A nonstationary description of depth migration

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ABSTRACT

Application of nonstationary filters in a depth-stepping manner allows an estimate of the angle-dependent reflectivity for a heterogeneous subsurface. For simplicity, a 2D geometry is adopted and scalar wave propagation is assumed. Migration of a single source gather proceeds by nonstationary *combination* filtering, followed by an offset dependent time shift and spreading correction. Source gather migration generalizes, for a seismic line, to nonstationary combination filtering of source gathers, followed by nonstationary *convolution* filtering of geophone gathers. Zero offset migration, when derived as a special case of source migration, uses a single nonstationary extrapolator that is a symmetric of combination and convolution filters.

INTRODUCTION

Many authors present descriptions of how to migrate seismic data by wavefield extrapolation. Notable among such descriptions are Berkhout (1984), and Wapenaar and Berkhout (1989), who often use matrix representations of the process of wavefield extrapolation. It is sometimes desirable to have a description of the migration process that uses the notation of integral calculus. Algorithms can then be easily constructed, and the merits of one migration method over another can be more directly shown. In this paper we provide such a notation to construct migration code and, because of its simplicity, to unify the many forms of seismic migration. The presentation is based on the theory of nonstationary filters (Margrave, 1998) and borrows some concepts (and some notation) from Berkhout (1984).

Here migration is shown to proceed by first filtering a source gather by nonstationary *combination*, followed by an offset-dependent time shift and spreading correction. Source-gather migration, when generalized to an entire seismic line, prescribes that, for one depth step, source gathers are filtered by nonstationary combination, and geophone gathers by nonstationary *convolution*. Zero offset migration is then derived as a special case of full prestack migration, and stationary (constant velocity) migration is presented as it's stationary zero-offset limit.

MODEL OF REFLECTIVITY

Reflectivity can be defined as the ratio between the reflected wavefield ψ_R and the incident wavefield ψ_i evaluated at the same instant in time

$$\mathrm{r}(x_{\Delta z}) = \frac{\Psi_R(x_{\Delta z}, t_r^+)}{\Psi_i(x_{\Delta z}, t_r^-)} \tag{1}$$

where Ψ_i and Ψ_R represent wavefields incident to and reflected from a point ($x_{\Delta z}$) in the subsurface. Times t_r^- and t_r^+ indicate immediately before and immediately after

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reflection. Replacing and Ψ_I and Ψ_R with their temporal Fourier transforms and rearranging gives

$$\mathrm{r}(x_{\Delta z})\int \psi_i(x_{\Delta z},\omega)\exp(-i\omega t_r^-)d\omega = \int \psi_R(x_{\Delta z},\omega)\exp(-i\omega t_r^+)d\omega \tag{2}$$

and, assuming reflectivity r is independent of ω and $t_r^- = t_r^+$

$$\int \left[\mathrm{r}(x_{\Delta z})\psi_i(x_{\Delta z},\omega)-\psi_R(x_{\Delta z},\omega)\right]\exp(-i\omega t_r)d\omega = 0 \, . \tag{3}$$

Equation (3) implies that the product of r and the spectrum ψ_i is equal to the spectrum ψ_R

$$\mathrm{r}(x_{\Delta z})\psi_i(x_{\Delta z},\omega) = \psi_R(x_{\Delta z},\omega) \tag{4}$$

Note that for a monochromatic wavefield the imaging condition is independent of time, and r is simply the ratio of the incident and reflected wavefields

$$\mathrm{r}(x_{\Delta z}) = \psi^{-1}{}_i(x_{\Delta z},\omega)\psi_R(x_{\Delta z},\omega) \, . \tag{5}$$

In equation (4) ψ_i and ψ_R have identical spreading and phase delays, so (5) removes these effects, and thus estimates reflectivity. The common notion of applying a $t = 0$ imaging condition is equivalent to averaging the monochromatic reflectivity estimates over all frequencies.

A general, monochromatic, description for the incident wavefield can assume a surface ($z = 0$) distribution of sources (source array) that are superimposed according to

$$\psi_i(x_{\Delta z}) = \int S(x)W^+(x, x_{\Delta z} - x)dx \tag{6}$$

where S describes the surface distribution of sources, and the extrapolator W^+ takes down-going waves down and up-going waves up. The operator W^- will be used to extrapolate down-going waves up and up-going waves down. (Note that, for constant velocity phase shift, W^+ and W^- are the complex conjugates of each other.) Also, W^+ will be a function of velocity, and we use its first coordinate to describe lateral velocity dependence. We also have chosen (arbitrarily) to assign velocity dependence to x and we are therefore using an NSPS extrapolator as defined by Margrave and Ferguson, (1997). (For more discussion on extrapolators, see Margrave and Ferguson, 1998a.) For one source at x_s equation (6) becomes

$$\psi_i(x_{\Delta z}, x_s) = \int \delta(x_s - x)W^+(x, x_{\Delta z} - x)dx = W^+(x_s, x_{\Delta z} - x_s) \tag{7}$$

where dependence on source coordinate x_s has been added to the incident wavefield ψ_i. Velocity in the construction of W^+ is taken from the source location x_s or, in figure (1), velocity is held constant over the aperture indicated.

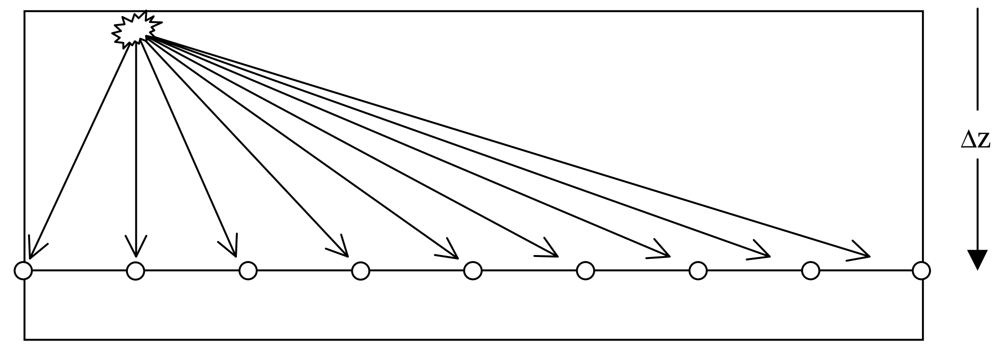

Fig. 1. The incident wavefield ψ_i for a single source at the surface. Velocity is held constant over the aperture indicated by the rays and is a function of the source point coordinate.

A monochromatic description of the reflected wavefield ψ_R is

$$\psi_R\left(x_{\Delta z}, x_s\right) = \int \psi_g\left(x, x_s\right) W^{-}\left(x_{\Delta z}, x_{\Delta z} - x\right) dx \tag{8}$$

which describes the reflected wavefield ψ_R as the downward continuation of geophone recordings ψ_g according to the extrapolator W^-. Velocity dependence is on the reflection point coordinate $x_{\Delta z}$ and, as in figure (2), velocity is held constant over the aperture indicated. This is again an arbitrary assumption and means that we use the PSPI algorithm (Gazdag and Squazerro, 1984, Margrave and Ferguson, 1997) in equation (8).

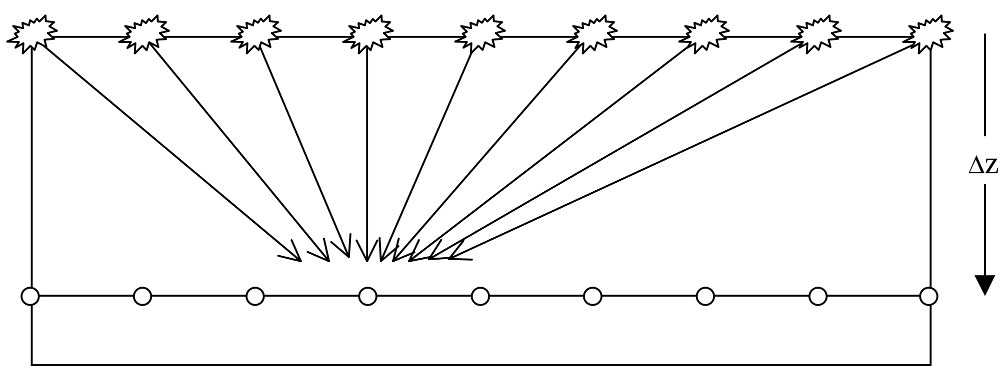

Fig. 2. The reflected wavefield ψ_R at a single reflection point. Velocity is held constant over the aperture indicated by the rays and is a function of reflection point coordinate.

For a single monochromatic source at x_s, and using equations (7) and (8), equation (5) becomes

$$\mathrm{r}(x_{\Delta z}, x_s) = \left[W^+(x_s, x_{\Delta z} - x_s)\right]^{-1} \int \psi_g(x, x_s) W^-(x_{\Delta z}, x_{\Delta z} - x) dx \quad . \tag{9}$$

For constant velocity, W^+ and W^- are symmetric under the exchange of x and x_{Δ_s} and are the complex conjugates of one another. Furthermore, $W^+W^- = 1$ for nonevanescent energy. These conditions are only approximated by the NSPS and PSPI extrapolators used here. Wapenaar and Grimbergen (1998) argue that these conditions should be met, even for lateral inhomogeneity, if source-receiver reciprocity is to be preserved. Margrave and Ferguson (1998b) show that NSPS and PSPI approximately invert themselves under a wide range of lateral gradients. In extreme cases only PSPI inverts NSPS and vice-versa. Margrave and Ferguson (1998a) show that NSPS and PSPI can be combined into a symmetric form which more closely obeys the conditions above. Here, we assume that

$$\left[W^+(x_s, x_{\Delta z} - x_s)\right]^{-1} \approx W^-(x_s, x_{\Delta z} - x_s) \tag{10}$$

so that a symmetric operator results in our zero offset theory. Using equation (10) equation (9) gives

$$\mathrm{r}(x_{\Delta z}, x_s) = W^-(x_s, x_{\Delta z} - x_s) \int \psi_g(x, x_s) W^-(x_{\Delta z}, x_{\Delta z} - x) dx \quad . \tag{11}$$

If we average all of the monochromatic reflectivity estimates (11) for all of the sources in a seismic line we integrate over x_s

$$\mathrm{r}(x_{\Delta z}) = \int W^-(x_s, x_{\Delta z} - x_s) \int \psi_g(x, x_s) W^-(x_{\Delta z}, x_{\Delta z} - x) dx dx_s \quad . \tag{12}$$

Equation (12) consists of two filter operations that are known from nonstationary filter theory (Margrave, 1998). The filter corresponding to integration over x is a nonstationary *combination* filter – recognizable by it's velocity dependence on output coordinate x_{Δ_z}. The other filter is a nonstationary *convolution* filter recognizable by it's velocity dependence on input coordinate x_s.

SINGLE SOURCE MIGRATION

A very effective way to generate an image of the subsurface is to apply equation (12) or (11) in a downward stepping algorithm. At each depth level the monochromatic estimates are averaged into an estimate of reflectivity, thus an image is constructed. Equation (11) is equivalent to single-source migration, and (12) is equivalent to migration of many sources.

There are advantages of choosing single-source migration, over multi-source migration that impact amplitude variation with offset (AVO) and computational effort. For AVO, the output image of a source migration is an offset dependent reflectivity that can be grouped with other source migrations for analysis. Multi-source migration averages AVO. Computationally, a source gather is small compared to an entire seismic line and can be migrated on a single computational node (Ferguson and Margrave, 1998). Distribution of a migration among a large number of nodes can result in a tremendous reduction in runtime (Ferguson and Margrave, 1998). Additionally, multi-source migration is a very sort-intensive process; at each depth level, outputs of all of the source gathers and all of the reflection point gathers must be available for imaging.

CONSTANT OFFSET MIGRATION

If the acquisition geometry is such that the recorded wavefield ψ_R is composed of single sources x_s and single receivers x at a common offset c, where $x - x_s = c$, then equation (12) becomes

$$\begin{aligned} \mathrm{r}(x_{\Delta z}) &= \int W^-(x_s, x_{\Delta z} - x_s)\int \psi_g(x, x_s)\delta(x - x_s - c)W^-(x_{\Delta z}, x_{\Delta z} - x)\,dx\,dx_s \\ &= \int W^-(x_s, x_{\Delta z} - x_s)\psi_g(x_s + c, x_s)W^-(x_{\Delta z}, x_{\Delta z} - [x_s + c])\,dx_s \quad . \end{aligned} \tag{13}$$

If the offset $c = 0$ then the constant offset equation (13) becomes

$$\mathrm{r}(x_{\Delta z}) = \int W^-(x_s, x_{\Delta z} - x_s)\psi_g(x_s)W^-(x_{\Delta z}, x_{\Delta z} - x_s)\,dx_s \quad . \tag{14}$$

Equation (14) represents a zero-offset, or post-stack, migration consisting of two functions W^-. Since these functions can commute within the integral they combine into a single extrapolator

$$W^-_{sym}(x_s, x_{\Delta z}) = W^-(x_s, x_{\Delta z} - x_s)W^-(x_{\Delta z}, x_{\Delta z} - x_s) \quad . \tag{15}$$

in the Fourier domain (15) becomes

$$\begin{aligned} W^-_{sym}(x_s, x_{\Delta z}) &= \int\int \alpha^-(x_s, k_x)\alpha^-(x_{\Delta z}, k'_x)\exp(i[x_{\Delta z} - x_s][k_x + k'_x])\,dk_x\,dk'_x \\ &= \int \exp(i\xi[x_{\Delta z} - x_s])\int \alpha^-(x_s, k_x)\alpha^-(x_{\Delta z}, \xi - k_x)\,dk_x\,d\xi \end{aligned} \tag{16}$$

where ξ is a replacement variable for $k_x - k_x'$. Analysis of (16) reveals that the Fourier domain representations α^- of operators W^- are convolved with each other. Margrave and Ferguson (1998a) show that equations (15) and (16) prescribe an operator that is symmetric which, according to Wapenaar and Grimbergen (1998) is required to satisfy reciprocity. The nonstationary form of operator α is

$$\alpha(x_s, k_x) = \exp\left(i\Delta z\sqrt{\frac{\omega^2}{v(x_s)^2} - k_x^2}\right). \tag{17}$$

For ordinary phase shift, the combined operator in W_{syn}^- reduces to $[W^-(x_{\Delta_z} - x_s)]^2$. A common approximation to $[W^-(x_{\Delta_z} - x_s)]^2$ is to invoke the model of exploding reflectors and replace $[W^-(k_x)]^2$ with a single extrapolator. Propagation then occurs at ½ the velocity

$$\left[W^{-}(k_x)\right]^2 \approx \exp i\Delta z\sqrt{\frac{\omega^2}{v_c^2}-k_x^2} \tag{18}$$

where v_c is equal to ½ the average velocity in the lateral coordinate. Clearly, this 'exploding reflector' approximation, though kinematically accurate does not apply the correct spreading.

CONCLUSIONS

Nonstationary filters can be used in depth migration to provide a mathematical description that uses integral calculus and is flexible in describing the migration process. Application of nonstationary filters in a depth stepping manner allows an estimate of the angle dependent reflectivity to be constructed when source gathers are migrated. Migration of source gathers is also easy to implement on a distributed (multi-node) computer and, because each source gather is small compared to the entire seismic line, each node can be of modest size. Migration of a single source is formulated using nonstationary combination filtering followed by an offset dependent time shift and spreading correction. Migration of an entire seismic line uses nonstationary combination filtering of source gathers, followed by nonstationary convolution filtering of reflection point gathers. Zero-offset migration uses a single nonstationary extrapolator that is a symmetric product of combination and convolution filters.

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