Prestack v(z) f-k migration for VTI media

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ABSTRACT

Transverse isotropy with a vertical symmetry axis (VTI) can be implemented in a prestack migration algorithm that handles the vertical velocity variation. An extension of the prestack v(z) f-k migration algorithm in CDP-offset domain is formulated into a migration scheme that considers the VTI properties of the subsurface materials. By proper tabulation of related parameters, the algorithm is essentially not much more expensive than conventional prestack phase-shift migration methods. Experimentation shows that the migration algorithm produces theoretically expected results.

INTRODUCTION

Li and Margrave (1998, 1999) presented examples of the depth migration results by the prestack v(z) f-k method using Blackfoot multi-component data. The results from vertical component data show some contradictions that are difficult to explain without introducing the angle-dependence of the migration velocity, which leads to the consideration of the anisotropic properties of the formations in this area. It was indicated in Li and Margrave (1998) that, when the image of the target sand-channel is focused, the target depth is about 200 meters deeper than its geological depth. Further experiments also showed that, by adjusting the migration velocity to ensure the correct target depth, the image became blurred, and the sand-channel was not clearly recognizable. Even the velocity obtained from well log information could not create satisfactory images with correct depth information.

These contradictions imply that, even for almost laterally homogeneous media such as in the Blackfoot area, southern Canada, it is difficult to find a proper velocity field to convert the time sections into depth sections.

For areas with mainly vertical velocity variation such as sedimentary basins, the theory of transverse isotropy with vertical symmetry axis (VTI) may provide a better description of the wave propagation. The difference between the vertical velocity and the horizontal velocity suggests why the prestack migration focussing velocity (and the stacking velocity) can not be used for time-to-depth conversion.

This paper formulates a prestack migration algorithm for VTI media based on the v(z) f-k migration method presented by Margrave (1998b) and Li and Margrave (1998, 1999). Details of the computer implementation are also presented.

FORMULAE FOR ANISOTROPIC VELOCITIES IN VTI MEDIA

Thomsen (1986) expressed the TI property by five parameters, called Thomsen's parameters. These parameters are more interpretable in terms of applied exploration seismology because they are easier to relate to seismic measurements. The parameters are usually denoted as α_0, β_0, ε, γ, and δ^*, where α_0 and β_0 are the sound speeds for

P- and S-waves in the symmetric axis direction. The parameters ε and γ can be understood as quantitative indicators of the "strength" or "weakness" of the anisotropy because when ε and γ tend to zero the materials tend to be isotropic. From Thomsen (1986), the angle dependent P-wave and Sv-wave velocities can be expressed as functions of these parameters:

$$v_P^2(\theta) = \alpha_0^2 \left[1 + \varepsilon \cdot \sin^2\theta + D^*(\theta)\right] \tag{1a}$$

$$v_{SV}^2(\theta) = \beta_0^2 \left[1 + \frac{\alpha_0^2}{\beta_0^2}\varepsilon \cdot \sin^2\theta - \frac{\alpha_0^2}{\beta_0^2} D^*(\theta)\right]; \tag{1b}$$

where

$$D^*(\theta) \equiv \frac{1}{2}\left(1 - \frac{\beta_0^2}{\alpha_0^2}\right)\left\{\left[1 + \frac{4\delta^*}{\left(1 - \beta_0^2/\alpha_0^2\right)^2}\sin^2\theta \cdot \cos^2\theta + \frac{4\left(1 - \beta_0^2/\alpha_0^2 + \varepsilon\right)\varepsilon}{\left(1 - \beta_0^2/\alpha_0^2\right)^2}\sin^4\theta\right]^{\frac{1}{2}} - 1\right\}. \tag{2}$$

The angle θ is the angle between the wave propagation direction and the symmetric axis. In VTI cases, it is the same as the incident angle of the wave propagation in each layer. In above equations (1a), (1b) and (2), only four parameters, α_0, β_0, ε, and δ^*, are involved. The fifth parameter γ is only involved with SH-wave velocities.

In the migration scheme presented in this paper, the four parameters, α_0, β_0, ε, and δ^*, are considered as known quantities that are allowed to change arbitrarily in depth.

V(Z) F-K FORMULATION FOR VTI MEDIA

For the prestack v(z) f-k migration, the direct wavefield extrapolation from zero depth to any depth z without considering anisotropy (velocities are independent to incident angles) can be written as (Li and Margrave, 1998, 1999)

$$\Psi(x,h,z,t) = \iiint \psi(k_x,k_h,z,\omega) exp\left(i\left(\omega t - k_x x - k_h h\right)\right) dk_x dk_h d\omega, \tag{3}$$

and

$$\psi(k_x,k_h,z,\omega) = \psi(k_x,k_h,0,\omega) exp\left(-i\int_0^z \left\{\sqrt{\frac{\omega^2}{v_S^2(z')} - \left(\frac{k_x - k_h}{2}\right)^2} + \sqrt{\frac{\omega^2}{v_R^2(z')} - \left(\frac{k_x + k_h}{2}\right)^2}\right\} dz'\right), \tag{4}$$

where the source-side velocity v_S and receiver-side velocity v_R only change with depth. Equation (4) expresses the process of wavefield extrapolation, the imaging process is to obtain the wavefield expressed as in equation (3) at $t=0$ and $h=0$.

The phase term in equation (4) is an integral over depth from zero depth (the surface) down to target level z, and it is seen that these phase terms are the only places where velocity information is involved. Therefore, the difficulties introduced by anisotropic property, the velocity angle-dependence, may just involve different ways to compute the phase terms. For VTI media, the phase term can be simply written as

$$\int_0^z \left\{ \sqrt{\frac{\omega^2}{v_S^2(z',\theta_S)} - \left(\frac{k_x - k_h}{2}\right)^2} + \sqrt{\frac{\omega^2}{v_R^2(z',\theta_R)} - \left(\frac{k_x + k_h}{2}\right)^2} \right\} dz' , \tag{5}$$

where, in each depth layer, θ_S and θ_R represent the incident angles in the source side and the receiver side.

The incident angles, the spatial wavenumbers of k_x, k_h, and the temporal frequency ω are related through their common relations with horizontal slownesses (or ray parameters). It is these relationships amongst these variables that build up the prestack v(z) f-k migration scheme for VTI media.

Because Fourier transform is directly related to wavefield plane-wave decomposition, ray parameters, p_S and p_R, can be expressed as

$$p_S(k_x, k_h, \omega) = \frac{k_x - k_h}{2\omega} \tag{6a}$$

and

$$p_R(k_x, k_h, \omega) = \frac{k_x + k_h}{2\omega} . \tag{6b}$$

Therefore, the phase term in equation (5) can be re-expressed as

$$\omega \int_0^z \left\{ \sqrt{\frac{1}{v_S^2(z',\theta_S)} - p_S^2(k_x, k_h, \omega)} + \sqrt{\frac{1}{v_R^2(z',\theta_R)} - p_R^2(k_x, k_h, \omega)} \right\} dz' . \tag{7}$$

The relation between horizontal slowness and the incident angle is based on Snell's law, i.e.,

$$p_S(\theta_S) = \frac{\sin\theta_S}{v_S(\theta_S)} \tag{8a}$$

and

$$p_R(\theta_R) = \frac{\sin\theta_R}{v_R(\theta_R)} . \tag{8b}$$

By also introducing the vertical slownesses q_S and q_R as,

$$q_S(z',k_x,k_h,\omega) = q_S(z',\theta_S) = \frac{\cos\theta_S}{v_S(z',\theta_S)}, \tag{9a}$$

and

$$q_R(z',k_x,k_h,\omega) = q_R(z',\theta_R) = \frac{\cos\theta_R}{v_R(z',\theta_R)}, \tag{9b}$$

The phase term can be finally expressed in a very simple form,

$$\omega\int_0^z \{q_S(z',k_x,k_h,\omega) + q_R(z',k_x,k_h,\omega)\}dz' . \tag{10}$$

The computation of the phase term in equation (10) is the main task of the migration scheme. If the phase terms are computed, the imaging process with wavefield extrapolation is a typical nonstationary filtering process in the mixed domain (Margrave, 1998a), which maps the spectrum of the unmigrated data into migrated data spectrum.

The computation of the phase terms in equation (10) may be done as follows:

Step 1: for each depth sample, z', compute p_S and p_R using (6a) and (6b) as the wavenumbers k_x, k_h and the frequency ω are known through a 3D Fourier transform of the prestack data volume.

Step 2: using equations (8a) and (8b), find θ_S and θ_R from the p_S and p_R values computed in Step1.

Step 3: compute anisotropy velocities v_S and v_R using equations (1a), (1b) and (2) at present z' and θ_S and θ_R.

Step 4: compute vertical slownesses q_S and q_R by equations (9a) and (9b).

Step 5: sum the two slownesses and integrate from $z'=0$ through $z'=z$ (summations in practice) as expressed by equation (10).

PRACTICAL COMPUTER IMPLEMENTATION

The above five-step scheme shows clearly how the velocity angle-dependence is accommodated in the phase-term computation in the migration process. However, this scheme is practically very inefficient, as many computations in the scheme can be very redundant through the main loops (over z', k_x, k_h and ω) of the v(z) f-k migration procedure. The following pre-loop tabulations of key variables can significantly increase the efficiency of the migration algorithm.

Angle-dependent velocity tables

For each sampled depth level z' and each pair of incident angles θ_S and θ_R, 2-D velocity tables $v_S(z', \theta_S)$ and $v_R(z', \theta_R)$ can be built using equations (1a), (1b), and (2).

When the velocity values at specific θ_S and θ_R are needed, simple interpolations in the velocity tables will be much more efficient, because the very complicated computations involved with equations (1a), (1b), and (2) are removed from the main computation loop over the Fourier transformed traces.

Angle tables versus horizontal slownesses

At each sampled depth level z', and sampled θ_S and θ_R, a series of values of the horizontal slownesses, p_S and p_R, can be computed using Snell's law (eqautions (8a) and (8b)). Therefore, by polynomial fitting algorithm (Ferguson and Margrave, 1997) (or other preferred fitting method), the θ-to-p function can be inverted into p-to-θ functions. That is, the values of the angles θ_S and θ_R can be computed from given values of p_S and p_R, which are practically computed by the wavenumbers k_x, k_h, and the temporal frequency ω. The resultant polynomial coefficients form 2D tables for all depth levels.

Integrated vertical slowness tables

For sampled horizontal slownesses p_S and p_R, and each depth level z, phase-term tables can be built using equation (9a), (9b), and (10) as follows (note the difference between z and z'),

- At each depth level z', for each pair of sampled p_S and p_R, use the coefficient tables to compute θ_S and θ_R, and use the velocity tables to calculate values of q_S and q_R, which form 2D tables at values of z' and p_S *or* p_R.
- For each depth level z, and each pair of p_S and p_R, integrate the q_S and q_R table from $z'=0$ to $z'=z$. Then, 2D tables of q_S-integral and q_R-integral are built and they automatically take the memory for q_S and q_R tables built in the previous step.

After building these tables, the actual migration process can be expressed as following:

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Fourier transform in x, h, and t direction, into kx, kh, and ω domain
Loop over kx
    Loop over kh
        Loop over ω
            Get pS and pR using equations (7a) and (7b)
            Loop over depth level z
                Get q-integrals from tables with known z, pS, pR, and ω
                Get the migration filter
            End loop z
        End loop ω
        Apply the migration filter as in equation (4) (including condition t=0)
        Sum over kh (imaging condition h=0)
    End loop kh
End loop kx
Inverse Fourier transform over kx at each depth level z.
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INITIAL RESULTS

The prestack v(z) f-k migration for VTI media is implemented in ProMAX 2D. Although the implementation is prepared for migration of PP and PS seismic data, the results shown here are from the migration of synthetic PP data and Blackfoot vertical component 2D data. More experiments are planed in the future work.

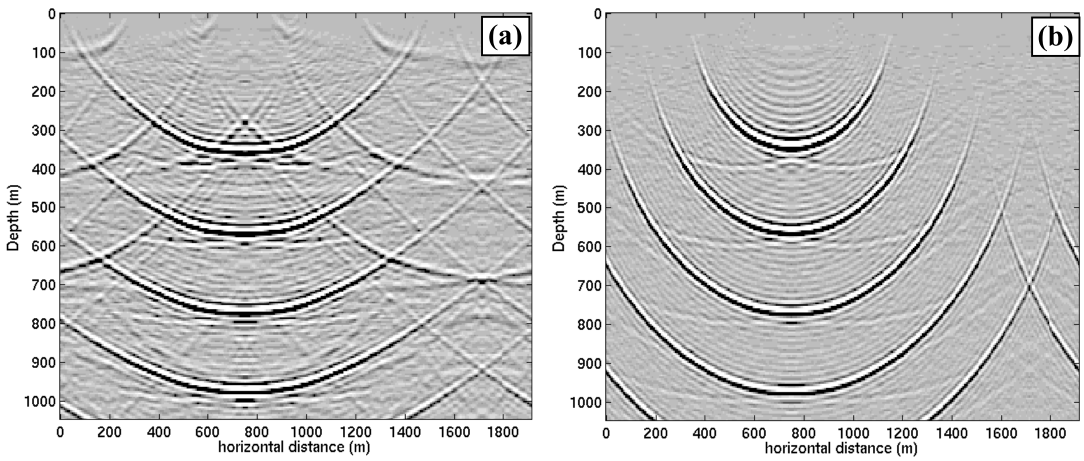

Figure 1: Migration impulse responses from v(z) f-k migration for VTI media, where (a) with non-zero anisotropic parameters and (b) with zero anisotropic parameters.

Figure 1 shows some impulse responses from the v(z) f-k migration for VTI media. The only non-zero input trace contains five impulses. The anisotropic parameters were taken from the parameter list in Thomsen (1986) for Gypsum, and are: the P-wave velocity α_0=1991 m/s, the Sv-wave velocity β_0,=795 m/s, ε=1.161 and δ^*=1.075. Figure 1(a) shows the migration result of this input trace, which contains five impulse responses. For comparison, the same input trace was also migrated with the same P-wave and S-wave velocities, but the parameters, ε and δ^*, were set to zero. The result is shown in Figure 1(b). As expected, the impulse responses in Figure 1(b) are the same as the responses from PP migration process without anisotropic properties considered.

The migration result with anisotropy included (Figure 1(a)) contains stronger artifacts than the one without anisotropy (Figure 1(b)). This is mainly because the VTI migration responses have larger migration aperture and wrap-around effects in time and offset directions. In this respect, the migration scheme for VTI media may require more considerations to attenuate wrap-around effects.

To demonstrate that the responses shown in Figure 1 are reasonable, Figure 2 shows the superposition of the migration results shown in Figure 1(a) and Figure 1(b). It can been seen that the responses with and without anisotropy coincide only at one location which corresponding to the responses for vertical incident waves in which the waves propagate with the sound speed. At non-zero incident angles, migration responses with anisotropy correspond to larger velocity values, which is theoretically expected for gypsum.

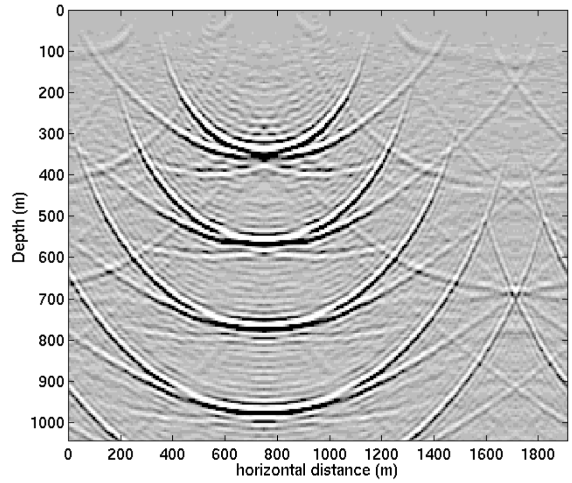

Figure 2: Superposition of the two migration results shown in Figure 1.

As mentioned in the beginning of this paper, highly focused images from Blackfoot vertical component data resulted in a target depth about 200 meters deeper than the geological depth. With migration for VTI media, it is possible to obtain good image and the correct depth information. Figure 3 (a) shows a migration image from the v(z) f-k migration with VTI property considered. The required parameters were given as:

- Vertically variant P-wave sound speed used was 90% of the P-wave depth-interval velocity converted from best stacking velocity;
- Vertically variant S-wave sound speed used was the S-wave velocity obtained from well logging information;
- The anisotropic parameters, ε and δ^* were 0.189 and 0.154 respectively, which were taken from the parameters for one kind of clayshales listed in Thomsen (1986).

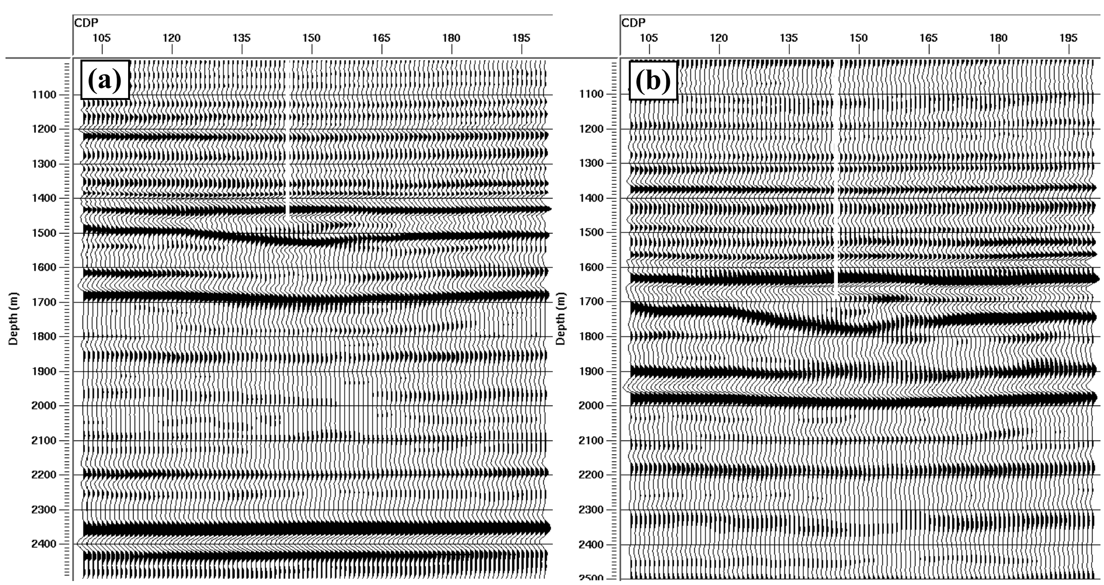

Figure 3: Migration images for Blackfoot vertical component data from (a) VTI v(z) f-k algorithm and (b) isotropic v(z) f-k algorithm. The white vertical lines indicate the well location.

As expected, the recognizable sand-channel is much closer to its geological depth (about 50 meters too shallow in Figure 1(a)). Surprisingly, the image quality is quite comparable to the image obtained using the best focusing velocity through the isotropic migration algorithm. We make no claim to have used the correct Thomsen parameters for the stratigraphy at Blackfoot. This subject is still under investigation.

CONCLUSION AND FUTURE WORK

The Prestack v(z) f-k migration algorithm can be extended to handle migration for VTI media. The computer implementation can be very efficient by proper tabulation of the related velocity, horizontal and vertical slownesses. The cost of the current implementation of the VTI v(z) f-k algorithm is similar to the prestack v(z) f-k algorithm without considering anisotropy. Promising results have been obtained from the experiments with the Blackfoot vertical component data.

The practical application of the VTI migration algorithm usually requires more detailed subsurface information. Apart from the conventional requirement of P-wave and/or S-wave velocity information, the other two Thomsen's parameters may need more research and experiments in the area where the data is acquired.

The Blackfoot area is one of the areas where lateral velocity variation can be approximately ignored, and the subsurface materials could be considered as VTI media. More detailed formation analysis for the Blackfoot area will, hopefully, provide high quality images with correct depth information.

The converted wave migration algorithm is, theoretically, not more difficult than the pure-mode data migration algorithm, even with VTI media assumption. In practice, obtaining a reasonable image from converted wave data is not an easy task, and it may involve more experiments and more interpretation of the subsurface lithological information as well.

ACKNOWLEDGMENTS

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