An approximate relationship between R^{ps} and R^{ss}

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ABSTRACT

The approximate equations for converted-wave reflectivity R^{ps} and pure-shear reflectivity R^{ss} can be combined to give a simple relationship between the two: $R^{ps}(\theta) \approx 4\frac{\beta}{\alpha}\sin(\theta)R^{ss}(0)$, where θ is the incidence angle of the shear wave. Simple interface examples indicate that the relationship is reasonable up to about a 15° angle of incidence. The equation can also be used with P-S data to infer the zero-offset S-wave reflectivity.

INTRODUCTION

We may have a converted-wave reflectivity section or a pure shear section, depending on what type of survey was conducted. Because both surveys relate to shear-wave properties it is natural to ask several questions: Which is better? Cheaper? How do they relate to each other? The first question of relative quality of S-wave sections is the subject of current interest and study. Simin (1997) finds that the P-S section has a broader band than the S-S section in a nine-component study conducted near Olds, Alberta. This is likely due to the two-way S-wave travel path through near surface. The P-S survey is typically cheaper than the S-S survey for several reasons, including: a conventional source can be used in a P-S survey as opposed to an S-wave vibrator (or Marthor, Omnipulse, etc.) and there is a shorter recording time for P-S versus S-S. What the approximate relationship is between R^{ps} and R^{ss} can be shown as below.

The equations from Aki and Richards (1980) that approximate the converted-wave reflectivity R^{ps} and pure-S reflectivity R^{ss} are given as:

$$R^{ps}(\theta) = \frac{-p\alpha}{2\cos\theta}[\left(1 - 2\beta^2p^2 + 2\beta^2\frac{\cos\psi}{\alpha}\frac{\cos\theta}{\beta}\right)\frac{\Delta\rho}{\rho} - \left(4\beta^2p^2 - 4\beta^2\frac{\cos\psi}{\alpha}\frac{\cos\theta}{\beta}\right)\frac{\Delta\beta}{\beta}], \tag{1}$$

and

$$R^{ss}(\theta) = -\frac{1}{2}\left(1 - 4\beta^2p^2\right)\frac{\Delta\rho}{\rho} - \left(\frac{1}{2\cos^2\theta} - 4\beta^2p^2\right)\frac{\Delta\beta}{\beta}, \tag{2}$$

where α, β, ρ are the average P-velocity, S-velocity, and density across the interface; p is the ray parameter; θ and ψ are the angles of incidence and reflection.

Suppose that θ and thus p are small and that $\frac{\beta}{\alpha} \approx \frac{1}{2}$, then as shown by Frasier (1995) and Stewart et al. (1995):

$$R^{ps}(\theta) \approx 4\frac{\beta}{\alpha}\sin\theta\, R^{ss}(0) \qquad (3)$$

Furthermore, we note that eqn. (3) can be inverted to give an estimate of the zero-offset S- wave reflection by

$$R^{ss}(0) \approx \frac{\alpha}{4\beta}\csc\theta\, R^{ps}(\theta). \qquad (4)$$

EXAMPLES

Let’s take several cases to examine the validity of eqn. (3). In the first, we use properties relevant to a clastic section as outlined in Table 1. We calculate true amplitude-versus-angle reflectivities exactly from the Zoeppritz equations. The converted-wave reflectivity is computed using eqn. (3).

We note that the approximation in eqn. (3) seems to predict the true reflectivity reasonably well up to about 15°.

In addition, we use the true P-S reflectivity with eqn. (4) to estimate $R^{ss}(0)$. Again, for angles less than about 15°, this provides a reasonable estimate. Thus, eqn. (4) shows promise as a basis to develop a weighting factor for actual $R^{ps}(\theta)$ data conversion to $R^{ss}(0)$.

In the second case, as given in Table 2 and Figure 2, we use a high-to-lower velocity case. Once again the approximations are reasonable to about 15°.

Table 1. Material properties at an interface typical to south-central Alberta.

Incidence Material	
Density	2512
P- velocity	3562
S- velocity	1837

Transmission Material	
Density	2528
P- velocity	3862
S- velocity	2011

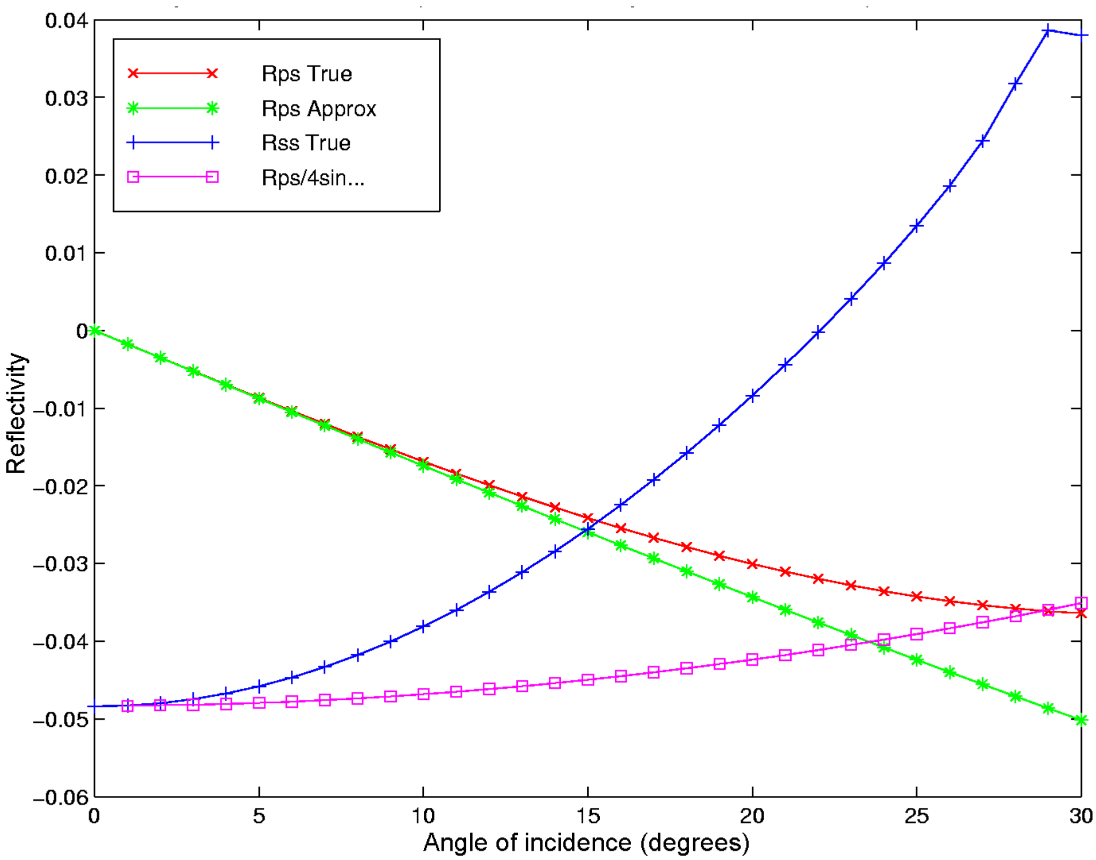

Figure 1. P-S and S-S reflectivities for a clastic interface using the properties in table 1.

Table 2. Material properties used in case two. These materials form an interface typical to south-central Alberta.

Incidence medium	
density	2528
P- velocity	3862
S- velocity	2011

Transmission medium	
density	2305
P- velocity	3416
S- velocity	1659

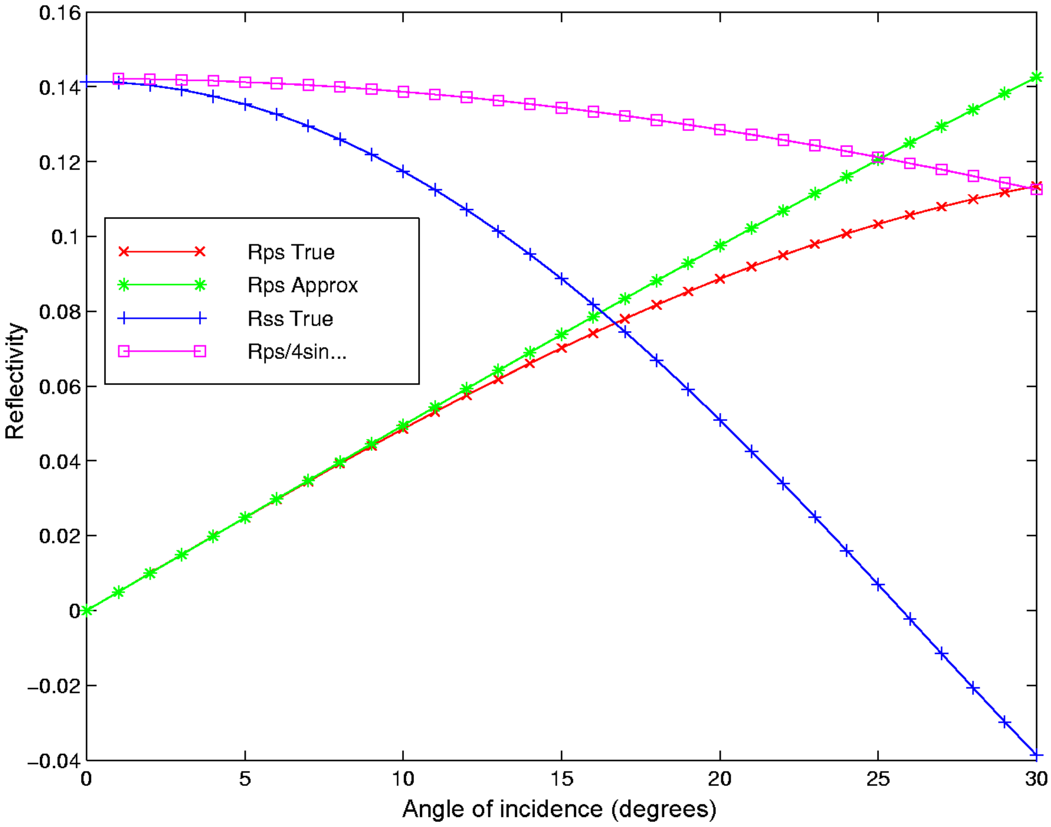

Figure 2. P-S and S-S reflectivities for a clastic interface using the properties in table 2.

CONCLUSIONS

The shear reflectivities (R^{ps} and R^{ss}) are approximately related by a simple sine function. Thus, we can use smaller offset converted-wave reflectivity to predict pure-shear reflectivity and vice versa. This promises to provide a technique for converting between reflectivity types.

ACKNOWLEDGEMENT

Dr. Clint W. Frasier of Chevron Petroleum Technology Corporation derived the result to equation (3) independently in an internal Chevron memo dated May 10, 1995 and also suggested its use as a method to extract $R^{ss}(0)$.

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