P-S ray construction using the method of images

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ABSTRACT

Finding the path of a simple ray, from arbitrary source and receiver positions, reflecting from a dipping interface can be accomplished by the method of images. This method can be extended to converted (P-S) waves, for the small offset case, by scaling the distance of the image receiver from the interface by the α/β value (the P-wave to S-wave velocity ratio). This moves the conversion point closer to the receiver position. The technique can be further generalized by introducing anisotropy (weak vertical transverse isotropy). Anisotropy perturbs the conversion point toward the receiver when Thomsen's (1986) 2δ-ε parameter is positive and toward the source when it is negative.

INTRODUCTION

Method of images for P-wave reflections

It is sometimes useful in analysing *P-P* (or *S-S*) waves to find the reflection point for an arbitrarily placed source and receiver with respect to a single interface. Often a quick drawing can help assess, say, a survey design change or give intuitive insight into a wave propagation problem. We can find the reflection point by extending a perpendicular line from the interface to the receiver (or source) and to an equally distant point on the other side of the interface (Figure 1). We then connect a line from the point below the interface (the "image" receiver) to the source. The location where this line intersects the interface is the reflection point. This is the point where the angle of incidence θ is equal to the angle of reflection. This is true from equivalent triangles in basic trigonometry.

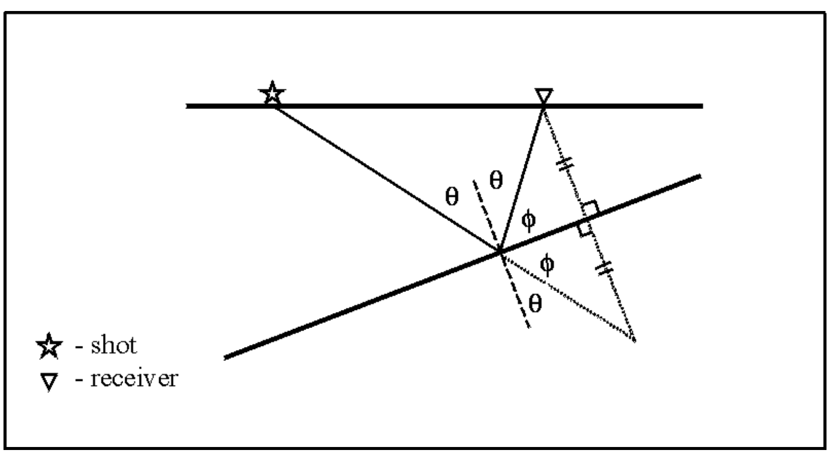

Fig. 1. Geometry for the method of images construction of *P-P* reflection points.

P-S reflection construction

We ask, is there a similar construction for a *P-S* reflection? Indeed, there is an analogous method that approximates the *P-S* conversion point, if θ is small.

From Snell's Law, with *P* and *S* velocities of α and β, we know:

$$\frac{\sin\theta}{\alpha} = \frac{\sin\phi}{\beta}, \tag{1}$$

and from Figure 2:

$$\tan\phi = \frac{x}{z_1}, \qquad \tan\theta = \frac{x}{z_2}. \tag{2}$$

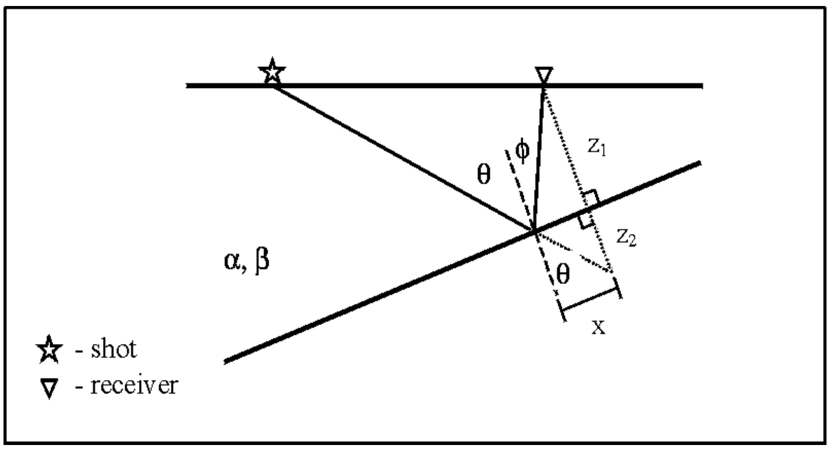

Fig. 2. Geometry for the method of images construction for *P-S* waves.

If θ is small, then:

$$\tan\theta \sim \sin\theta \text{ and also } \tan\phi \sim \sin\phi . \tag{3}$$

Thus:

$$z_2 \sin\theta \sim z_1 \sin\theta , \tag{4}$$

and

$$\frac{z_1}{z_2} \sim \frac{\alpha}{\beta}. \tag{5}$$

So, the construction proceeds similarly to the *P-P* case, except that the receiver image distance is scaled by the α/β value: that is, we extend a perpendicular from the interface to the receiver having a length z_1. Then, on the other side of the interface, we construct another perpendicular line with length z_2 . We join the end of this new perpendicular line to the shot location. The intersection point of this new line with the interface is the conversion point. Note that because α/β is greater than one, then z_1 is greater than z_2 and the conversion point is shifted toward the receiver. Thus the asymmetry of the reflection point position.

P-S reflection in anisotropic material

Let's also consider the case of a horizontal interface with weak vertical transverse isotropy (VTI) and small angles of incidence. Suppose the source-receiver offset is X_0 and the depth to the reflector is z_1.

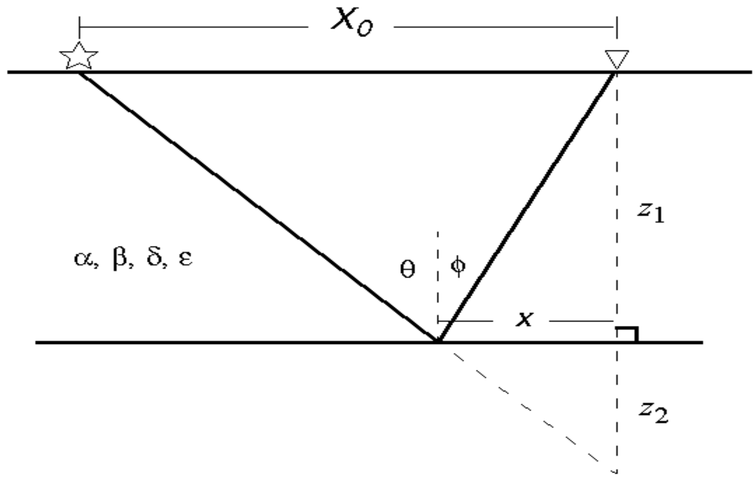

Fig. 3. Geometry for conversion point in a weakly anisotropic material.

Using Thomsen's (1986) anisotropy formulation and Snell's Law:

$$\frac{\sin\theta}{\alpha\left(1+\delta\sin^2\theta\cos^2\theta\right)}=\frac{\sin\phi}{\beta\left[1+\frac{\alpha^2}{\beta^2}(\varepsilon-\delta)\sin^2\phi\cos^2\phi\right]}. \tag{6}$$

With $\cos\theta \sim 1$ and $\sin\phi \sim \frac{z_2}{z_1}\sin\theta$, then:

$$1+\left(\frac{\alpha z_2}{\beta z_1}\right)^2(\varepsilon-\delta)\left(\frac{X_0}{z_1+z_2}\right)^2=\frac{\alpha z_2}{\beta z_1}\left[1+\delta\left(\frac{X_0}{z_1+z_2}\right)^2\right] \tag{7}$$

But $\frac{\alpha z_2}{\beta z_1} \sim 1$ from the isotropic case. So:

$$\frac{z_1}{z_2} \sim \frac{\alpha}{\beta}\left[1+(2\delta-\varepsilon)\left[\frac{X_0}{z_1(1+\beta/\alpha)}\right]^2\right]. \tag{8}$$

We can see that equation (8) is a perturbation of the isotropic case in equation (5).

Equation (8) gives an approximate, asymptotic position for the trace conversion point in a VTI medium. When 2δ-ε is positive then the conversion point is shifted toward the receiver even farther. If 2δ-ε negative then the conversion point is moved toward the shot position (or mid-point).

Let's develop a rapid mapping of a recorded trace using the above analysis. From Figure 3 and similar triangles, we see that

$$x = \frac{z_2 X_0}{z_1 + z_2} = \frac{\frac{z_2}{z_1} X_0}{1 + \frac{z_2}{z_1}} \tag{9}$$

where $\frac{z_2}{z_1} \sim \frac{\beta}{\alpha}\left[1 + (\varepsilon - 2\delta)\left[\frac{X_0}{z_1(1+\beta/\alpha)}\right]^2\right]$

But we take

$$z_1 = \frac{\alpha t_{pp}}{2}, \tag{10}$$

where t_{pp} is the two-way, normal-incidence, *P*-wave time to the reflector at depth z_1. This maps an NMO-corrected trace at time t_{pp} into its offset conversion position at location x.

CONCLUSIONS

Derived here is a simple technique, based on the method of images, of constructing the P-S conversion point position in a dipping reflector for small shot-receiver separations. The conversion point is shifted toward the receiver, with the amount depending on the α/β value. In the vertical transverse isotropy case, the conversion point is further perturbed from this position toward the receiver (if 2δ-ε is positive) or toward the source when 2δ-ε is negative.

REFERENCES

Thomsen, L. 1986, Weak elastic anisotropy: Geophysics, 51, 1954-1956.