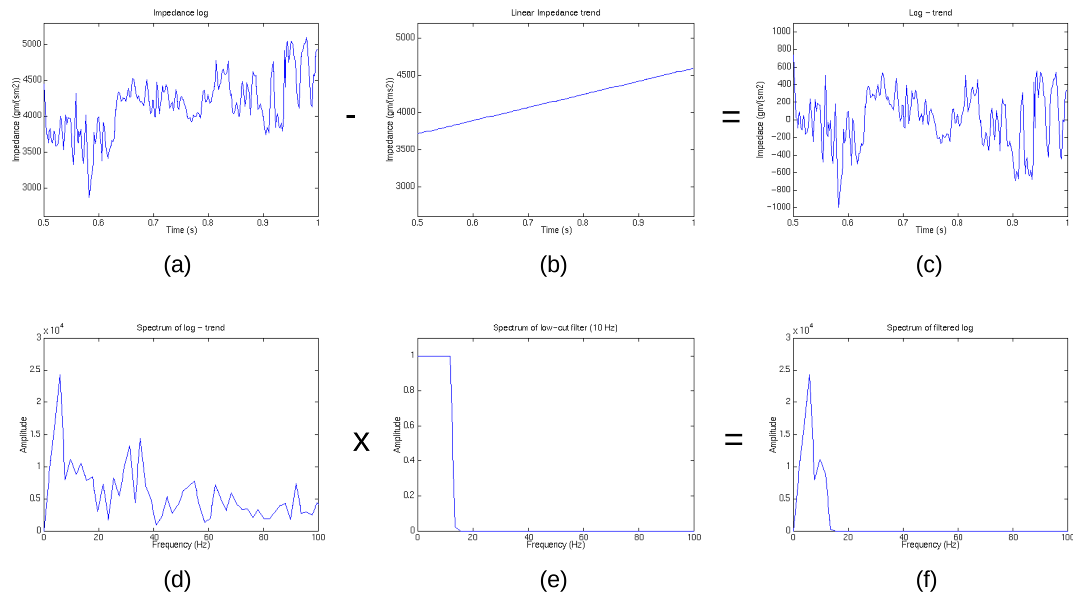

Fig 4. Conditioning of the impedance log prior to inversion. The Impedance log (a) has a linear trend (b) removed to leave a mid- to high-frequency log (c). The amplitude spectrum (d) of the log is high-cut filtered (e), to provide low-frequency (f) (0 - 10 Hz in this example).

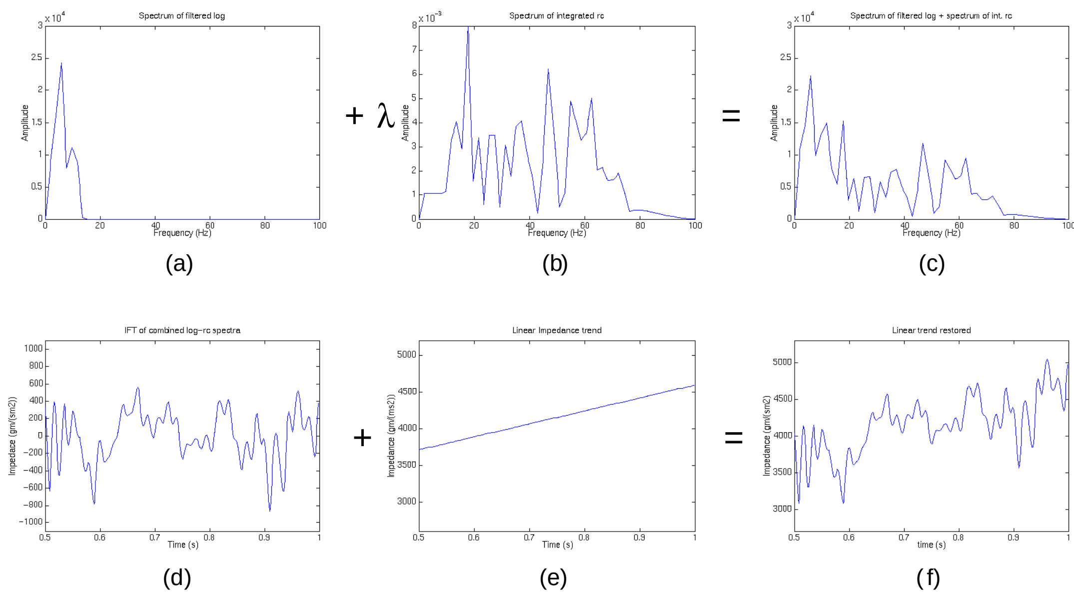

Fig 5. Computation of impedance. The spectrum of the low-frequency trend from the log (a) is added to the scaled spectrum of the integrated rc (b). λ is a numerically estimated scale factor. The combined spectra (c) is inverse Fourier transformed (d) and the linear trend (e) is restored. Figure (f) is the resulting impedance.

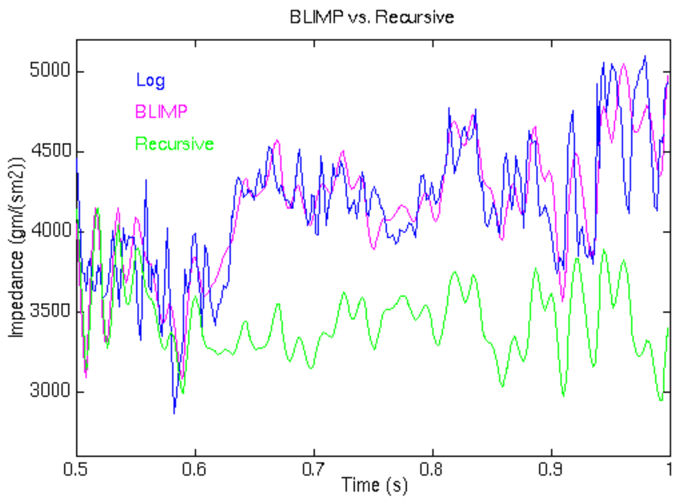

Fig 6. Comparison of the log-impedance and inversion impedances for the 10 Hz trace. Note the very good correlation of the BLIMP estimate (the next-to-lightest curve), with the true impedance (darkest curve) compared to the recursive impedance (lightest curve).

Examination of Figure 6 shows how BLIMP has improved the impedance estimate of the 10 Hz data over the recursive inversion. The mid to high frequencies of the seismically derived impedance (next to lightest curve) are still present, and direct comparisons to the log-impedance (darkest curve) are now possible (the curves overlay each other). Additionally, the BLIMP procedure is completely insensitive to the overall scale of the seismic data while recursion is not.

Figure 7 shows the impedance estimates from the equivalent 2 Hz trace. Note the much improved correlation of the recursive inversion to the log-impedance, especially the impedance jump at 0.82 seconds. This high quality of correlation is due to the reliability of the recorded low-frequency, in other words, usable data has been recorded down to 2 Hz. The band-limited inversion therefore, adds very little log-impedance information (0 - 2 Hz) to the inversion result. The inversion result thus has very little bias, compared to the 10 Hz trace, to the log-impedance.

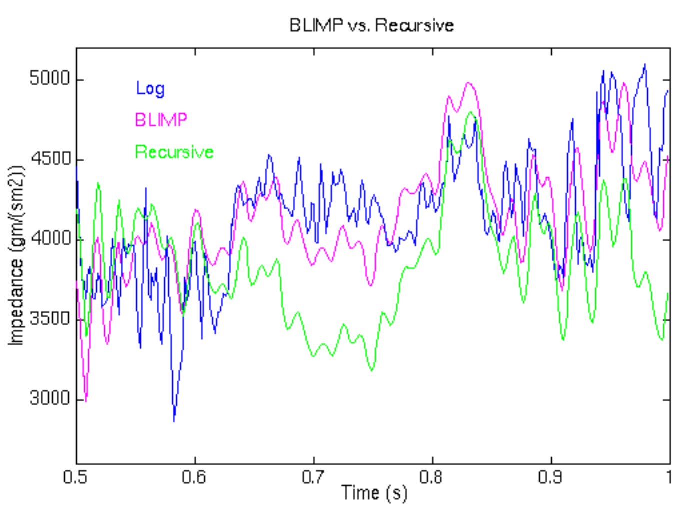

Fig 7. Comparison of the log-impedance and inversion impedances for the 2 Hz data. Note the very good correlation of both the BLIMP impedance (the next-to-lightest curve) and the

recursive impedance (lightest curve) to the true impedance (darkest curve). The high quality of the recursive estimate is due to the fact that reliable low-frequency was recorded during acquisition.

INVERSION OF 10 HZ AND 2 HZ VERTICAL COMPONENTS

The target lithology in the Blackfoot prospect are oil-producing channel-sands, located just above the Mississippian marker. A possible channel, between shotpoints 171 and 181, may be seen on Figures 1 (a) and (b) (Miller et al., 1995). The effect that these sands have on seismic propagation is to increase the S-wave velocity to about 2400 m/s from 2200 m/s, and decrease the P-wave velocity to 4000 m/s from 4050 m/s. Vp/Vs, therefore, decreases to 1.67 from 1.89 when the oil-producing sands are encountered. To identify zones of P-wave velocity decrease, the 10 Hz and 2 Hz data (Figure 1) were inverted using BLIMP. Figures 8 and 9 show the resulting impedances converted to P-wave velocity. Conversion to velocity was done using a best-fit Gardner approximation (Gardner et al., 1974).

Large differences between the 10 Hz and the 2 Hz data sets are not apparent. However, the 10 Hz velocities show more ambiguity at the Mississippian marker. This difference is due to having more of the log-impedance (0 - 10 Hz) included in the 10 Hz inversion, than for the 2 Hz. The result, due to a regional dip (down to the left), is that 0 - 10 Hz from the well log is ‘smeared’ across the section

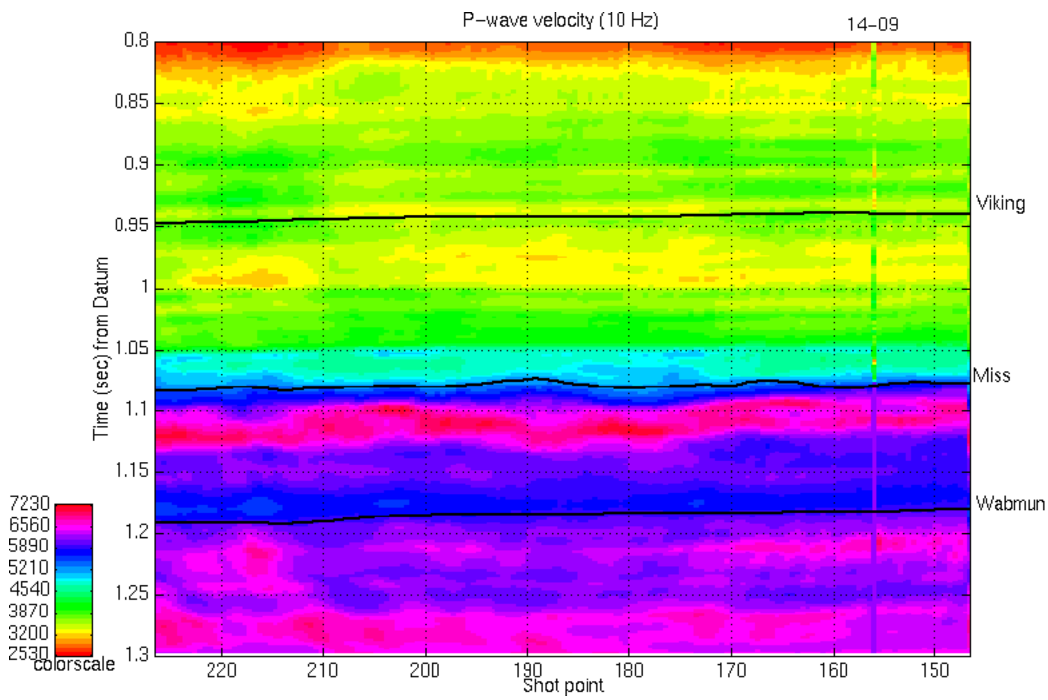

Fig 8. Band-limited inversion of 10 Hz data. Conversion of impedance to velocity was done using a Gardner approximation (Gardner et al., 1974). Note ambiguity of the Mississippian (Miss) level. This is due to ‘smearing’ of 0 - 10 Hz of the log-impedance.

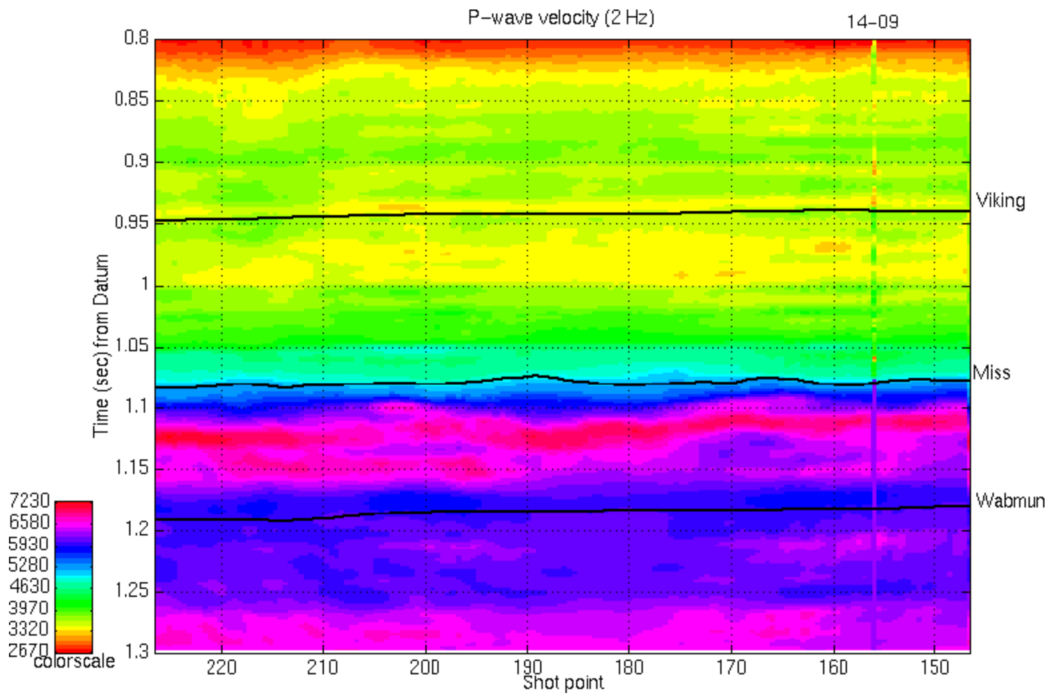

Fig 9. Band-limited inversion of 2 Hz data. Conversion of impedance to velocity was done using a Gardner approximation (Gardner et al., 1974). Note the sharpness, compared to Figure 8, of the Mississippian (Miss) level.

The velocity decrease anticipated, when encountering channel-sands, is not apparent on either the 2 Hz or 10 Hz velocity sections. It must be questioned, however, whether it is reasonable to expect a 50 m/s velocity change to be resolvable. In the Blackfoot prospect, one must look to the inversion of the PS-wave data (Ferguson and Stewart, 1996) where the expected S-wave velocity changes are much larger (~ 200 m/s).

CONCLUSIONS

The impedance inversion described by Waters (1984) has been cast as a MATLAB algorithm called BLIMP. This algorithm includes some modifications to the Waters approach: (1) The Fourier transform of the impedance-log is stabilized by removing a linear-impedance trend. (2) The spectra of the integrated seismic traces are scaled, prior to combining with the low-frequency impedance trend, using a least-squares approach.

BLIMP was used to estimate impedances for the 10 Hz and 2 Hz vertical-component data from the Blackfoot broad-band 2D 3C seismic survey. A single 10 Hz trace was inverted and compared to a log-impedance which tied the line. This result was compared to the inversion of a single 2 Hz trace. BLIMP was found to provide reliable impedance estimates for both the 10 Hz and 2 Hz traces. The 2 Hz traces were found to contain reliable low-frequency information.

All of the traces, for the 10 Hz and the 2 Hz data, were then inverted using BLIMP. It was found that the anticipated velocity decrease, when encountering oil-productive channel-sands, was not apparent on either velocity section. The question was raised whether it is reasonable to expect a 50 m/s velocity change to be resolvable using seismic inversion. It was suggested that, for the Blackfoot prospect, one look to the inversion of the PS-wave data (Ferguson and Stewart, 1996) to find more robust velocity changes indicative of the target lithology.

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