Suppression of multiples by wavefield separation techniques

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ABSTRACT

A major part of the multiple energy in seismograms is caused by reflections from the free surface, which often contaminate the primary reflections that propagate from the subsurface as upgoing waves. Since such energy come down from the ocean surface and is contained in the downgoing wavefield, it can naturally be suppressed by attenuating the downgoing wavefield, provided the wavefield can successfully be separated into the upgoing and downgoing parts. Generally, wavefield separation is performed on multicomponent ocean-bottom seismic (OBS) data. However, as a simplification of this method, this decomposition also can be realized on dual-sensor OBS data. In this report, the wavefield separation techniques on multicomponent or dual-sensor data are discussed in detail and the algorithms for attenuating the downgoing wavefield are presented.

INTRODUCTION

Seismic events in OBS data can consist of direct waves, primary reflections, source-side ghost, water-column reverberations, free-surface multiples and internal multiples. According to the difference in wave arrival directions, the recorded wavefields in OBS data can be grouped into downgoing and upgoing wavefields. The downgoing wavefield contains the direct wave, receiver-side surface multiples and water-column reverberations, while the up-going wavefield contains all the primaries and source-side ghosts and internal multiples. Since the primary reflections are contained only in the upgoing wavefield, it is naturally to consider wavefield separation, i.e. decomposing wavefield into the upgoing and downgoing wavefields.

Many researchers have worked on the wavefield separation technique, e.g. Amundsen and Reitan (1995), Osen et al. (1999), and Schalkwijk et al. (1999). The procedure of this method in essence is to combine the pressure, the horizontal and vertical velocity components in proper proportions to obtain the upgoing and downgoing *P*- and *S*-wavefields. This method can easily be implemented in a layered earth model and is quite a common application in OBS data processing.

Generally, the wavefield separation involves multicomponent data, because only with all of these components can the wavefield be completely described. However, in practice, not all components are always recorded, and it can be shown that the pressure and vertical velocity components can represent the wavefield satisfactorily. Therefore, we can also realize the wavefield separation using only these two components. Based on this principle, the dual-sensor method was introduced into the OBS survey. The dual-sensor method (Barr and Sanders, 1989; Bale, 1998; Dragoset and Barr, 1994; Barr et al., 1997; Ball, 1996;) is based on the fact that hydrophones and vertical component geophones record signals with the same polarity for upgoing waves, but with the opposite polarity for downgoing waves. So, with proper scaling, the summation of the hydrophones and vertical geophone data can attenuate the downgoing waves.

Based on inverse-scattering theory, Weglein et al. (1997) introduced a multiple suppression method by expanding the wavefield into a Born series. Each term of this series corresponds to a particular scattering path. If the terms corresponding to multiples in forward series can be characterized, described and distinguished from the terms corresponding to the primaries, then suppression of multiples can be realized in inverse series by removing those terms corresponding to multiples. However, the validation of this basic assumption needs to be further proved. Using a similar technique, Berkhout (1982) presented the feedback-loop approach. Although inverse-scattering series can attenuate the free-surface and internal multiples theoretically, this method has some problems: it requires the source signature and is computationally time-consuming. Another problem is the convergence of the series expansion, which assumes the weak scattering media. For strong scattering media, applying this method seems need more theoretical research.

In this paper, we are going to analyze the multicomponent wavefield separation method for multiple attenuation and describe how this method can be simply realized on dual-sensor data. Numerical examples are provided to demonstrate the performance of wavefield-separation techniques.

REVIEW OF METHODS

The wave equation can be expressed in the form of a first-order ordinary differential equation in stress and velocity (Aki and Richards, 1980):

$$\frac{d}{dz}\mathbf{B} = -i\omega\mathbf{A}\mathbf{B}, \tag{1}$$

where, **B** is the vector that contains the stress and velocity variables across a plane elastic/elastic interface, and

$$\mathbf{B} = (V_3, S_1, S_2, S_3, V_1, V_2)^T, \tag{2}$$

where, V_i are transformed components of particle velocity. V_1 and V_2 are horizontal particle velocity, V_3 is the vertical particle velocity. S_3 is the normal component of the traction in the solid, and S_1, S_2 are the shear component of the traction in the solid. z is depth, positive downward, ω is angular frequency. **A** is the elastic-system matrix defined as

$$\mathbf{A} = \begin{bmatrix} 0 & \mathbf{A_1} \\ \mathbf{A_2} & 0 \end{bmatrix}, \tag{3}$$

where

$$\mathbf{A_1} = \begin{bmatrix} \frac{1}{\lambda+2\mu} & \frac{\lambda p_1}{\lambda+2\mu} & \frac{\lambda p_2}{\lambda+2\mu} \\ \frac{\lambda p_1}{\lambda+2\mu} & \rho - \frac{4\mu(\lambda+\mu)p_1^2}{\lambda+2\mu} - \mu p_2^2 & -\frac{\mu(3\lambda+2\mu)p_1 p_2}{\lambda+2\mu} \\ \frac{\lambda p_2}{\lambda+2\mu} & -\frac{\mu(3\lambda+2\mu)p_1 p_2}{\lambda+2\mu} & \rho - \frac{4\mu(\lambda+\mu)p_1^2}{\lambda+2\mu} - \mu p_1^2 \end{bmatrix}, \tag{4}$$

and

$$\mathbf{A_2} = \begin{bmatrix} \rho & p_1 & p_2 \\ p_1 & \mu^{-1} & 0 \\ p_2 & 0 & \mu^{-1} \end{bmatrix}, \tag{5}$$

where λ and μ are the lame′ coefficients, p_1 and p_2 are horizontal slowness, i.e. $p_1 = k_1/\omega$ and $p_2 = k_2/\omega$, the p_1 and p_2 have to satisfy the relation:

$$p^2 = p_1^2 + p_2^2. \tag{6}$$

The wavefield separation can be obtained by eigen decomposition of the matrix **A**, i.e. $\mathbf{A} = \mathbf{L}^{-1}\,\mathbf{\Lambda}\,\mathbf{L}$, where **L** is the matrix composed of eigenvectors of matrix **A** and **Λ** is the diagonal matrix composed of the eigenvalues of **A**. Then, equation (1) can be written as

$$\frac{d}{dz}\mathbf{B} = -i\omega\mathbf{L}^{-1}\mathbf{\Lambda}\mathbf{L}\mathbf{B}. \tag{7}$$

It can be shown that **Λ** can be written as:

$$\mathbf{\Lambda} = diag\left(q_P, q_{SV}, q_{SH}, -q_P, -q_{SV}, -q_{SH}\right). \tag{8}$$

The physical meaning of equation (8) is that eigenvalues q_P, q_{SV}, q_{SH} correspond to the upgoing waves, whereas eigenvalues $-q_P$, $-q_{SV}$, $-q_{SH}$ correspond to the downgoing waves. Therefore, equation (7) can be further decomposed into two equations that correspond to up- and downgoing waves, respectively, i.e.

$$\frac{d}{dz}\mathbf{B}^U = -i\omega\mathbf{L}^{-1}\mathbf{\Lambda_1}\mathbf{L}\mathbf{B}$$

and

$$\frac{d}{dz}\mathbf{B}^D = -i\omega\mathbf{L}^{-1}\mathbf{\Lambda_2}\mathbf{L}\mathbf{B}, \tag{9}$$

where the superscripts U and D indicate upgoing and downgoing.

In order to solve equation (9), a boundary condition is needed. According to Aki and Richards (1980), at the boundary between two solid media in welded contact, the components of particle velocity (or displacement) and traction are continuous over the boundary. However, across the boundary between an inviscid fluid and a solid (e.g. the ocean-bottom), only the vertical component of the particle-velocity is continuous, the horizontal components of particle velocity can be discontinuous, implying that slip can occur parallel to the boundary. Further, the pressure in the fluid is equal to the negative of the vertical component of the traction in the solid, while the horizontal components of the traction in the solid vanish at the interface. So, at the sea-floor, $z = z_1 + \varepsilon, \varepsilon \to 0$, we have

$$S_1(z_1^+) = S_1(z_1^-) = 0, \quad S_2(z_1^+) = S_2(z_1^-) = 0,$$
$$S_3(z_1^+) = -W(z_1^-), \quad V_3(z_1^+) = V_3(z_1^-), \tag{10}$$

where, z_1^- denotes a depth level just above the sea-floor, and z_1^+ denotes a depth level just below the sea-floor.

Moreover, in the system of equations (1), all the types of wave (P, SV and SH) are included. We can obtain the plane P, *S*V waves when $S_2 = V_2 = 0$ and $p_2 = 0$, and obtain SH waves when $S_1 = S_3 = V_1 = V_3 = 0$ and $p_1 = 0$. We can also obtain the plane P waves when $S_1 = S_2 = V_1 = V_2 = 0$ and $p_1 = p_2 = 0$ (Gilbert and Backus, 1966). Therefore, the upgoing and downgoing wavefield for P, SV and SH just below the seafloor can be obtained correspondingly from equation (9). Following the derivation given by Amundsen and Reitan (1995), we have:

$$U_p(z_1^+) = \left(\frac{\rho(1-2p^2\beta^2)}{q_\alpha} V_3(z_1^+) + S_3(z_1^+) - 2\rho\beta^2 (p_1V_1(z_1^+) + p_2V_2(z_1^+))\right) / \left(\frac{\rho}{q_\varepsilon}\right)^{1/2},$$

$$D_p(z_1^+) = \left(-\frac{\rho(1-2p^2\beta^2)}{q_\alpha} V_3(z_1^+) + S_3(z_1^+) - 2\rho\beta^2 (p_1V_1(z_1^+) + p_2V_2(z_1^+))\right) / \left(\frac{\rho}{q_\varepsilon}\right)^{1/2},$$

$$U_{sv}(z_1^+) = \left(2\mu(\rho q_\beta)^{1/2} V_3(z_1^+) + pS_3(z_1^+) + \frac{\mu(q_\beta^2 - P^2)}{p}(p_1V_1(z_1^+) + p_2V_2(z_1^+))\right) / (\rho q_\beta)^{1/2},$$

$$D_{sv}(z_1^+) = \left(-2\mu(\rho q_\beta)^{1/2} V_3(z_1^+) + pS_3(z_1^+) + \frac{\mu(q_\beta^2 - P^2)}{p}(p_1V_1(z_1^+) + p_2V_2(z_1^+))\right) / (\rho q_\beta)^{1/2},$$

$$U_{SH}(z_1^+) = p_2V_1(z_1^+) - p_1V_2(z_1^+),$$

$$D_{SH}(z_1^+) = p_2V_1(z_1^+) - p_1V_2(z_1^+), \tag{11}$$

where, U is the upgoing wavefield, D is the downgoing wavefield and the subscript indicates the type of wave. α and β are P-wave and S-wave velocities in the solid and the vertical ray parameters are:

$$q_\alpha = \sqrt{(\alpha^{-2} - p^2)}, \quad q_\beta = \sqrt{(\beta^{-2} - p^2)}. \tag{12}$$

In equation (11), both the upgoing and downgoing wavefields are functions of the vertical traction component, horizontal particle-velocity component and vertical particle-velocity component. In practice, not all components are always recorded or processed. In OBS data, for example, we sometimes process only the hydrophone and vertical-component geophone data. For more practical application, Osen et al. (1999) extended the formulae above into the following form.

For the hydrophone (W), we have:

$$U^W(z_1^+) = \frac{1}{2}\{W(z_1^-) - \frac{\rho}{q_\alpha}[(1-2p^2\beta^2)^2 + 4p^2\beta^4 q_\alpha q_\beta]V_3(z_1^+)\}, \tag{13}$$

where, $U^W(z_1^+)$ is the upgoing pressure wavefield just below the seafloor, $W(z_1^-)$ is the pressure just above seafloor, and $V_3(z_1^+)$ is the vertical velocity component just below the seafloor. The superscript indicates the component type. Note that only pressure and vertical velocity components are involved in equation (13), which means that the upgoing wavefield can be obtained by combining only the pressure component with the particle velocity-component in such proportions.

For horizontal velocity components, we have:

$$\begin{aligned} U^{V_1}(z_1^+) &= \frac{1}{2}\{V_1(z_1^+) - \frac{p_1}{q_\alpha}[1-2\beta^2(p^2 + q_\alpha q_\beta)]V_3(z_1^+)\}, \\ U^{V_2}(z_1^+) &= \frac{1}{2}\{V_2(z_1^+) - \frac{p_2}{q_\alpha}[1-2\beta^2(p^2 + q_\alpha q_\beta)]V_3(z_1^+)\}, \end{aligned} \tag{14}$$

where, $U^{V_1}(z_1^+)$ and $U^{V_2}(z_1^+)$ are the upgoing wavefields just below the seafloor for the horizontal velocity components, and $V_3(z_1^+)$ is the vertical velocity component just below the seafloor. This equation states that the upgoing wavefield for horizontal particle velocity component can be obtained by combining the horizontal particle velocity component and the scaled vertical particle velocity component in these proportions.

For the vertical velocity components, we have:

$$\begin{aligned} U^{V_3}(z_1^+) = \frac{1}{2}\{V_3(z_1^+) + \frac{p_1}{q_\beta}[1-2\beta^2(p^2 + q_\alpha q_\beta)]V_1(z_1^+) - \\ \frac{p_2}{q_\beta}[1-2\beta^2(p^2 + q_\alpha q_\beta)]V_2(z_1^+) - \frac{1}{\rho q_\beta}(p^2 + q_\alpha q_\beta)W(z_1^-)\}, \end{aligned} \tag{15}$$

where, $U^{V_3}(z_1^+)$ is the upgoing wavefield just below the seafloor for the vertical velocity component. This equation shows that the upgoing wavefield for the vertical particle velocity component can be obtained by combining the vertical velocity, the scaled horizontal velocity component and the scaled vertical particle velocity components.

From equation (13), we can see that the pressure wavefield and the vertical velocity wavefield can be represented by each other. We can also see that the upgoing wavefield for pressure can be obtained just by combining the pressure and scaled vertical praticle velocity components. This demultiple scheme just uses a scaling relationship between the two components. This principle is the basic idea of the dual-sensor method. Therefore, the dual-sensor method can be seen as a special example of the wavefield decomposition method.

We can further simplify the decomposition (equation (13)), when the medium is acoustic, i.e., where no S-waves propagate. Then the upgoing and downgoing pressure wavefields $U^W(z_1^-)$ and $D^W(z_1^-)$ can be computed by:

$$\begin{aligned} U^W(z_1^-) &= \frac{1}{2}\{W(z_1^-) - \frac{\rho_1}{q_{\alpha 1}} V_3(z_1^+)\}, \\ D^W(z_1^-) &= W(z_1^-) - U^W(z_1^-), \end{aligned} \tag{16}$$

where ρ_1 is the density of the acoustic medium, q_{α_1} is vertical slowness in the acoustic medium. $q_{\alpha_1} = \sqrt{c^{-2} - p^2)}$, and c is the velocity in the acoustic medium.

Moveover, in equation (16), if a wave is incident vertically, the ray parameter p is zero; then the upgoing wavefield can be obtained by

$$U^W(z_1^-) = \frac{1}{2}\{W(z_1^-) - \frac{1}{\rho_1 c} V_3(z_1^+)\}. \tag{17}$$

Equation (17) is derived in the frequency-ray parameter (ω, p) domain. However, since the scaling factor now is a constant, this equation can also be applied in the space-time (x, t) domain. The problem now becomes how to design the scaling factor between the hydrophone and geophone components.

There are several methods for calculating the scaling factor; the following cross-correlation method is one of them. This method uses the cross-correlation between the records of hydrophone and geophone components to define the scaling factor.

We know that the hydrophone component data (W_i) and vertical velocity component data (Z_i) have the same polarity for upgoing waves, but the opposite polarity for downgoing waves. So, if we calculate the values of this expression:

$$\psi_{ZW}(j) = \frac{\sum_{i=1}^{L} Z_{i+j} W_i}{\sqrt{\sum_{i=1}^{L} Z_{i+j}^2 \sum_{i=1}^{L} W_i^2}}. \tag{18}$$

where, $\psi_{ZW}(j)$ is the cross-correlation of hydrophone data and geophone data in each window and L is the length of window. For downgoing waves, their polarities are opposite. If they are exactly phase-matched, the value of $\psi_{ZW}(j)$ should be –1. For upgoing waves, their polarities are same, so if they are exactly phase-matched, the value of $\psi_{ZW}(j)$ should be 1. If hydrophone data and geophone data are not phase matched, the values of $\psi_{ZW}(j)$ are between –1 and 1. So, according to the cross-correlation values, we can choose a threshold to determine the value of the scaling factor, F, for which $F \in \{-1, 1\}$ in each window.Then, we use this formula:

$$U^V = F \cdot V, \quad U^W = F \cdot W \tag{19}$$

to obtain the upgoing wavefield. This method doesn't require that the hydrophone data and geophone data are exactly phase-matched.

NUMERICAL EXAMPLES

To test the performance of these wave separation techniques, we use synthetic seismograms modelled in a plane-layered medium. The model is a 2-D model with a 500 m water layer and two further reflectors at depths of 1000 m and 1500 m. The velocities of P waves corresponding to each layer are 1500 m/s, 2100 m/s, and 2500 m/s, respectively. Both pressure (Figure1) and vertical velocity synthetic data are generated by OSIRIS Precise Seismic Modeling software. Note that primaries are present for events at approximately 0.81 and 1.23 s, they arrive with same polarities and are reinforced after application of multi-component wave-decomposition methods. The downgoing direct arrival and reverberations are present for events at approximately 0.33, 1.01 and 1.67 s, respectively. They arrive with opposite polarities and are attenuated. Also notice that multiples associated with primary reflections and arriving from both above and below, at approximately 1.5 and 1.9 s, are attenuated after the application of this method (Figure 2).

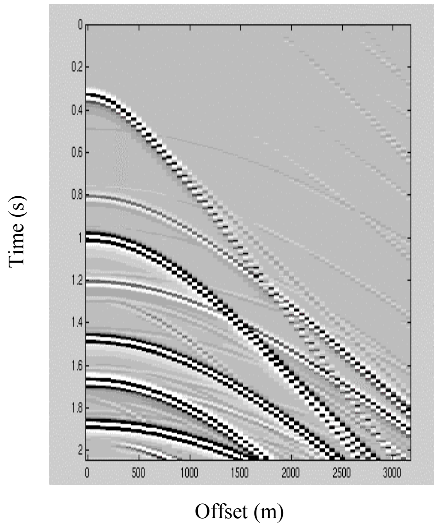


Figure1. Modelled hydrophone data.

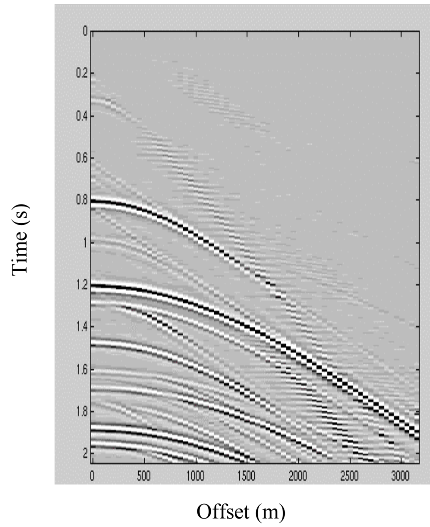


Figure 2. Decomposed upgoing wavefield.

For comparison, we also apply the dual-sensor wave separation method described in equation (18) to this model data (Figure 3). We can see that the method works very well when primary and multiple arrive separately, but when the primary and multiple intersect, it eliminates both primary and multiple.

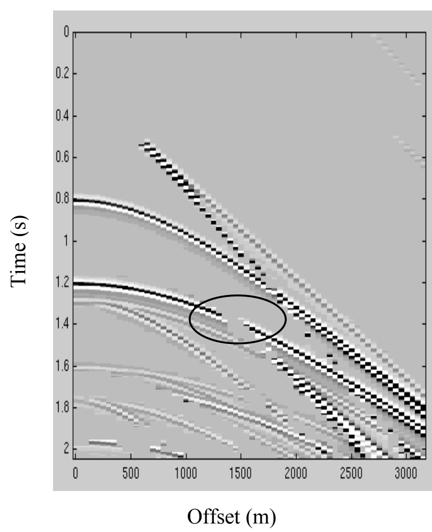
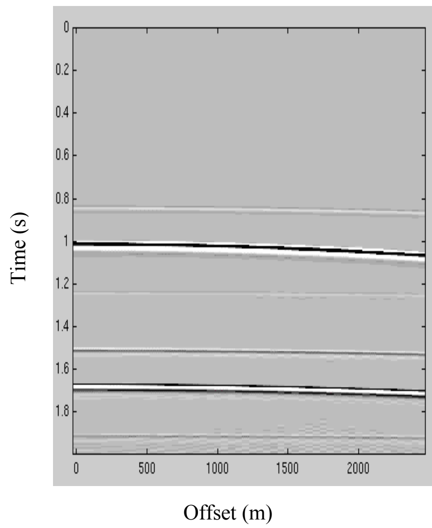


Figure 3: Upgoing wavefield by dual-sensor method.

To further show how the dual-sensor wave separation method works, we make some modification to the above model and obtain a new model result (Figure 4). Primaries are still present for events at approximately 0.81 and 1.23 s, the downgoing direct arrival and reverberations are present at approximately 0.33 s, 1.01s and 1.67 s, and the multiples, arriving from both above and below, are present at approximately

1.5 and 1.9 s. Now, the primaries and multiples arrive at different times and we see this method works very well (Figure 5).



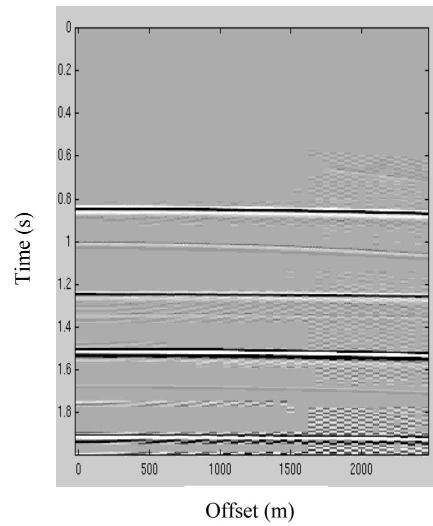


Figure 4. Modelled hydrophone data.

Figure 5. Upgoing wavefield by dual-sensor method.

DISCUSSION

The suppression of multiples can be realized by wavefield-separation techniques. The wavefield separation technique, in essence, combines the pressure, horizontal and vertical velocity components in proper proportions to gain the upgoing wavefield. Numerical examples show this method works well. Generally, the wavefield separation involves multicomponent OBS data. However, sometimes not all components are recorded, and then the pressure and vertical component of velocity can represent the wavefield. Therefore, using only these two data components, we can also realize the wavefield separation. Based on this principle, the dual-sensor method was introduced into the OBS survey. The dual-sensor method described in equation (18) is a more practical method, but as shown in numerical examples, this method has certain limitations: it cannot work well when primary and multiple intersect. However, it performs well when primaries and multiples arrive at different times. In any case, this method can be used together with other methods as a supplement.

The wave-separation method is computationally fast and doesn't require the source signature. But it only attenuates the downgoing waves and cannot totally remove the effect of the upgoing multiples, i.e. source-side and internal multiples. Current wavefield-decomposition techniques have implicitly been based on assumptions of a horizontally flat sea-floor with constant medium parameters.

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