Seismic source wavelet estimation and the random reflectivity assumption

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ABSTRACT

In the February 1991 issue of *GEOPHYSICS*, Anton Ziolkowski gives a scathing criticism of statistical wavelet estimation methods. Among other points, Ziolkowski questions the validity of the randomness assumption. This assumption allows statistical methods to estimate the seismic source wavelet autocorrelation from the seismic trace autocorrelation. In this study, we examine this traditional assumption of a random reflectivity sequence. The validity of this assumption is examined by using well-log synthetic seismograms and by using a procedure for evaluating the resulting deconvolutions. With real data, we compare our wavelet estimations with the in-situ recording of the wavelet from a vertical seismic profile (VSP). As a result of our examination of the randomness assumption, we present a fairly simple test that can be used to evaluate the validity of a randomness assumption.

INTRODUCTION

Ziolkowski's work criticizes the conventional assumptions of wavelet deconvolution as being generally invalid. These assumptions included wavelet stationarity, minimum phase and zero phase, and the assumption of random reflectivity. Ziolkowski advocates that seismic source signatures be measured rather than estimated when performing deconvolution. Although it is difficult to ever argue against using physical measurements, the expense and difficulty of the measurements may lead one to use statistical wavelet estimation. It is then worthwhile to consider how accurate these statistical measurements might be. In particular, we seek to evaluate the common assumption of reflectivity randomness by use of model synthetics and real data. Since one would expect this assumption to be dependent on geology, we have examined data from various oil fields.

Statistical wavelet estimation methods usually assume that the reflectivity is a random uncorrelated signal (that is, the reflectivity has an autocorrelation which approximates a delta function) and this effectively means that the seismic trace autocorrelation is approximately equal to the seismic wavelet autocorrelation, $X(z)X^*(z) \cong W(z)W^*(z)$. Consider the following figure from Russell (1994).

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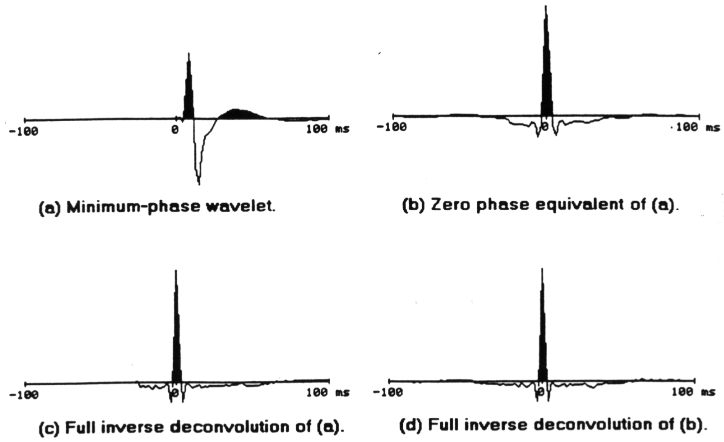

Fig. 1: The effect of deconvolution on known minimum-phase and zero-phase wavelets (from B. Russell, SEG inversion course notes).

Figure 1 shows that if the wavelet is known, then deconvolution will give consistently good results since the application of a Wiener deconvolution filter produces a good approximation to a bandpassed delta function. The deconvolution of data is only effective if the input wavelet is a reasonable approximation to the true wavelet. To establish the best, and most effective, means of wavelet extraction is the key to creating a more interpretable, high resolution, seismic section. This is precisely why we must investigate the assumptions upon which our commonly used statistical methods are based. The random reflectivity assumption allows one to estimate the wavelet's autocorrelation, and consequently the amplitude spectrum, from the trace autocorrelation. Hence, given an input seismic trace, it is possible to estimate the wavelet needed for deconvolution.

METHODOLOGY

Four synthetic seismic sections are generated for use in this investigation. Two are created with a minimum phase wavelet and the remaining two are created with a zero phase wavelet. We first look at the validity of the randomness assumption with a primaries only section derived from a well in central Alberta and then with a section that also has multiples, also derived from the same well. The use of synthetic data is done because we know what the wavelet used to generate the section is and, therefore, can make meaningful comparisons with the statistical estimates.

The estimation of minimum phase wavelets from dynamite data and the estimation of zero phase wavelets from vibroseis generally use the assumption of a random reflectivity sequence. This assumption is used so that the wavelet's autocorrelation can be derived from the seismic trace's autocorrelation. Given a wavelet's autocorrelation or its corresponding amplitude spectrum, we are then left with the problem of deriving its phase spectrum. For a zero phase spectrum, there is no problem. For the case of a minimum phase wavelet, its phase spectrum is uniquely defined by the amplitude spectrum and can be found using either the Wiener-

Levinson double inverse method or the Hilbert transform method. Both of these methods use the assumption of reflectivity randomness to estimate minimum phase wavelets. White and O'Brien (1974), Claerbout (1976), and Lines and Ulrych (1977) all give detailed descriptions of these methods.

The Wiener-Levinson double inverse method can use the wavelet autocorrelation rather than the wavelet itself. If the desired output is set to a spike at zero delay (which will be the optimum only if the wavelet is minimum phase), then the inverse filter for a minimum phase wavelet is obtained. To obtain the wavelet estimate, another Wiener filter is applied to invert the inverse filter and thereby estimating the minimum phase wavelet from its autocorrelation.

The Hilbert transform method (sometimes called the Kolmogorov method) also uses the autocorrelation and the amplitude spectrum which can be derived from this autocorrelation. If a wavelet is minimum phase, then its phase spectrum can be uniquely derived from its amplitude spectrum by taking the Hilbert transform of the log amplitude spectrum (Robinson, 1967).

In judging the accuracy of the randomness assumption, on model synthetic data, we develop a systematic approach. We can compare the trace autocorrelations with the wavelet autocorrelations and examine the degree of similarity. For a purely random reflectivity, these should be identical to within a scale factor. Subsequently, we can compare the estimated wavelet with the actual wavelet. However, it is really the deconvolution based on the wavelet estimate that we wish to evaluate. One way to do this is to compute the convolution of the actual wavelet with the deconvolution filter for the estimated wavelet. Hopefully, this will produce as close a representation of a spike as for the filter computed from the actual wavelet. We also convolve the estimated wavelet deconvolution filter with the trace and compare the result to the reflectivity response from the well log.

For the case of real data, our evaluation is somewhat more difficult since we do not know the wavelet or the reflectivity series. However, we do have access to in-situ measurements of the seismic wavelet if we have VSP data, and in some cases we have sonic well data to evaluate our deconvolutions. In our particular case, we have vibroseis surface recordings and VSPs recorded from a vibrator source.

RESULTS AND DISCUSSIONS

We now present a comprehensive set of results regarding our studies on the randomness assumption. Figures 2 through 19 illustrate the analysis on minimum phase synthetic sections. Following these are the analyses for the zero phase sections and then the work on real data is displayed. Shown in Figure 2 is a synthetic seismogram that contains only the primary reflections. That is to say, no multiples have been computed. Along side this seismic section is the minimum phase model wavelet used to create it. It is this wavelet that is convolved with a well log derived reflectivity sequence to generate the synthetic to be analyzed. Note several important reflections between 50 and 100, 100 and 150, and again between 150 and 200.

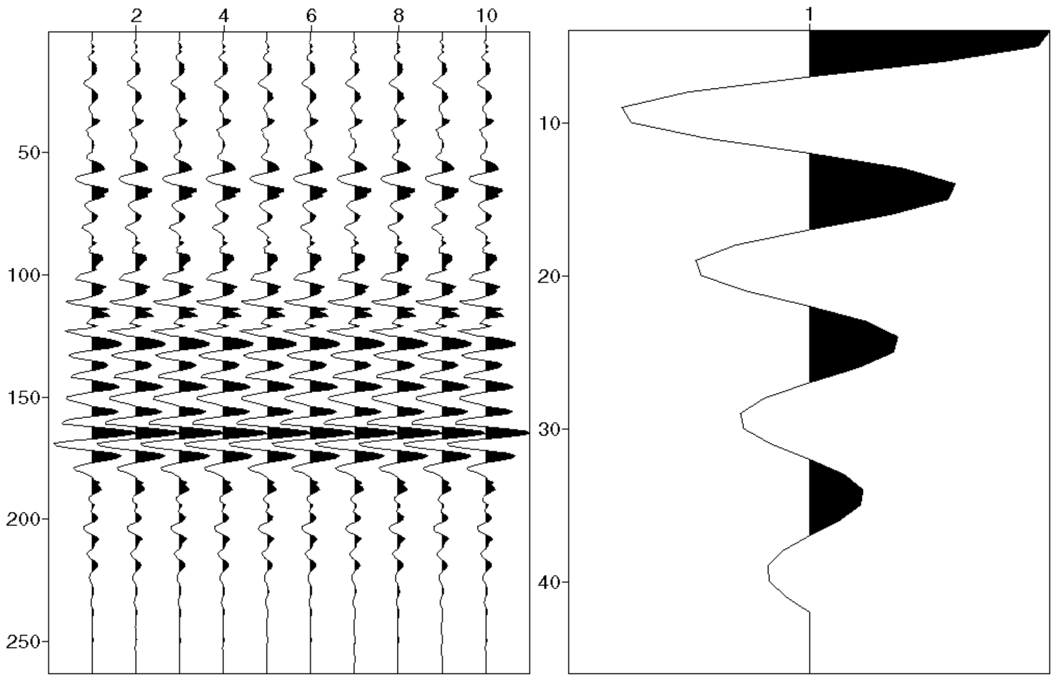

Fig. 2: A primaries only minimum-phase synthetic trace and the wavelet used to create it.

The Wiener-Levinson and Hilbert methods base their minimum phase statistical wavelet estimates on an input seismic section. These statistical estimation methods use the trace autocorrelation to reproduce the wavelet autocorrelation and assume that the trace autocorrelation is approximately equal to the wavelet autocorrelation. This requires that the reflectivity sequence be random.

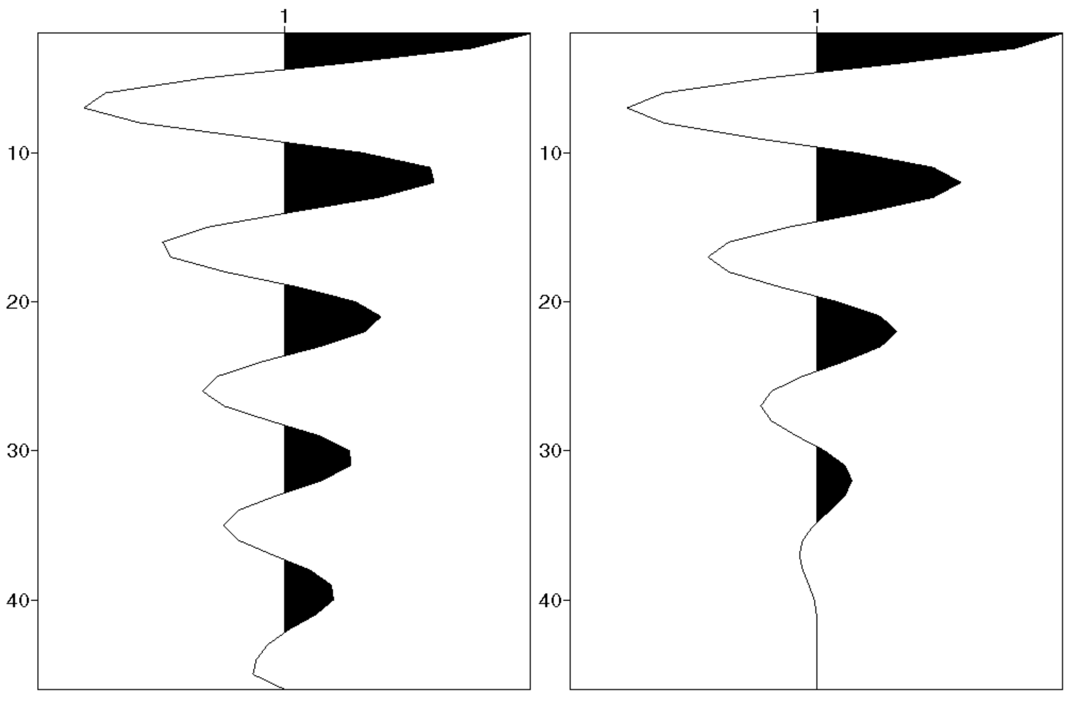

Fig. 3: The trace autocorrelation (left) and the minimum-phase wavelet autocorrelation (right).

There is significant correlation between the two plots shown in Figure 3. The close similarity lends support to the claim of randomness. Now consider the images in Figures 4 and 5. Here is a comparison of the actual model wavelet and two common minimum phase wavelets. The Hilbert estimate is created from a frequency domain method and the Wiener-Levinson estimate is generated from a time domain method.

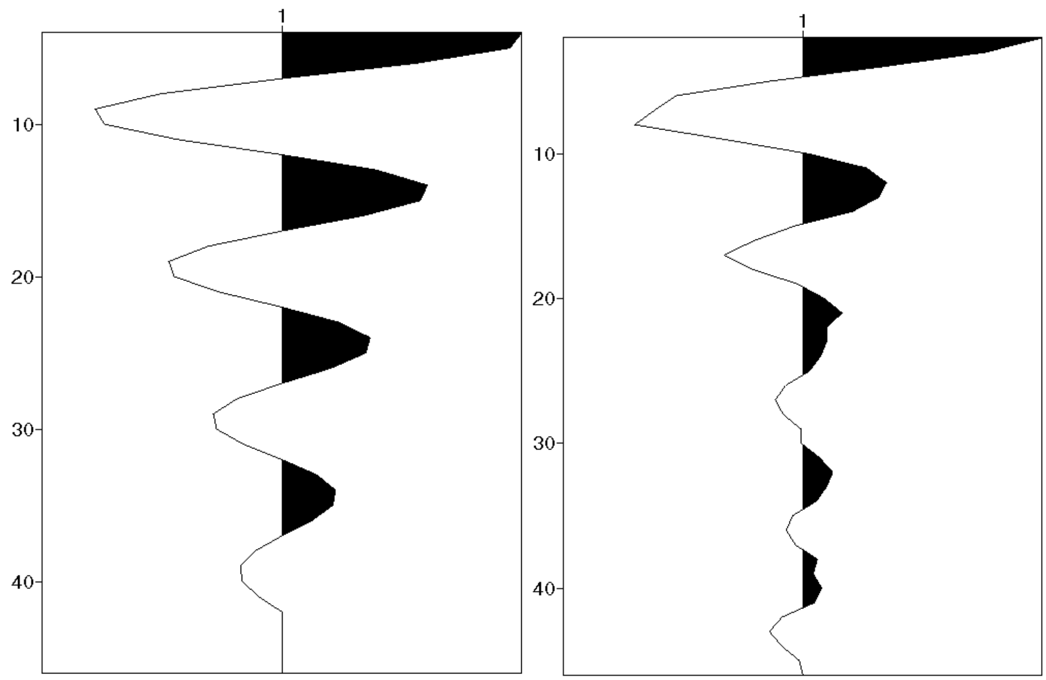

Fig. 4: The model wavelet (left) and a Wiener-Levinson wavelet estimate (right).

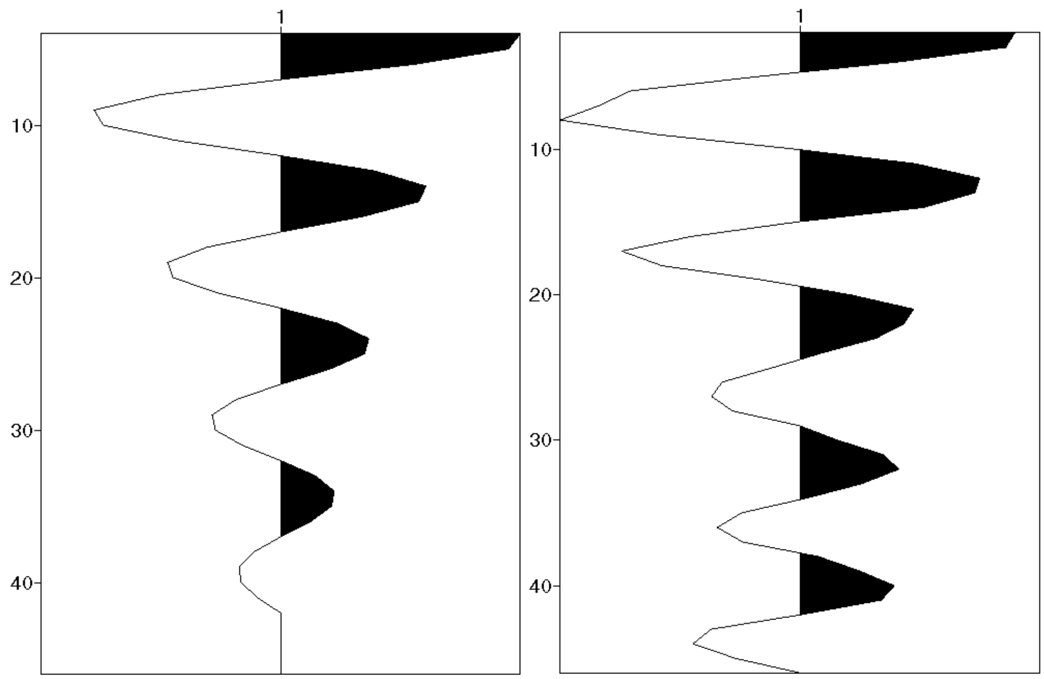

Fig. 5: The model wavelet (left) and a Hilbert transform wavelet estimate (right).

The proceeding two plots illustrate the concept of a *resolving kernel*. That is to say, a resolving kernel is the convolution of a model wavelet with a deconvolution filter to get an estimate of the ideal spike. This gives a measure of resolvability in that the convolution of a wavelet with a deconvolution filter should (in the ideal case of estimates that approximate the wavelet) give a spike as the response.

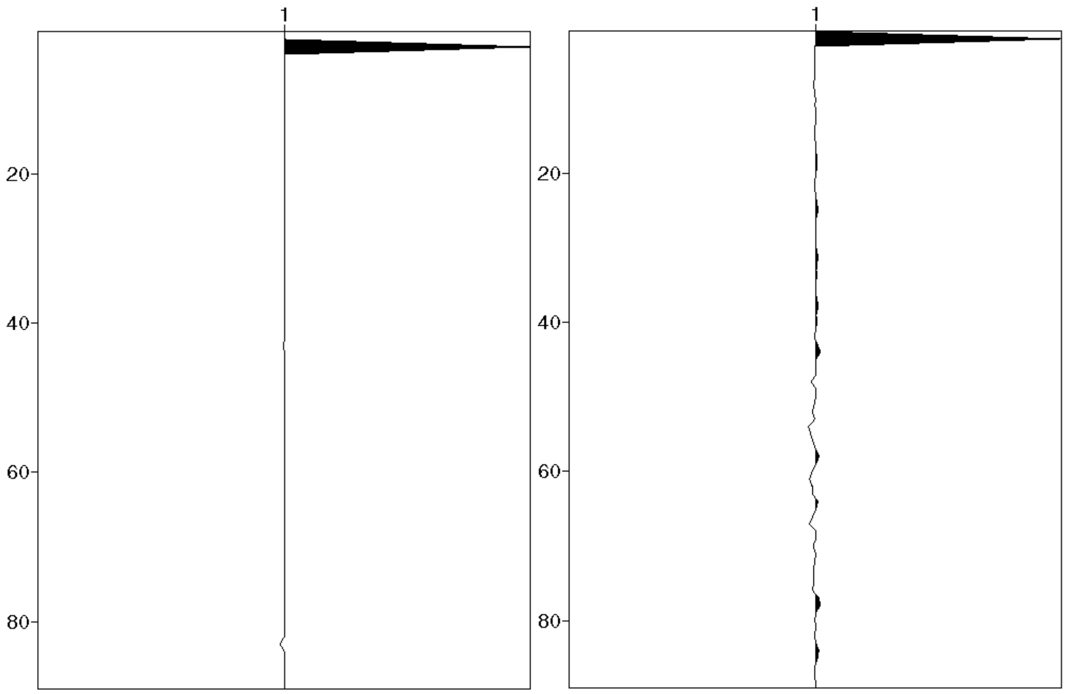

Fig. 6: Model wavelet deconvolution (left) and Wiener-Levinson wavelet deconvolution (right).

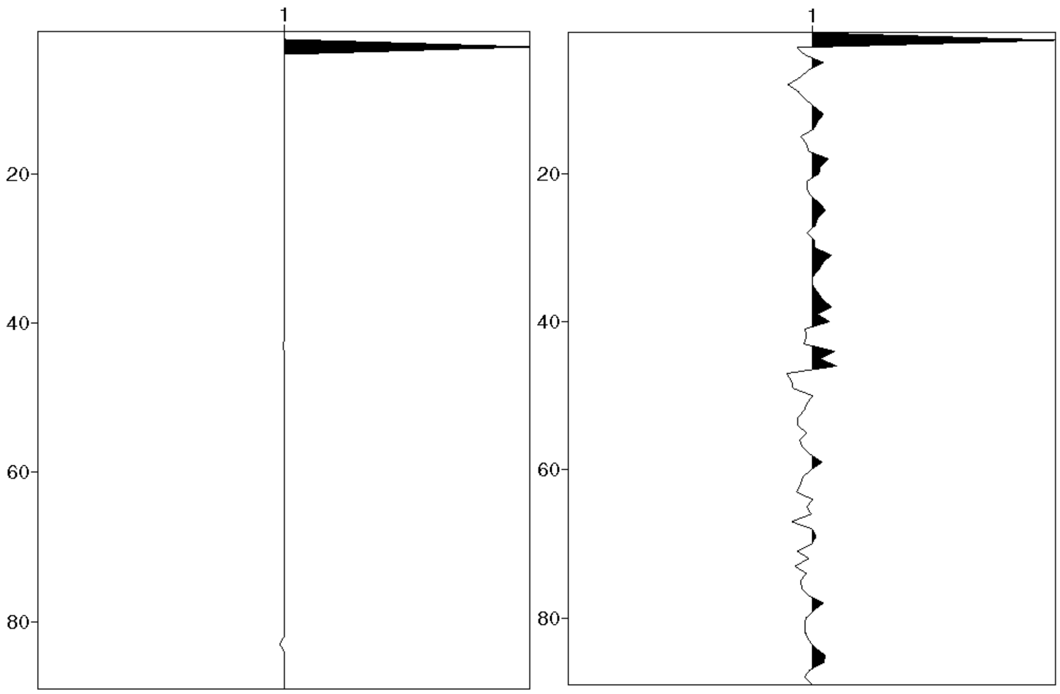

Fig. 7: Model wavelet deconvolution (left) and Hilbert wavelet deconvolution (right).

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The decon filter from the model wavelet gives a clean spike. This is to be expected. What is interesting is that the convolutions of the model wavelet with the statistically estimated filters give very good representations of a spike. The result with the Hilbert filter has a little noise in the tail and the result from the Wiener-Levinson filter has minute amounts of noise in the tail.

In Figures 8 and 9, we evaluate the ability of the statistical methods to reproduce the actual well log derived reflectivity sequence. Using the filters based on statistical estimation, we generate estimates of the reflectivity by convolving the synthetic seismic section with the various filters. This is simply the deconvolution of the input trace in that the process will remove the wavelet from the input data and the result should be the reflectivity. Notice that these deconvolutions are good reproductions of the actual sequence in question. Some things to note are that the estimated reflectivities are a bit smeared and that the estimates have a slight delay. Some significant events that correlate from the actual sequence to the estimated sequence can be noticed just before 50, between 50 and 100, and at about 150. As mentioned before, these events are a bit delayed on the estimates and appear out of phase.

After these figures is an examination of how good these reflectivity estimates are. An ideal cross-correlation of these reflectivities will give a spike as the result. Hence, this can provide a measure of goodness in that one can compare the similarity of the cross-correlations to a spike. Figure 10 shows such a result. Both of the cross-correlations show a spike that is many magnitudes greater than the minor amounts of noise that occur later in the trace. These are effectively spiking responses. Notice, again, that there appears to be a slight delay in the cross-correlation with the Hilbert estimate.

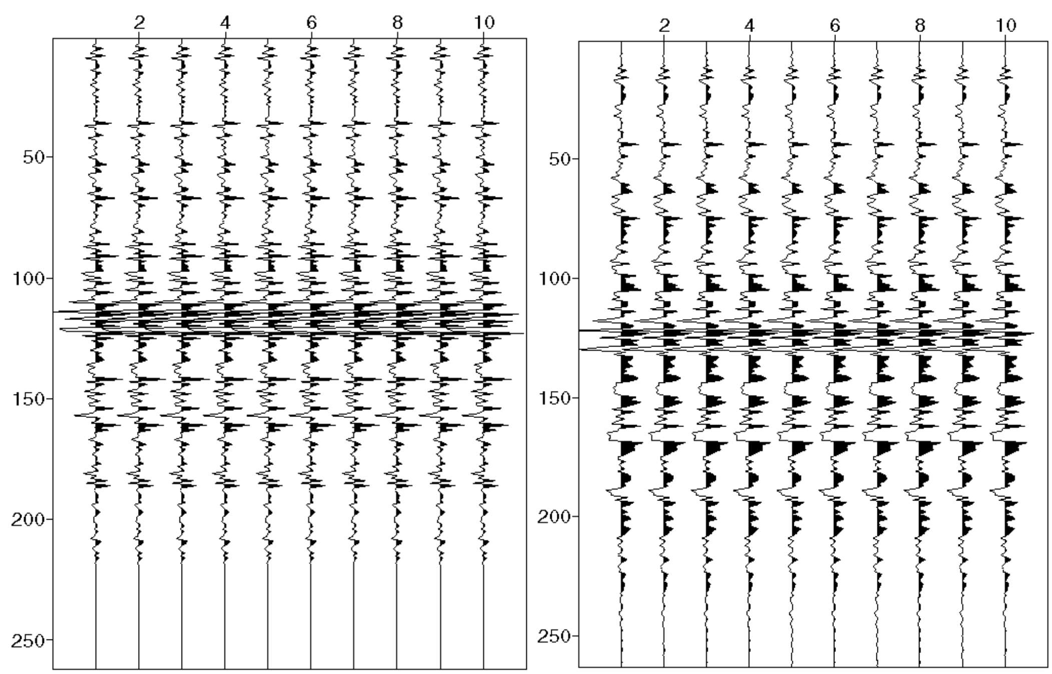

Fig. 8: The actual well-log derived reflectivity series (left) and the Wiener-Levinson reflectivity series estimate (right).

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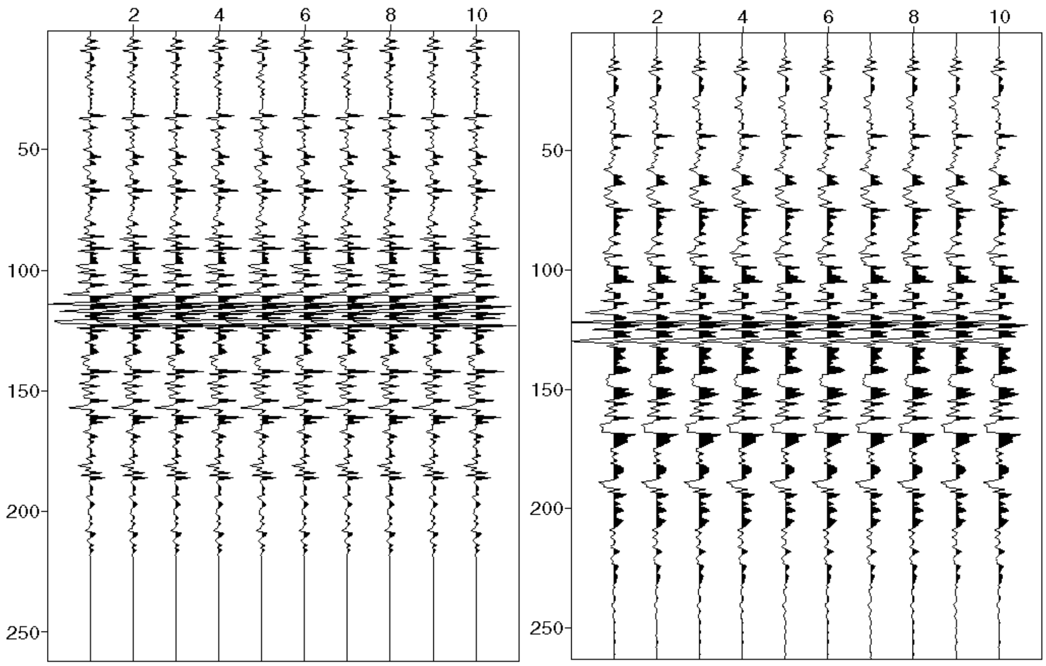

Fig. 9: The actual well-log derived reflectivity series (left) and the Hilbert transform reflectivity series estimate (right).

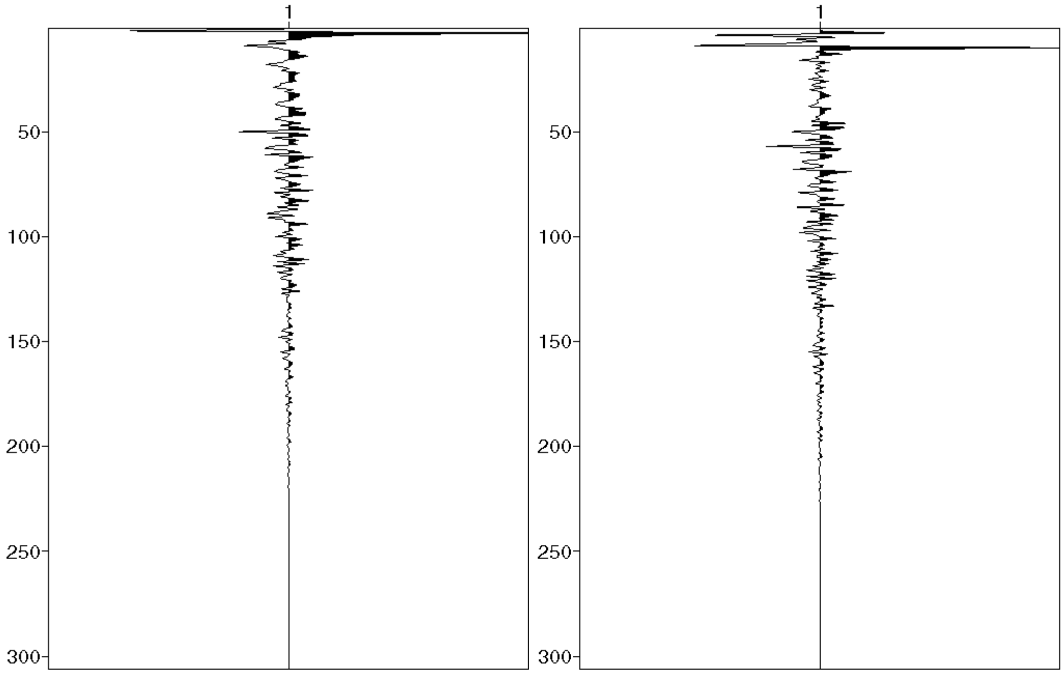

Fig. 10: Cross-correlation of the actual reflectivity series with the Wiener-Levinson reflectivity series (left) and the Hilbert transform reflectivity series (right).

With that concludes the analysis for a minimum phase primaries only section. We present the same analysis for a synthetic seismogram that contains multiples. This section is created in the same manner as above but multiples have been added by the method outlined by Robinson. The inclusion of multiples in a synthetic seismic section generally worsens the whiteness assumption. Again, as an initial test, the trace and wavelet autocorrelations are compared (Figure 12). There is no significant difference between the two except that the trace autocorrelation seems to have the onset of a doublet near its end. The very close similarity between the trace and wavelet autocorrelations lends further support to the randomness assumption. Figure 13 shows the Wiener-Levinson estimate based on an input trace with multiples. Likewise, the Hilbert transform estimate is shown in Figure 14. Even though the whiteness assumption is worsened with multiples in the section the resolving kernels, shown in Figures 15 and 16, for the estimated wavelets are very good approximations to the expected spike response. The final 3 plots concerning this seismic section relate to the reflectivity sequence. We compare the actual reflectivity to the estimated reflectivity in Figure 17 and Figure 18. These estimated reflectivity sequences are quite good reproductions of the actual reflectivity. Once again, the estimated results are a bit smeared but there is no delay present. This causes events at 50, between 50 and 100, and at 150 on the actual sequence to correlate better to the same events on the estimated sequences. As above, the final plot (Figure 19) is a comparison of cross-correlations. Both of these are very reasonable approximations to the ideal result of a spike.

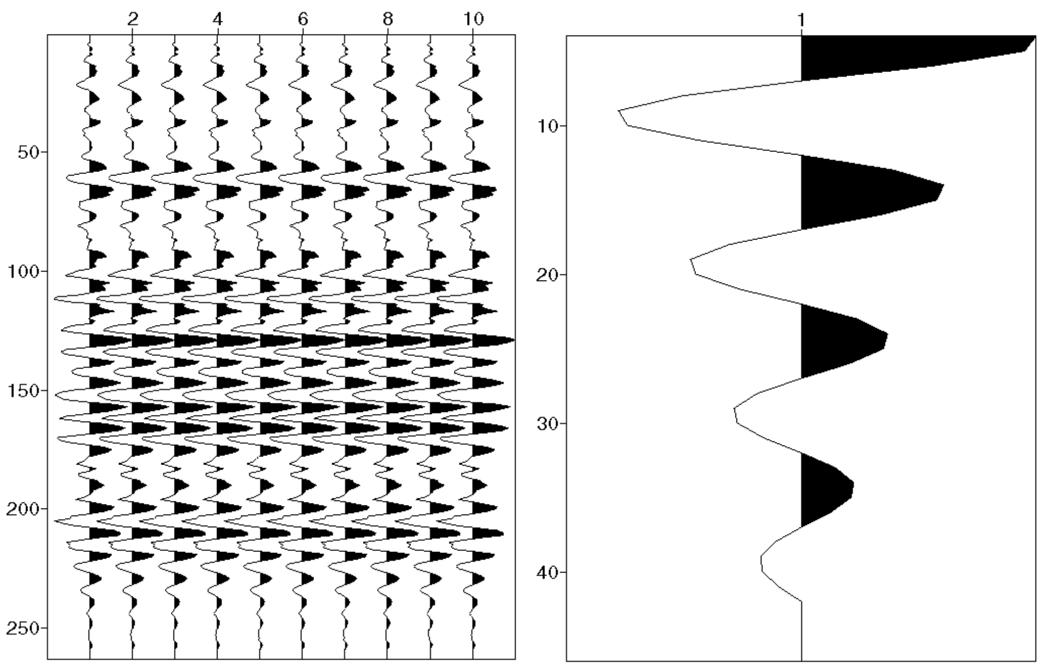

Fig. 11: A primaries with multiples minimum-phase synthetic trace and the model wavelet used to create it.

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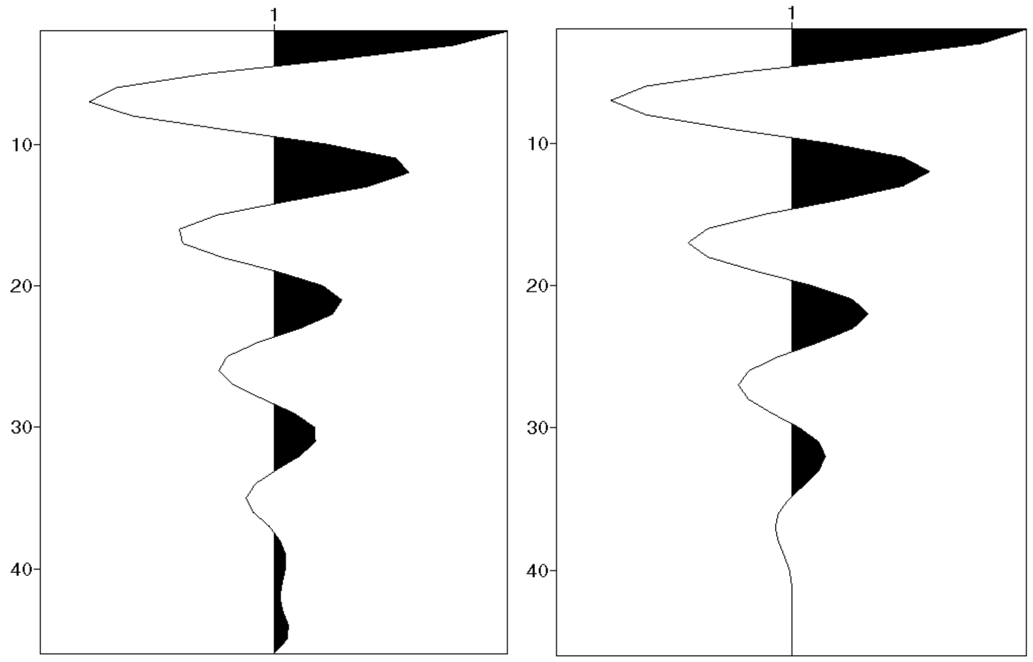

Fig. 12: The trace autocorrelation (left) and the wavelet autocorrelation (right).

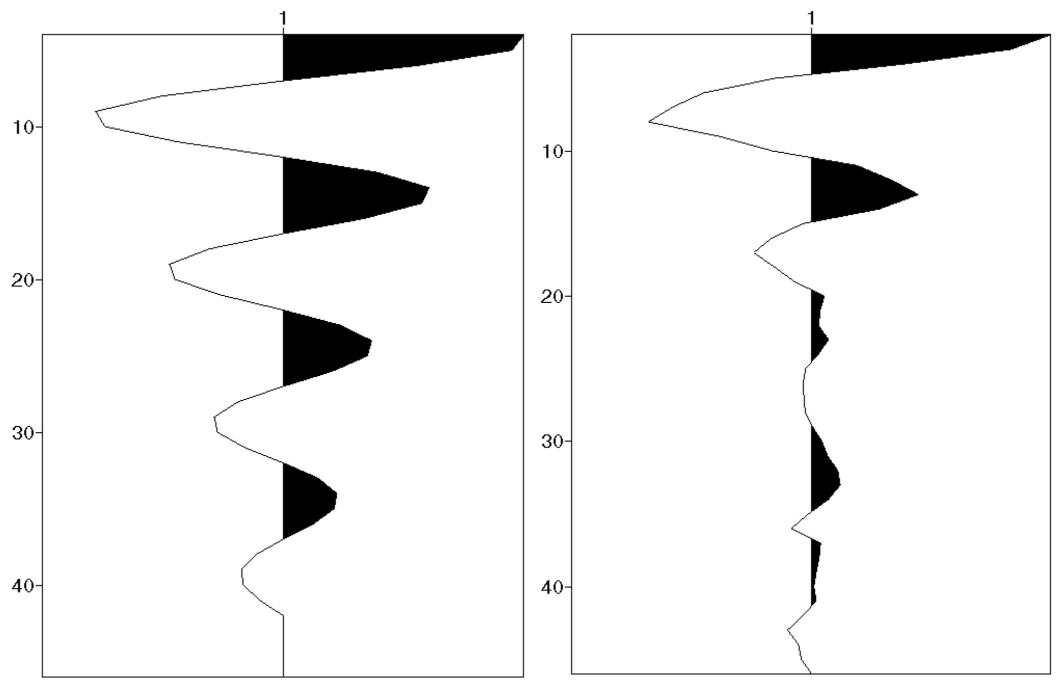

Fig. 13: The model wavelet (left) and a Wiener-Levinson wavelet estimate (right).

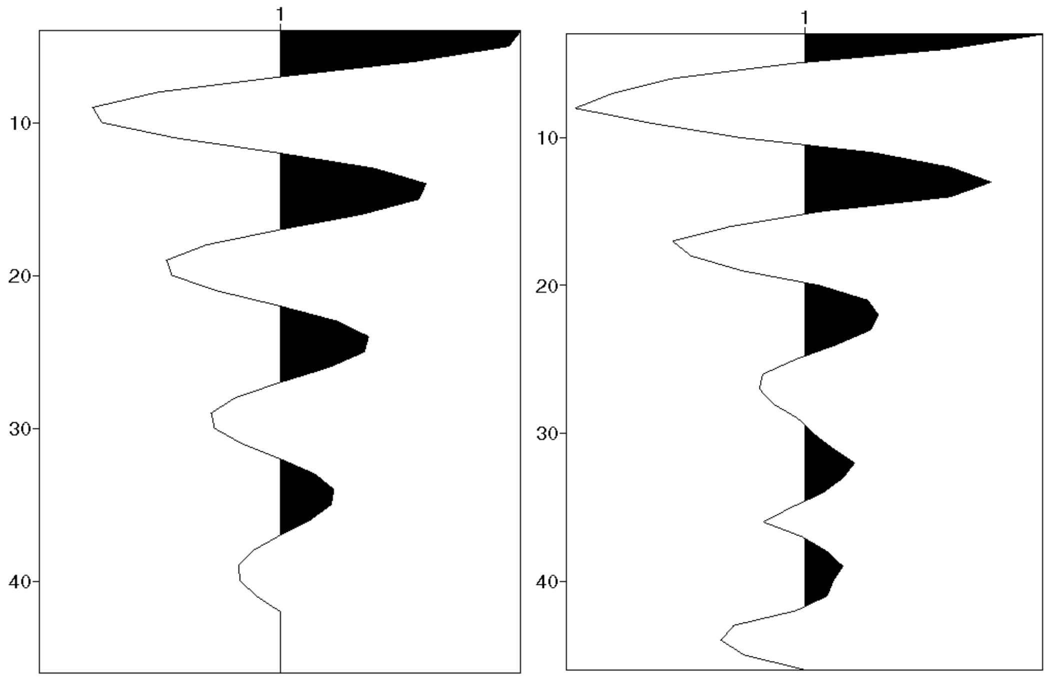

Fig. 14: The model wavelet (left) and a Hilbert transform wavelet estimate (right).

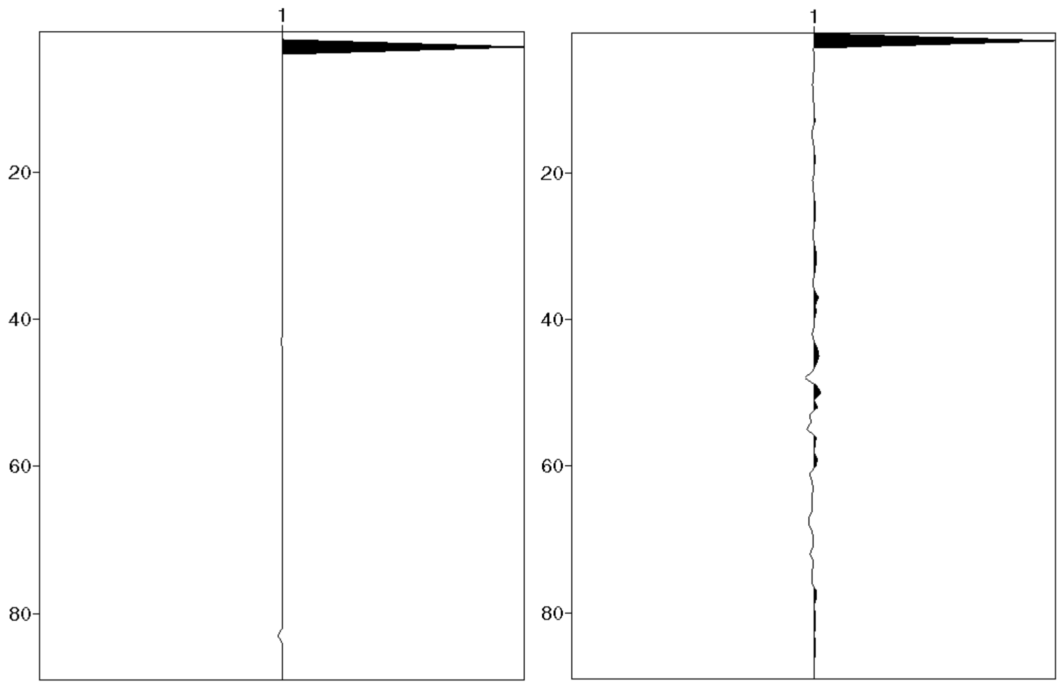

Fig. 15: Model wavelet deconvolution (left) and Wiener wavelet deconvolution (right).

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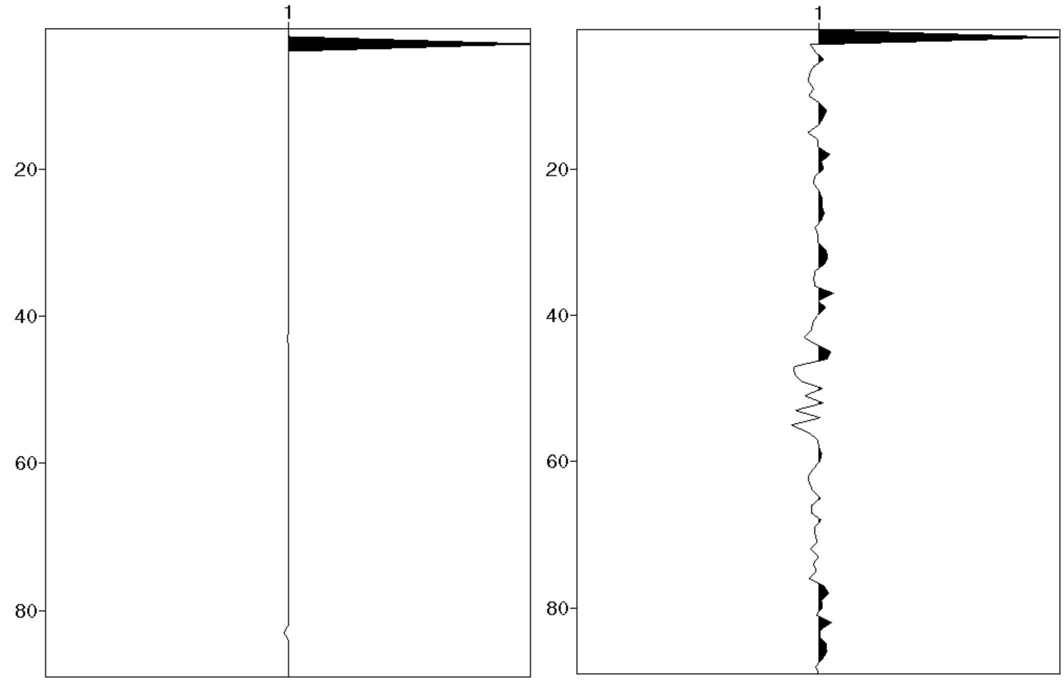

Fig. 16: Model wavelet deconvolution (left) and Hilbert wavelet deconvolution (right).

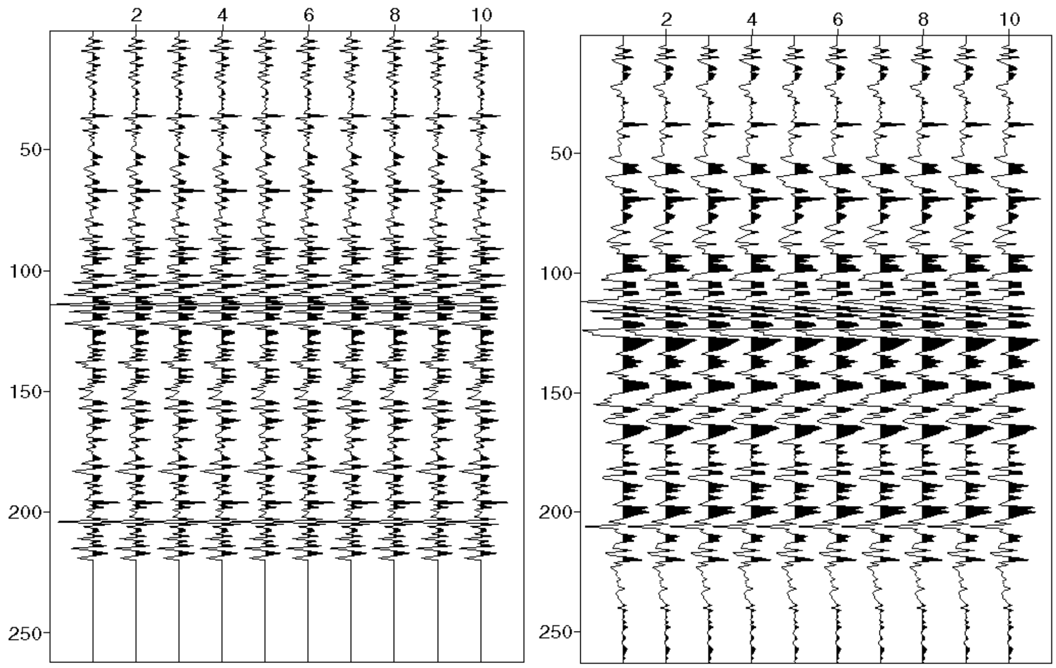

Fig. 17: The actual well-log derived reflectivity series (left) and the Wiener-Levinson reflectivity series estimate (right).

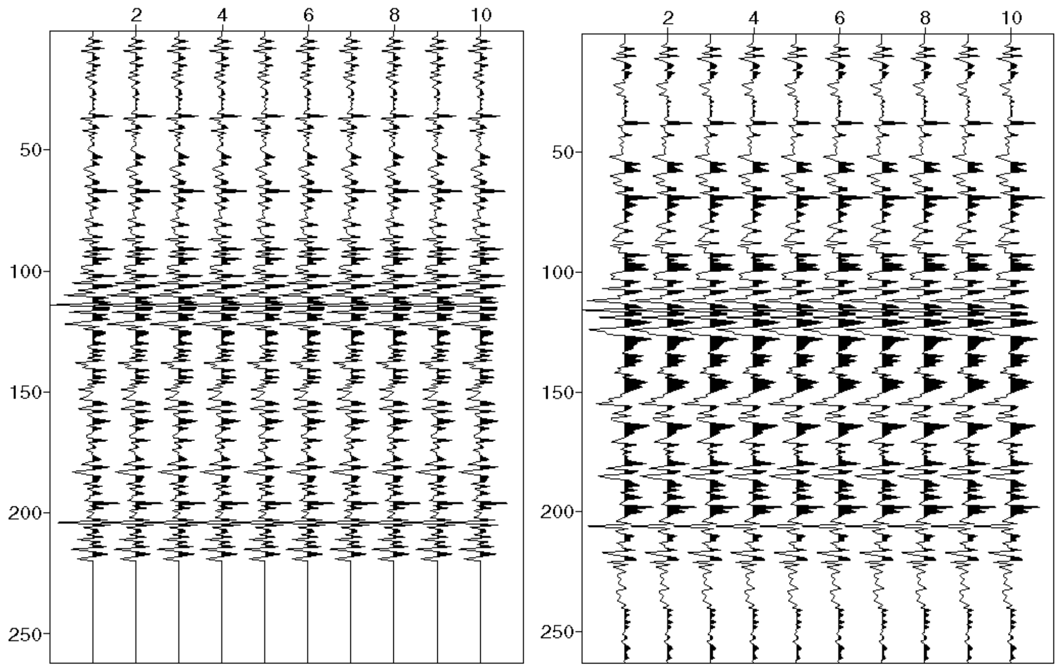

Fig. 18: The actual well-log derived reflectivity series (left) and the Hilbert transform reflectivity series estimate (right).

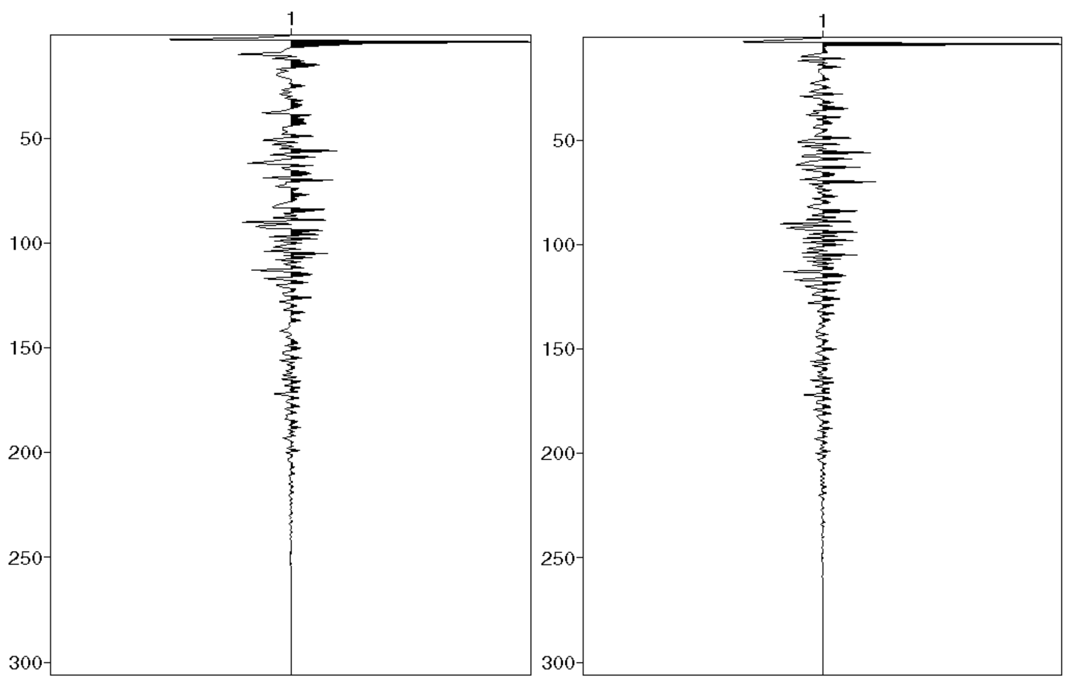

Fig. 19: Cross-correlation of the actual reflectivity series with the Wiener-Levinson reflectivity series (left) and the Hilbert transform reflectivity series (right).

The above discussion revolves around minimum-phase wavelets. Another common type of wavelet is the zero phase wavelet. When compared to minimum phase wavelets, these zero phase wavelets are symmetrical and are very broad banded in the frequency domain. That being the case, we investigate the changes that may occur when a zero phase wavelet is used in our reflectivity analysis. We create a zero phase Ricker wavelet that has a dominant frequency of 30Hz and is sampled at 2ms. Besides the fact that we are now dealing with a zero phase wavelet, nothing else has changed and the analysis proceeds as above. Figures 20 – 26 illustrate the investigations for a primaries only zero phase synthetic while figures 27 – 33 show results for a zero phase synthetic that contains primary and multiple arrivals.

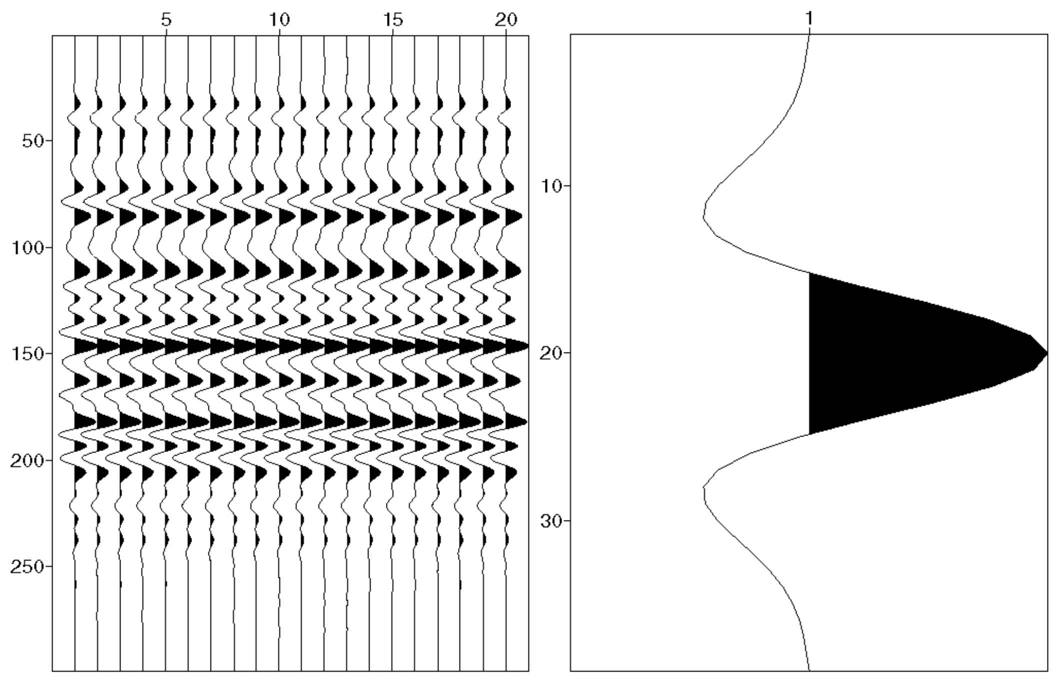

Fig. 20: primaries only zero-phase synthetic trace and the Ricker wavelet used to create it.

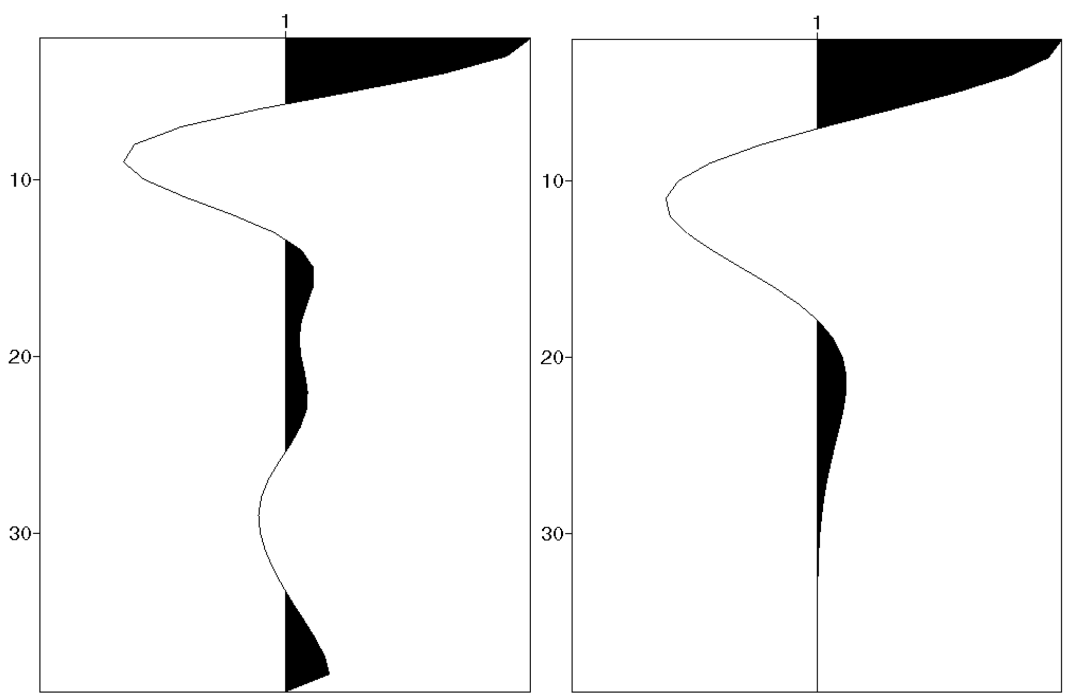

Fig. 21: The trace autocorrelation (left) and the Ricker wavelet autocorrelation (right).

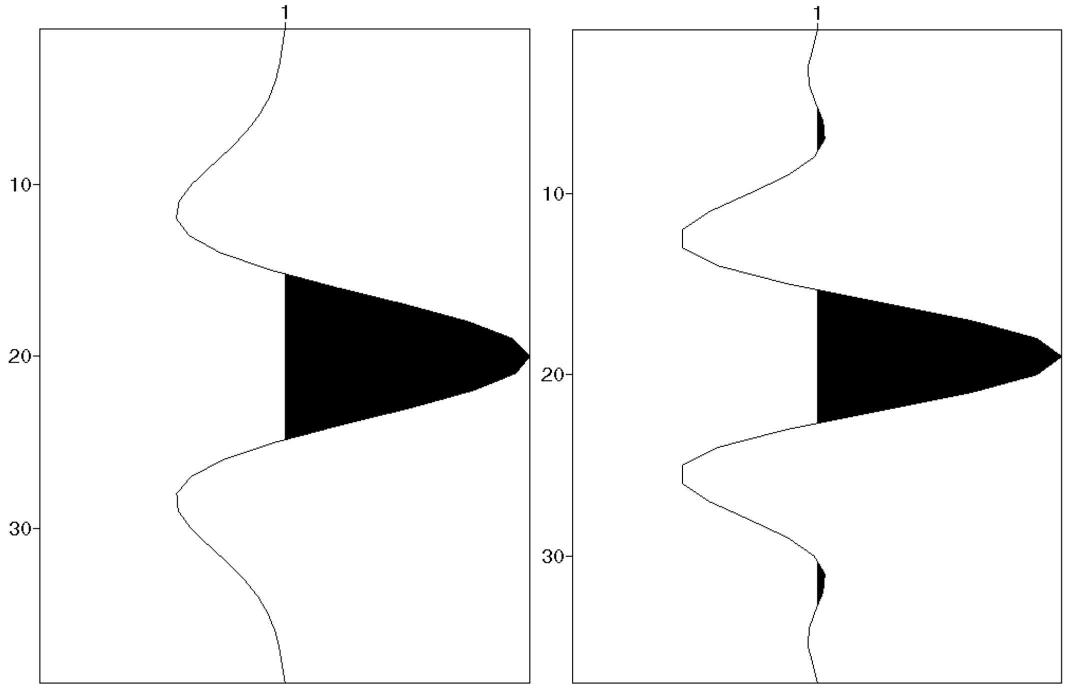

Fig. 22: The model Ricker wavelet (left) and a Klauder zero-phase wavelet estimate (right).

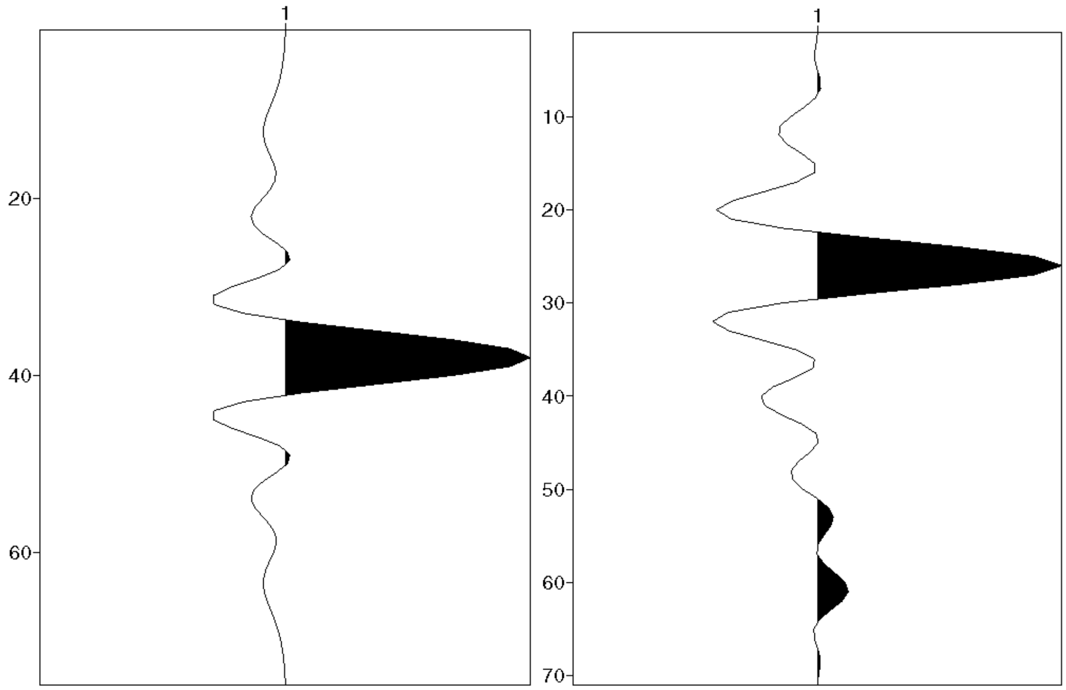

Fig. 23: Ricker wavelet deconvolution (left) and Klauder wavelet deconvolution (right).

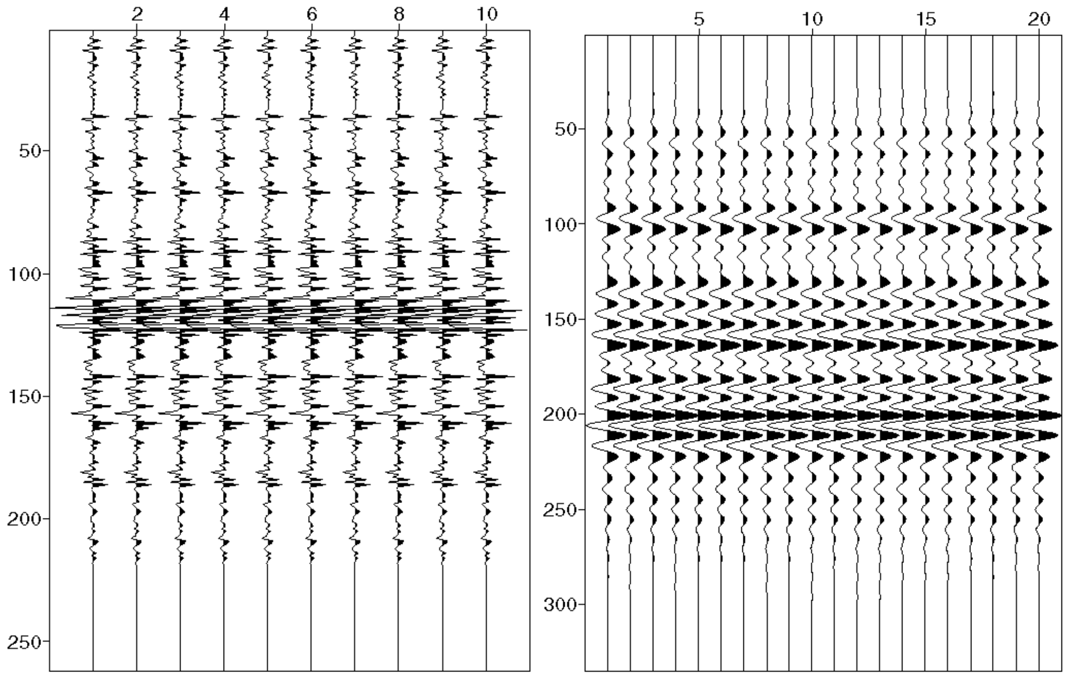

Fig. 24: The actual well-log derived reflectivity series (left) and the Ricker band-limited reflectivity series estimate (right).

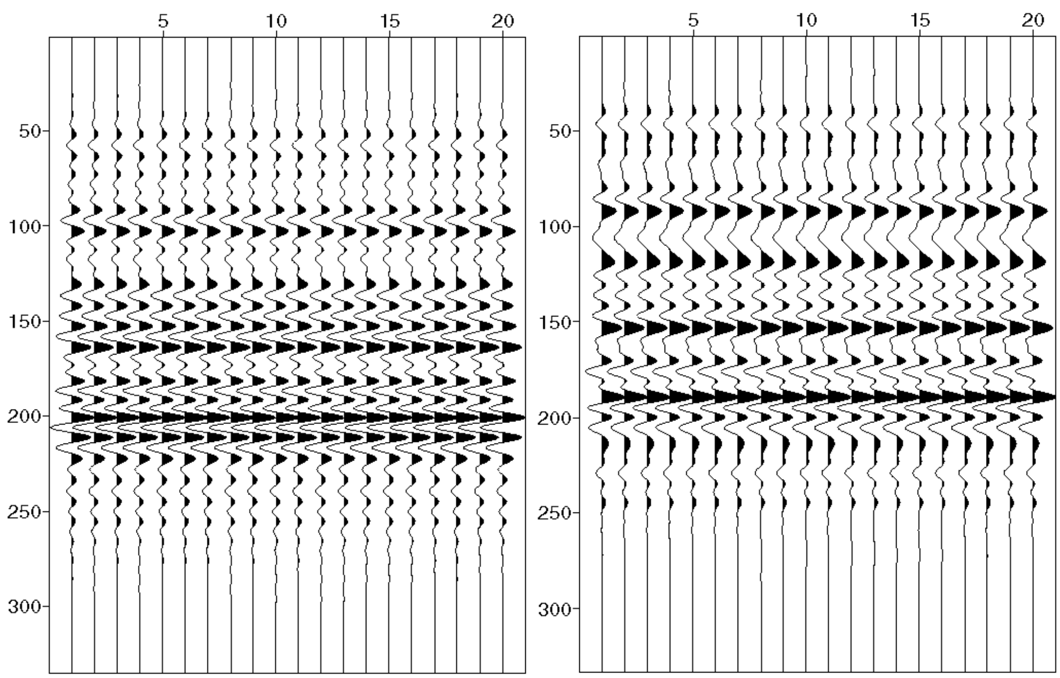

Fig. 25: The Ricker reflectivity estimate (left) and the Klauder reflectivity estimate (right).

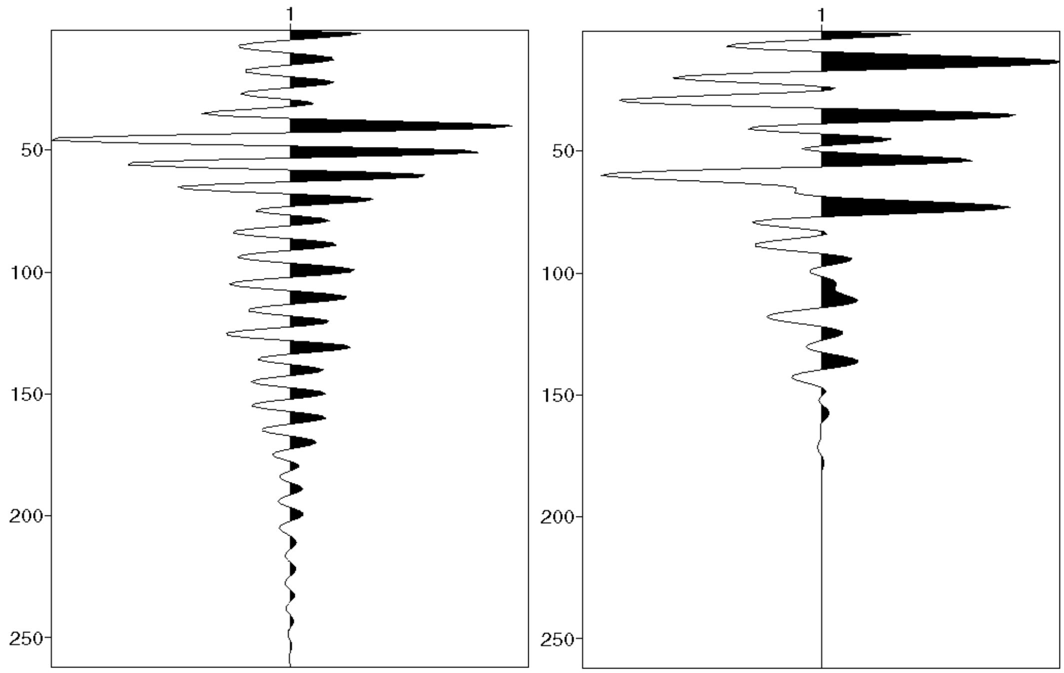

Fig. 26: Cross-correlation of the actual reflectivity series with the Ricker reflectivity series (left) and the Klauder reflectivity series (right).

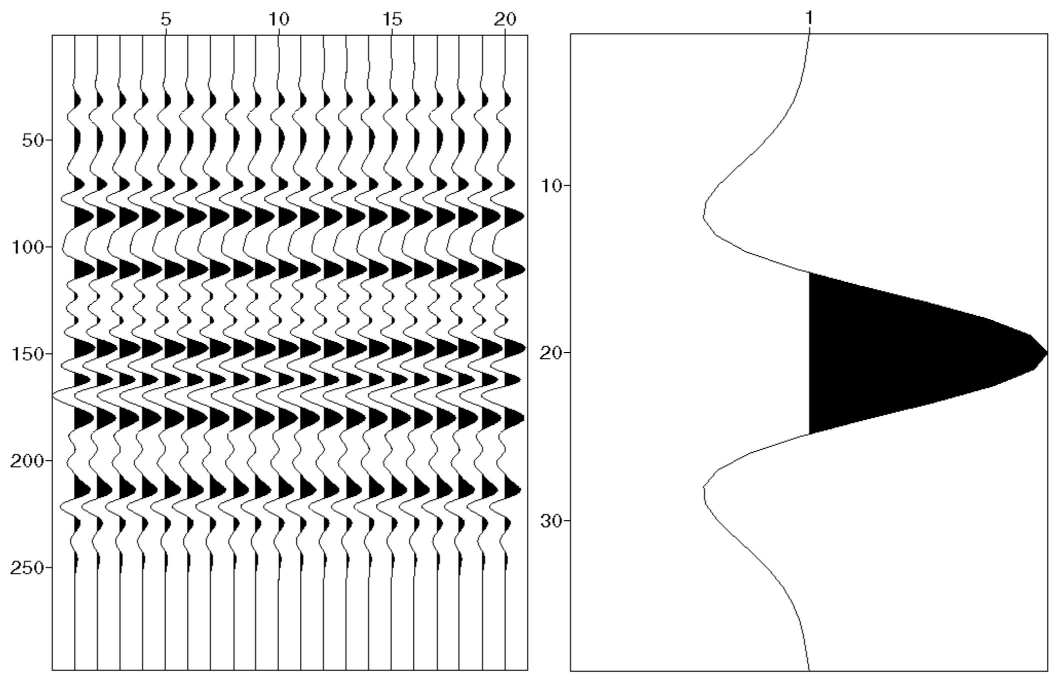

Fig. 27: A zero-phase synthetic trace, with primaries and multiples, and the Ricker wavelet used to create it.

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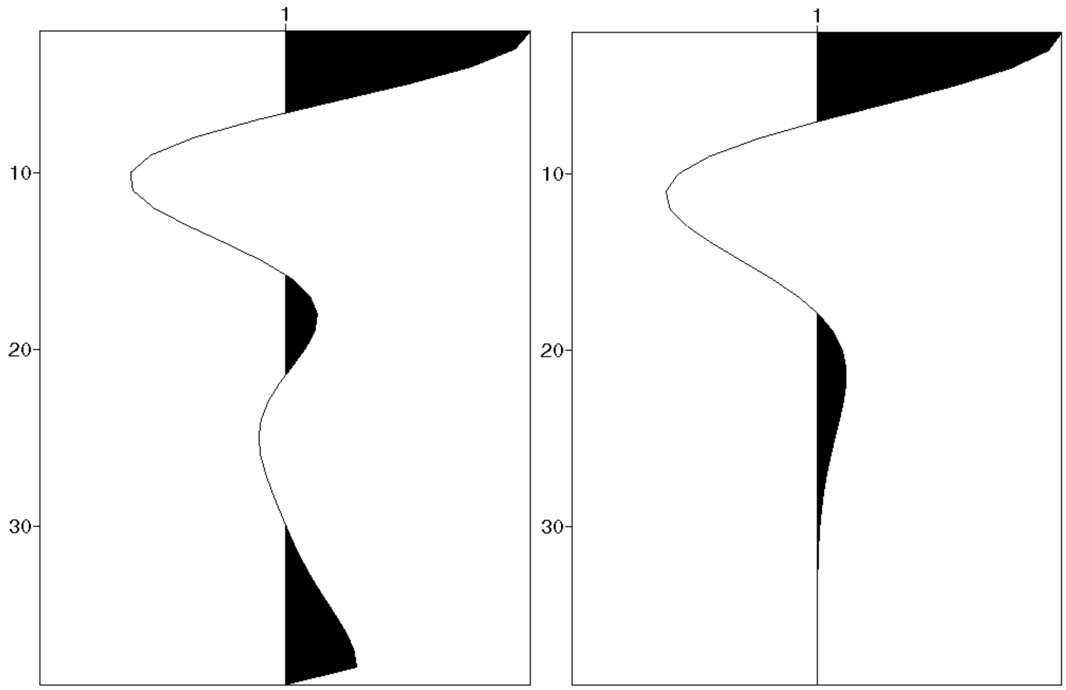

Fig. 28: The trace autocorrelation (left) and the Ricker wavelet autocorrelation (right).

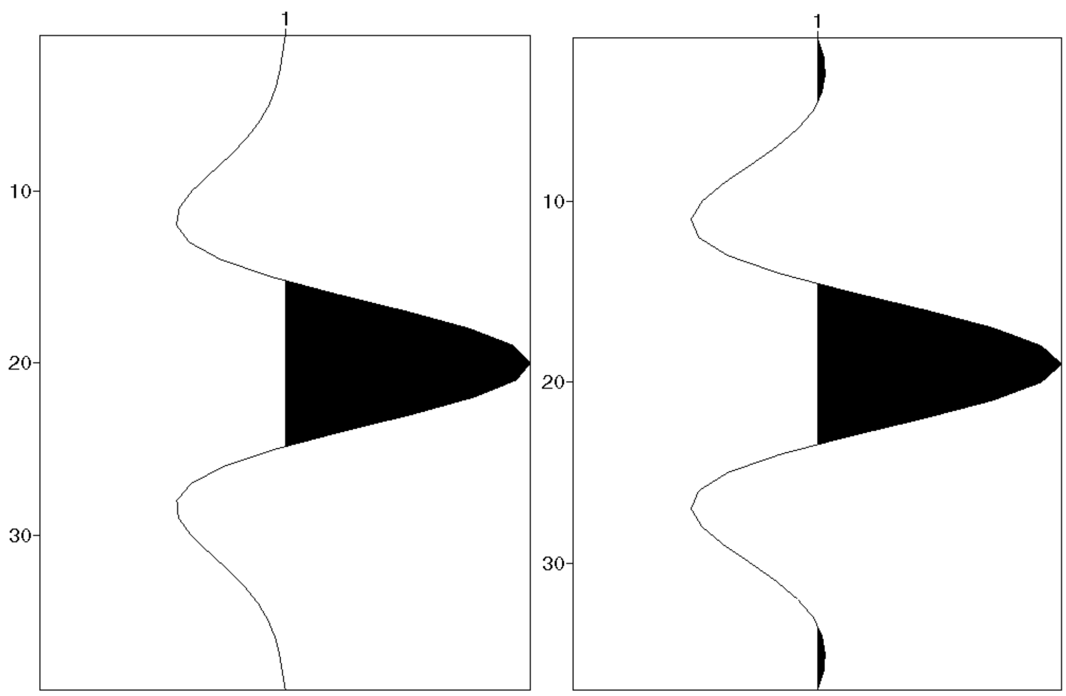

Fig. 29: The model Ricker wavelet (left) and a Klauder zero-phase wavelet estimate (right).

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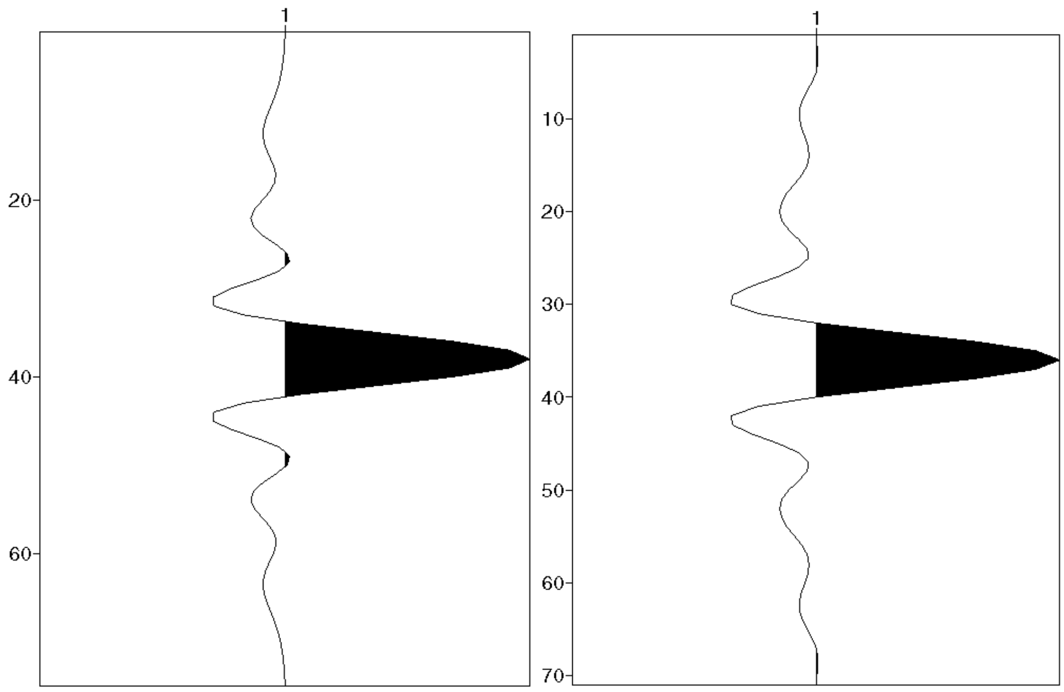

Fig. 30: Ricker wavelet deconvolution (left) and Klauder wavelet deconvolution (right).

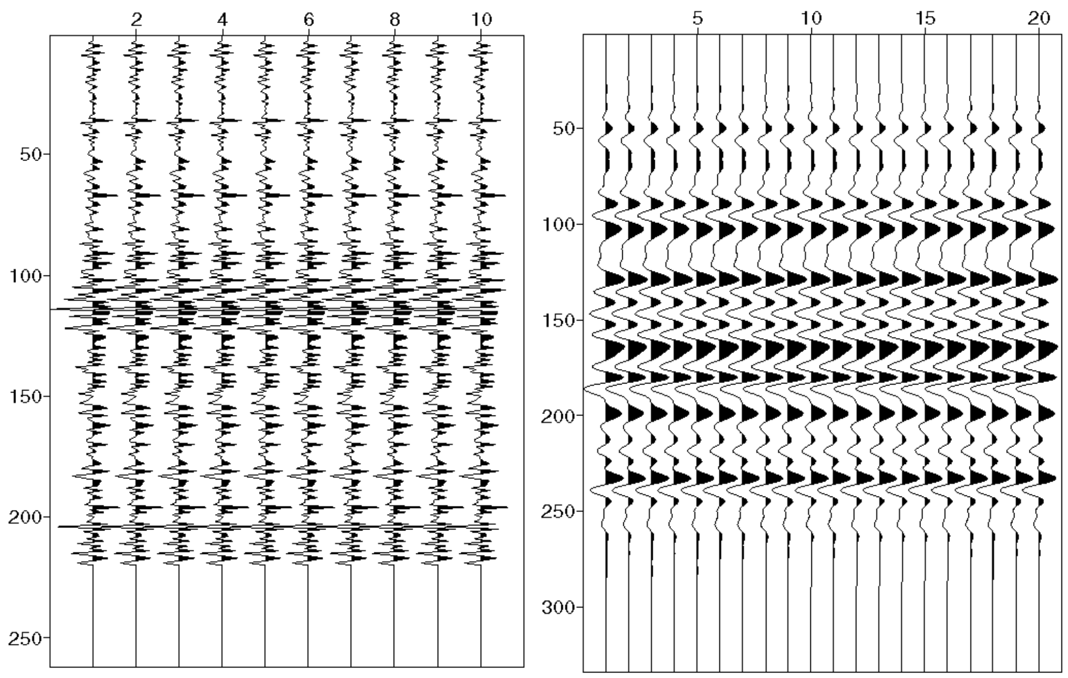

Fig. 31: The actual well-log derived reflectivity series (left) and the Ricker band-limited reflectivity series estimate (right).

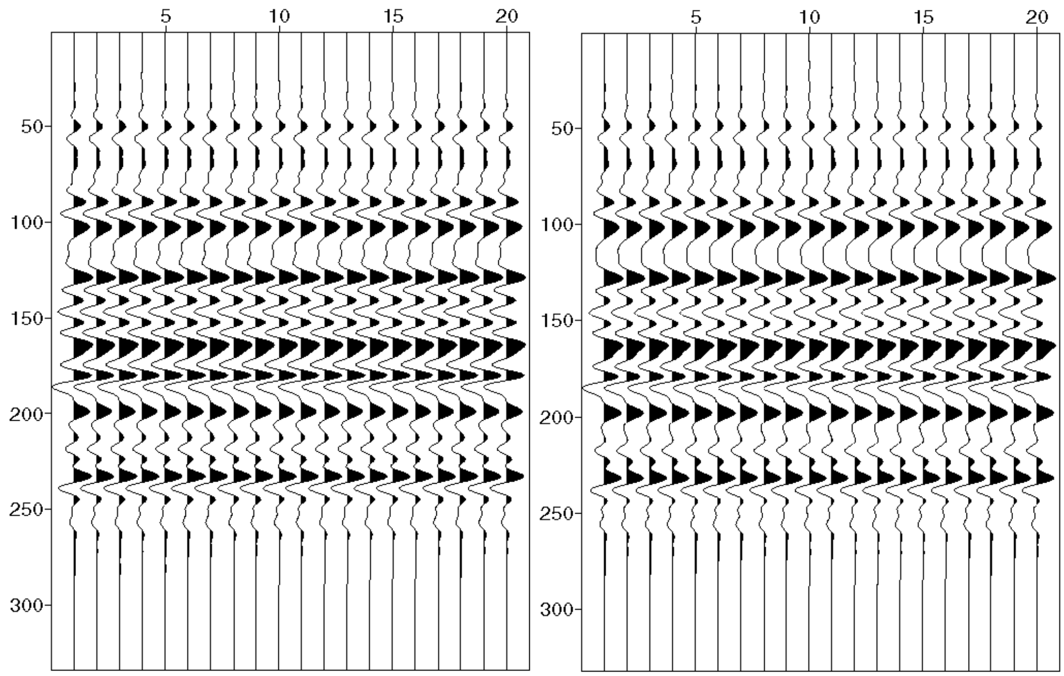

Fig. 32: The Ricker reflectivity estimate (left) and the Klauder reflectivity estimate (right).

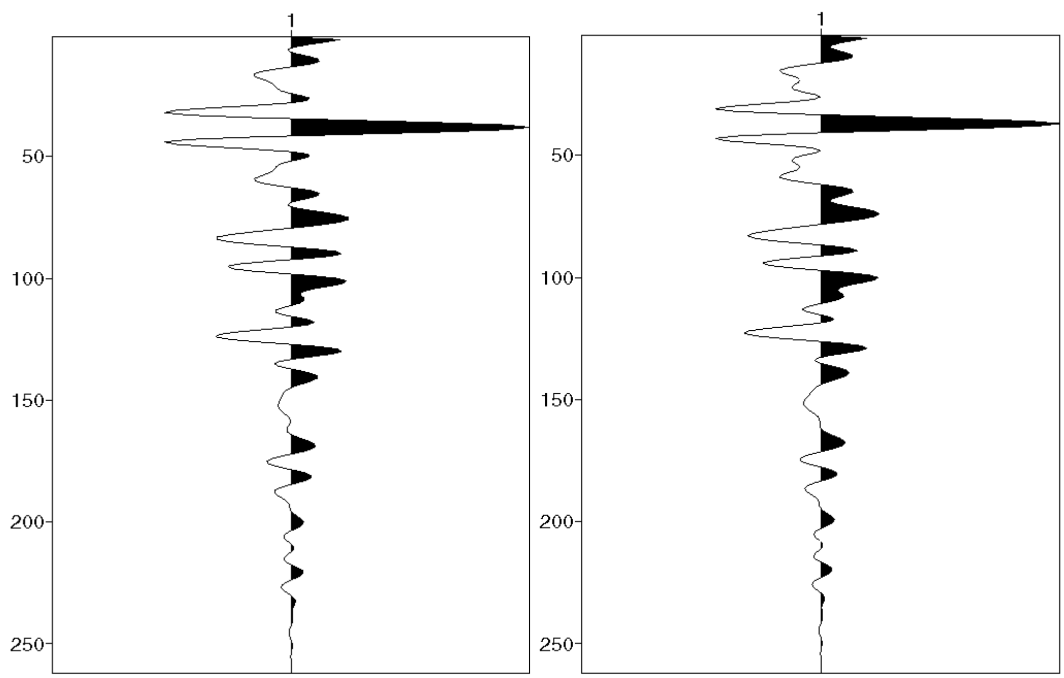

Fig. 33: Cross-correlation of the actual reflectivity series with the Ricker reflectivity series (left) and the Klauder reflectivity series (right).

From figures 20 – 33, we see that general trends discussed before continue. Notice, in Figure 21 and Figure 28, that the trace autocorrelation and the wavelet autocorrelation are quite similar. Also note that the Klauder zero phase wavelet estimates, shown in Figures 22 and 29, are nearly identical to the model Ricker wavelet. Despite these similarities some points of discussion arise. Figure 23 shows, for the 1st zero phase synthetic, that the Klauder and Ricker wavelet deconvolutions spike at different positions. Namely, the Klauder wavelet deconvolution has a spike that lags behind the spike seen for the Ricker wavelet. This lag is also evident in Figure 25, where the two band limited reflectivity estimates are compared. It shows that events on the Klauder reflectivity sequence lag behind events on the Ricker reflectivity sequence. Figure 29 illustrates the close correlation between the Ricker wavelet and the Klauder estimate based on the zero phase synthetic with multiples. Their deconvolutions (Figure 30) spike at the same positions and are effectively identical. This similarity is also reflected in Figure 32. Here, the band limited reflectivities are shown and they too are effectively identical.

With the battery of tests on synthetic data concluded, we turn our attention to an analysis on a real data set. The data set comes to CREWES through a gracious donation from PanCanadian Petroleum and is from a field in Alberta, Canada. Shown in Figure 34 is a near offset surface recording of real data with a vibroseis source and the downgoing VSP waveform. It is this downgoing VSP waveform that is our in-situ measured wavelet. One thing to note about the waveform is that it, apparently, is illustrating a time varying nature. This violates the major assumption of the wavelet being stationary.

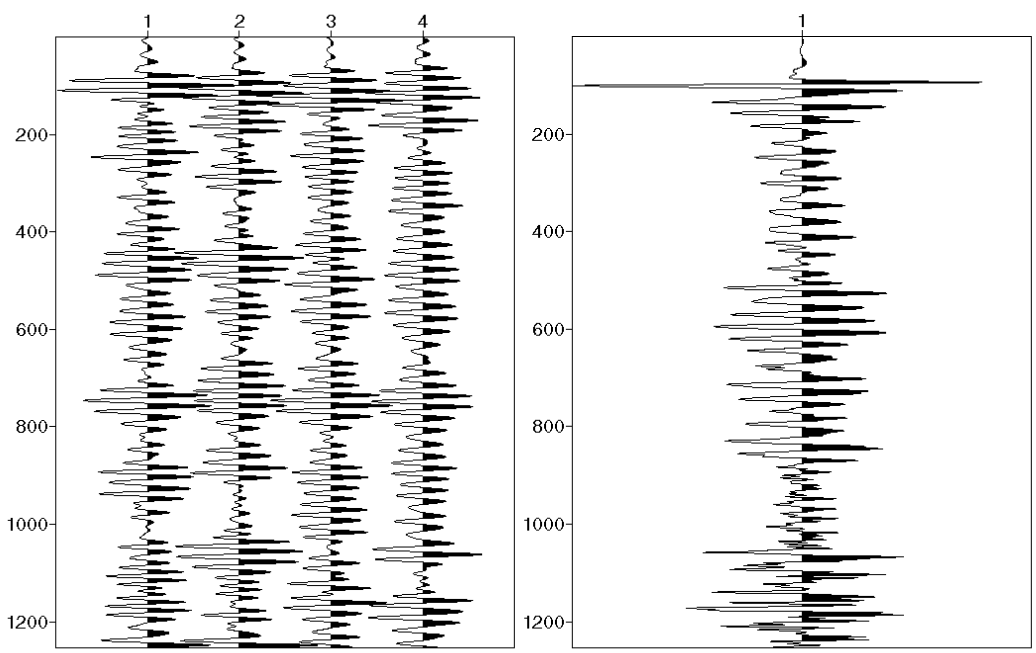

Fig. 34: Near offset surface data and the in-situ measured wavelet (downgoing VSP).

Shown below, Figure 35, is the autocorrelation of the reflected wave surface trace and the autocorrelation of the total downgoing VSP. The close similarity between the autocorrelations lends support to the assumption of a random reflectivity sequence.

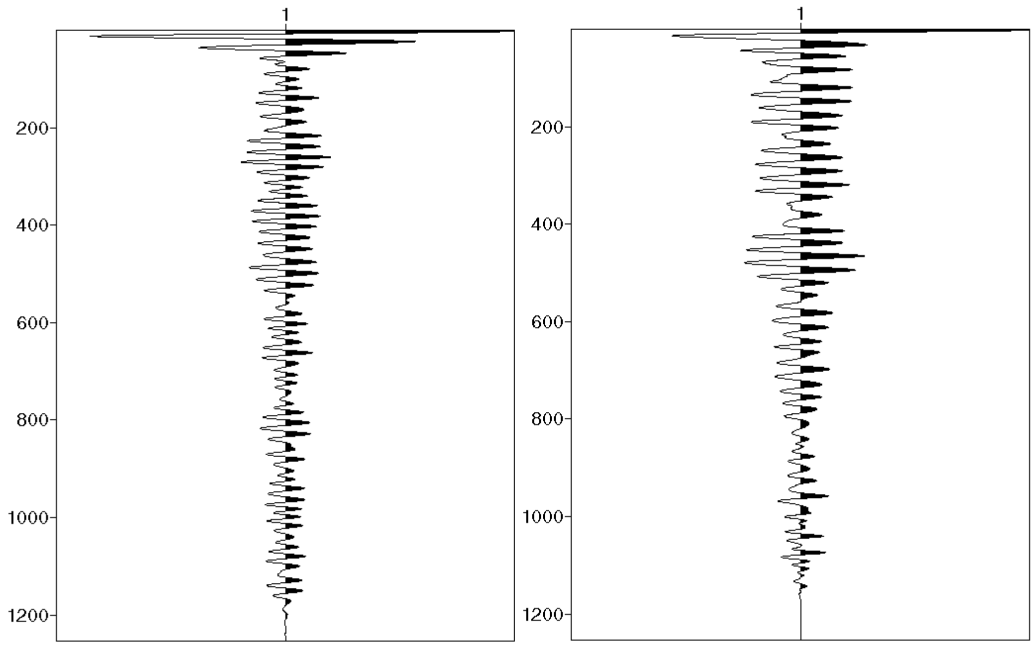

Fig. 35: Near offset trace autocorrelation (left) and in-situ wavelet autocorrelation (right).

The direct measured wavelet is truncated to the same number of samples as a zero phase wavelet estimate. This gives a measured wavelet with a more reasonable length. A comparison of the two is shown in Figure 36.

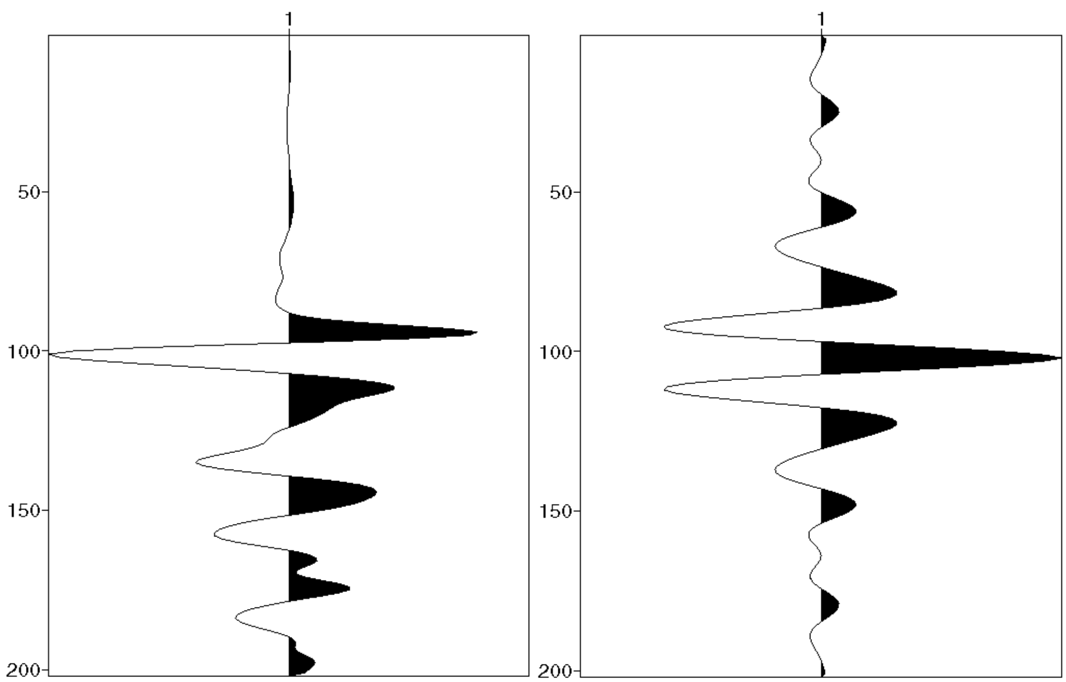

Fig. 36: Truncated measured wavelet (left) and a Klauder zero-phase wavelet (right).

When plotted on the same scale, noticeable differences exist between the measured wavelet and the Klauder zero phase wavelet estimate. The most pronounced of these differences are in wavelet phase. It appears that the estimated wavelet is about 180° out of phase from the in-situ wavelet. This means that whenever there is a trough in the measured wavelet, there is a peak in the estimate. Also note what can be best described as difference in symmetry. What is meant by this is that the estimate is a typical zero phase wavelet in that it is symmetric about its major peak. However, this is not what is seen in the truncated measured wavelet. There is no symmetry about the major trough. In fact, it appears as if the in-situ measurement is a delayed minimum phase wavelet of sorts.

Although there are these significant differences between the two, the autocorrelation of the real seismic trace data is quite similar to the autocorrelations of the truncated and estimated wavelets (see Figures 37 and 38). The wavelet autocorrelations themselves are also quite similar to each other. Collectively, this reinforces the belief that trace and wavelet autocorrelations are approximations of each other and, hence, the reflectivity is random.

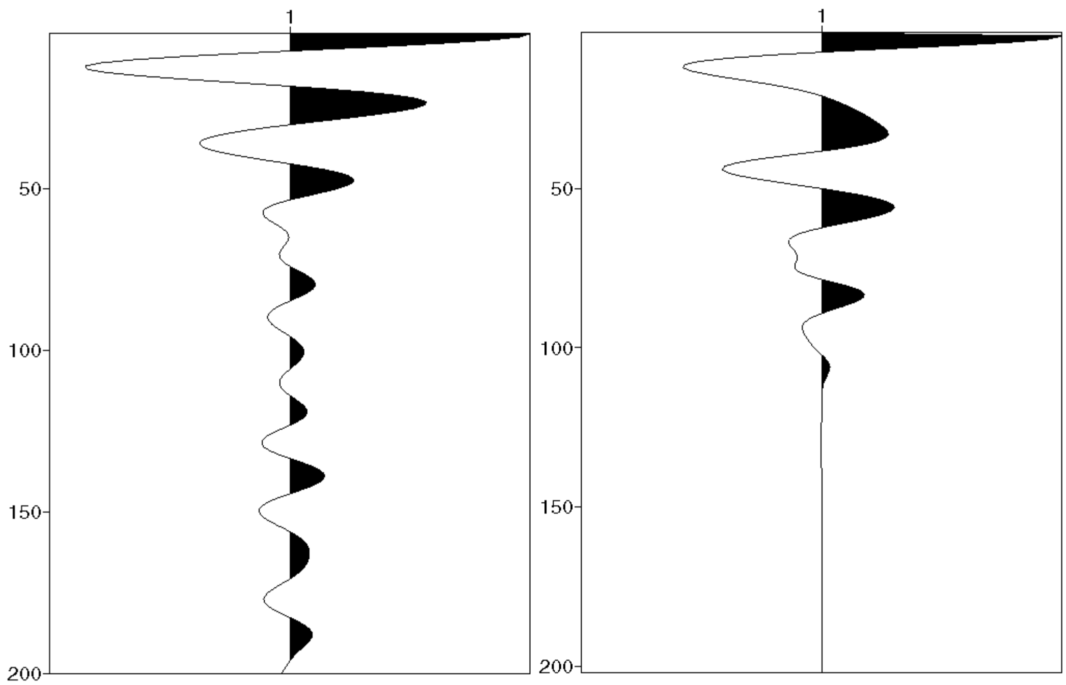

Fig. 37: Near offset trace autocorrelation (left) and truncated wavelet autocorrelation (right).

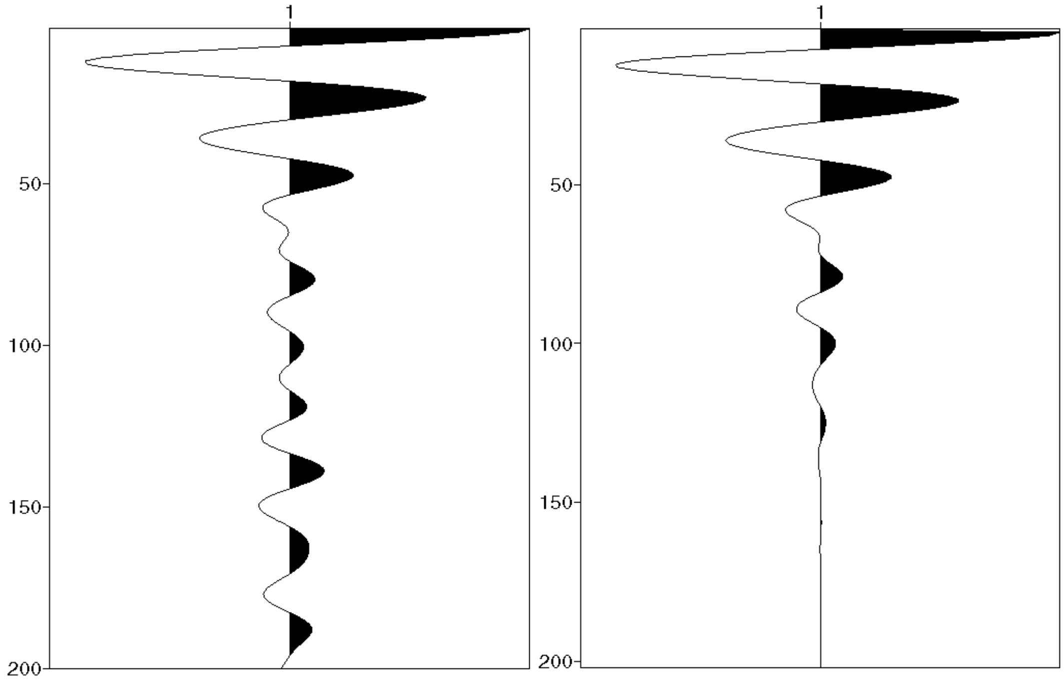

Fig. 38: Near offset trace autocorrelation (left) and Klauder wavelet autocorrelation (right).

Resolving kernels for the deconvolution of the different wavelets (in-situ, in-situ truncated, and statistical zero phase) are shown in the following two figures. All three give good, relatively clean, spikes but the spikes all occur at different positions. These different spiking positions may imply that the phase differences and nonstationarity of the wavelet itself are important. That is, the different spiking positions may be due to different phases and the nonstationary nature of the wavelet.

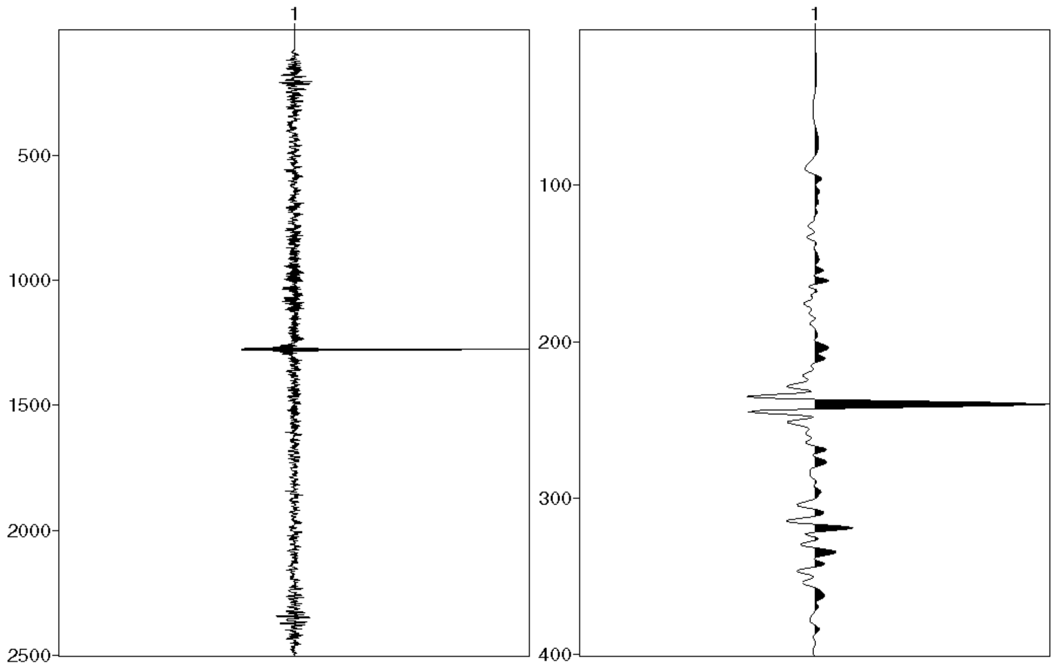

Fig. 39: Measured wavelet deconvolution (left) and truncated wavelet deconvolution (right).

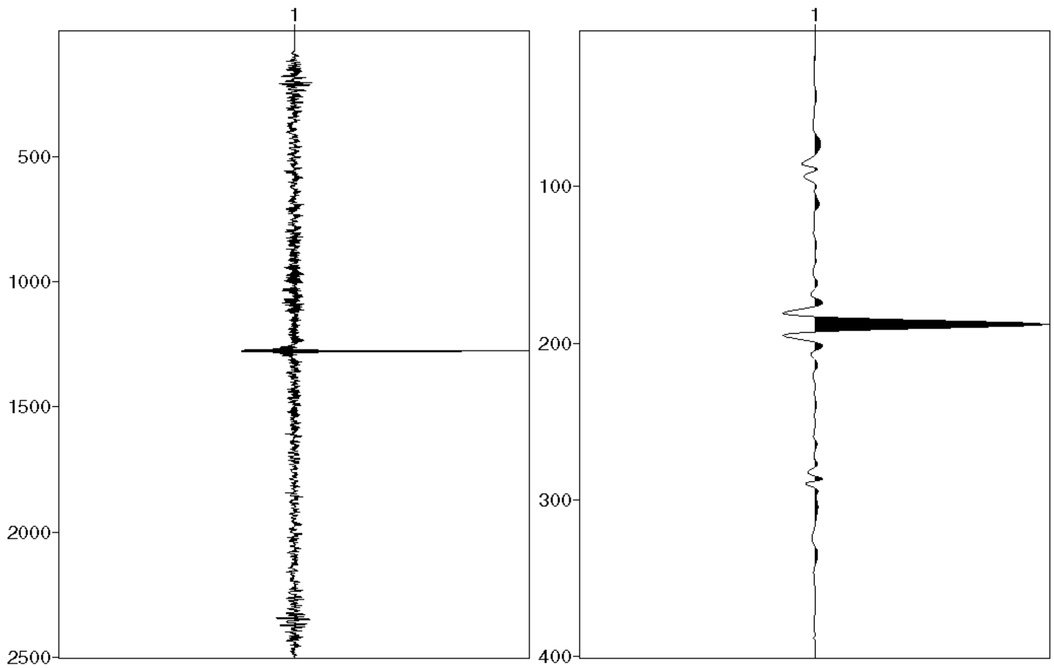

Fig. 40: Measured wavelet deconvolution (left) and Klauder wavelet deconvolution (right).

The final three figures relate to how well the deconvolution filters work. Since there is no well log derived reflectivity sequence to act as a control for the results, we proceed in a slightly different manner. Instead of comparing estimated sequences to the actual sequence, we evaluate the results with respect to how well responses are brought and if the ringy nature of the surface data can be surpressed. Note that the deconvolution result from a filter based on the Klauder zero phase wavelet estimate best reduces the ringiness and responses are very clear.

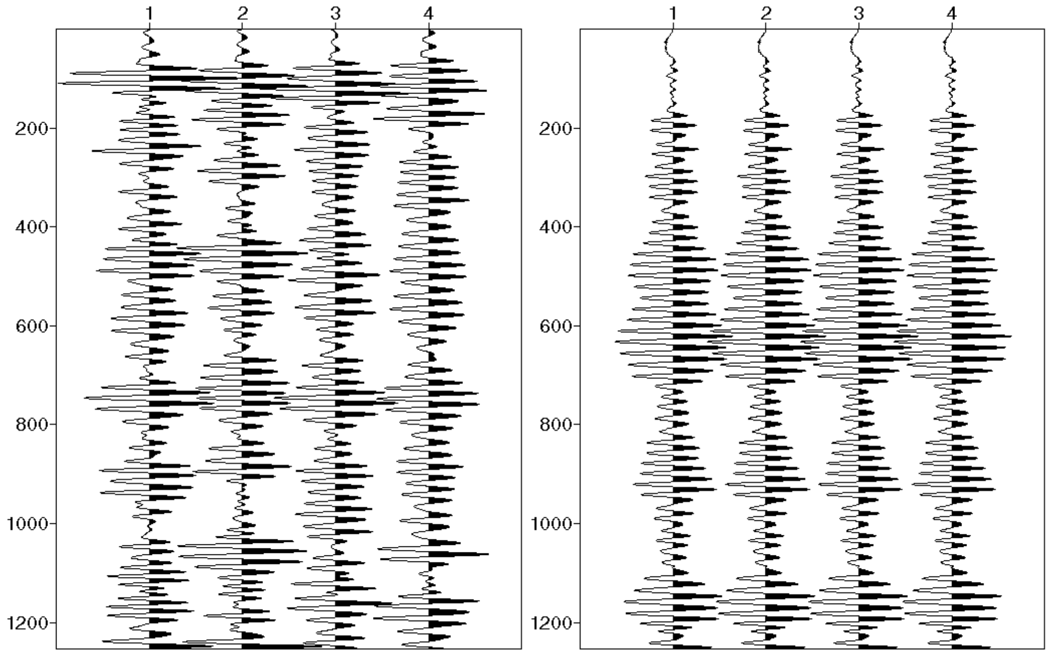

Fig. 41: Near offset traces (left) and their convolution with an in-situ filter (right).

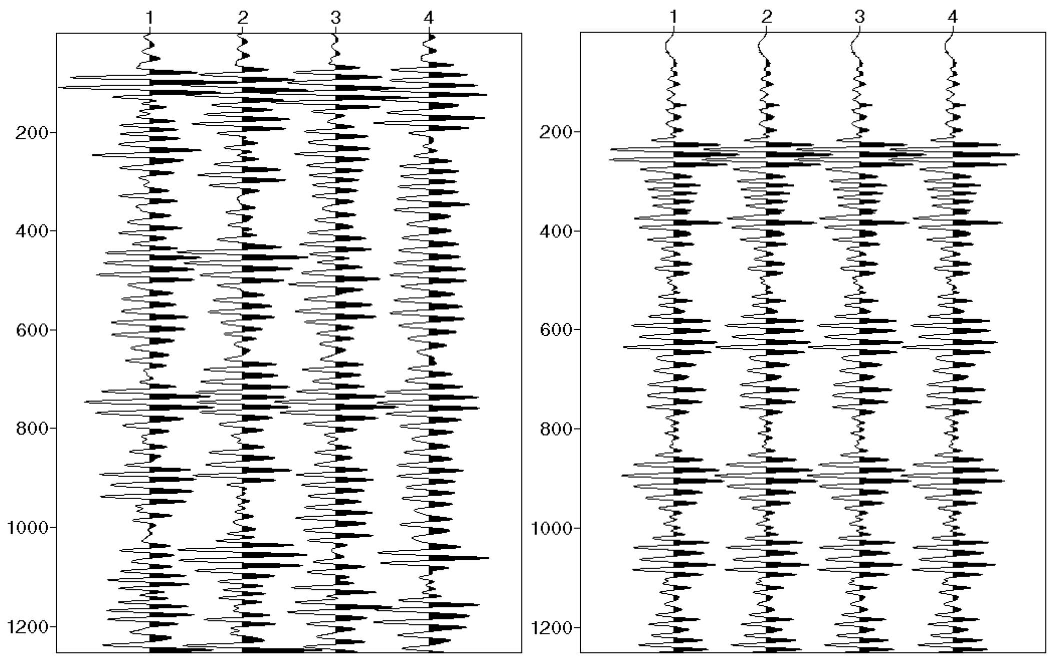

Fig. 42: Near offset traces (left) and their convolution with a truncated filter (right).

Contents

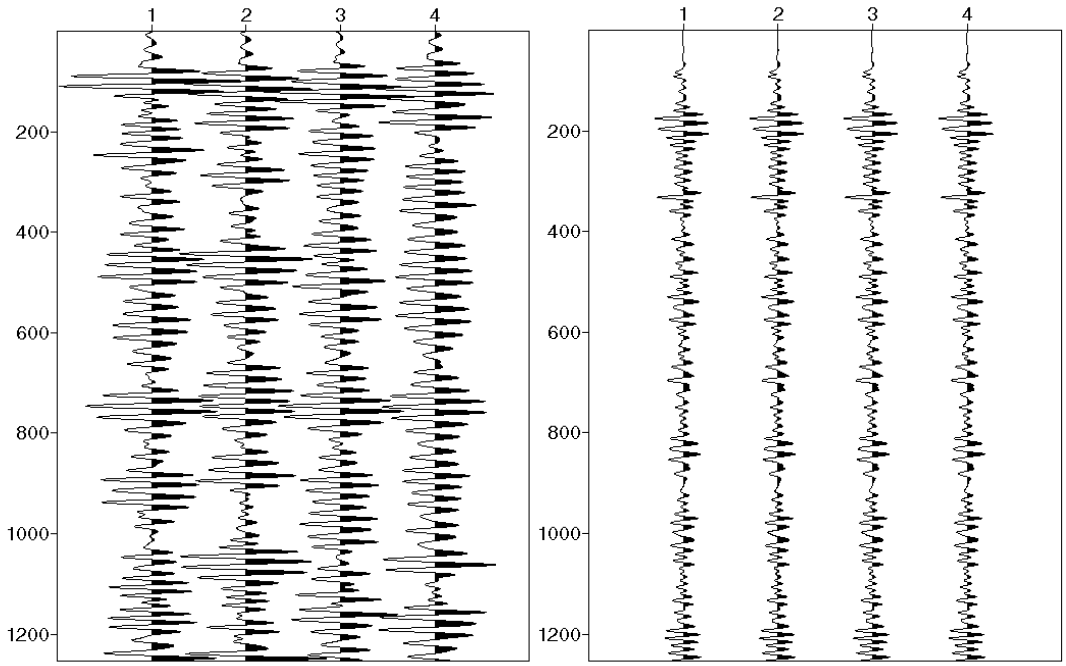

Fig. 43: Near offset traces (left) and their convolution with a Klauder zero-phase filter (right).

Through all of this extensive analysis of the assumption that the reflectivity sequence is random, we have developed two fairly simple and easy to apply tests to check the validity of the randomness assumption. Here is an algorithm to test the random reflectivity assumption.

- Convolve the wavelet with a deconvolution filter to obtain a resolving kernel.
- The closeness of the resolving kernel to a spike demonstrates the effectiveness of the wavelet deconvolution.

We end this section with an additional test that requires a well log derived reflectivity. The algorithm for this test is as follows.

- Convolve the input trace data with a deconvolution filter to obtain an estimate of the reflectivity sequence.
- Compare this estimate to the actual well log derived reflectivity sequence.

CONCLUSIONS

The preceding results give some mixed impressions regarding the randomness of the reflectivity sequence. Using sonic and density logs to compute a reflectivity sequence for a geological area and then using this information to measure the goodness of the estimates can test the validity of the assumption. In order to evaluate the effectiveness of the random reflectivity assumption on deconvolution, we propose a simple test that uses a sonic log. If we compute a seismic trace for a known wavelet and then compute an estimated wavelet, and corresponding deconvolution filter, then we can compute the convolution of the filter with the actual wavelet and examine the output or resolving kernel. The closeness of the resolving kernel to a spike demonstrates the deconvolution's effectiveness. A further test is to apply the estimated deconvolution filter to the trace and compare the deconvolved output to the reflectivity sequence. With real data, we can compare the wavelet estimate obtained from surface recorded data with in-situ VSP recordings. For the data that we examined, our results show that the other problems of wavelet phase and nonstationarity beset the wavelet estimation problem more so than the assumption of reflectivity randomness. Through all of the investigations it has become clear that the randomness assumption for a reflectivity sequence will be closely tied to the lithology of an area. That is to say, if an exploration area has periodic properties, then its reflectivity sequence will not have the required statistical property of randomness.

ACKNOWLEDGEMENTS

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