Angle of incidence estimation for converted-waves

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ABSTRACT

Amplitude-versus-angle (AVA) analysis represents a link between the geological properties of rock interfaces and their seismic signature with offset. This and other methods that combine seismic amplitudes with geological properties require the knowledge of the angles of incidence of the rays on the interface where the properties are being analyzed.

The objective of this work is to consider the advantages of using the three-term PS series instead of manipulating the two-term PP series (proposed by Todorov and Stewart, 1998) in the calculation of angles of incidence. A velocity model from Blackfoot area was used to indicate that the two-term series is a good approximation to the exact angles of incidence obtained by ray tracing, although it always overestimates the result. We find that the three-term series is a better approximation exact error for various cases.

INTRODUCTION

Taner and Koehler (1969) proposed a series for the calculation of traveltimes of a wave reflecting from an interface in a horizontally layered subsurface:

$$t_n^2(x) = c_1 + c_2 x^2 + c_3 x^4 + \dots \tag{1}$$

where

$$t_n(x) = \text{PP reflection traveltime for } x \text{ offset}$$

$$c_i = \text{constants depending on model}$$

$$x = \text{source - receiver offset}$$

They concluded that this series converges rapidly using the first two terms. In their work they also show that the third coefficient, C_3, is always negative, and can be zero when: $V_1 = V_2 = \dots = V_n$.

For some cases, the number inside the square root will be negative and hence produce complex traveltimes.

They also gave an explicit formula to calculate the coefficients of this series:

$$c_1 = A_1 \tag{2}$$

And the following coefficients can be computed in a recursive way from equation (3).

$$c_m B_{1,m-1} + c_{m-1} B_{2,m-2} + \dots c_2 B_{m-1,1} = A_m \quad (m = 2,3,4,\dots) \tag{3}$$

The B_{kn} coefficients are determined recursively by equation (4).

$$B_{kn} = B_{k,n-1} B_{11} + B_{k-1,n-1} B_{21} + B_{1,n-1} B_{k1} \quad (n = 2,3,4,\dots),(k = 1,2,3,\dots) \tag{4}$$

The definition for B_{kn} and A_k is given by equations (5) and (6) respectively.

$$B_{k1} = b_1 b_k + b_2 b_{k-1} + \dots + b_k b_1 \quad (k = 1,2,\dots) \tag{5}$$

$$A_k = \gamma_1 \gamma_k + \gamma_2 \gamma_{k-1} + \dots + \gamma_k \gamma_1 \quad (k = 1,2,\dots) \tag{6}$$

And finally the b_m and γ_m coefficients are determined as follows:

$$b_m = q_m a_{m+1} \quad (m = 1,2,3,\dots) \tag{7}$$

$$\gamma_m = q_m a_m \quad (m = 1,2,3,\dots) \tag{8}$$

$$a_m = 2\sum_{k=1}^{n} h_k V_{P_k}^{2m-3} \quad (m = 1,2,3,\dots) \tag{9}$$

$$q_1 = 1, \quad q_k = \frac{1 \cdot 3 \cdot \dots (2k-3)}{2 \cdot 4 \cdot \dots (2k-2)} \quad (k = 2,3,\dots) \tag{10}$$

Al-Chalabi (1973) analyzed some different velocity definitions for a horizontally layered medium. He shows numerically that including more terms in the traveltime series approximation does not necessarily improve the convergence, (Figure 1).

Additionally, he states that for the particular cases where the offset/depth ratio is small the series converges rapidly, but when this ratio is large, strong oscillations are seen (Figure 1).

In the seismic reflection case, we don't really have offset/depth ratios higher than two. From Figure 1, he states that the three term truncated series produced very accurate results.

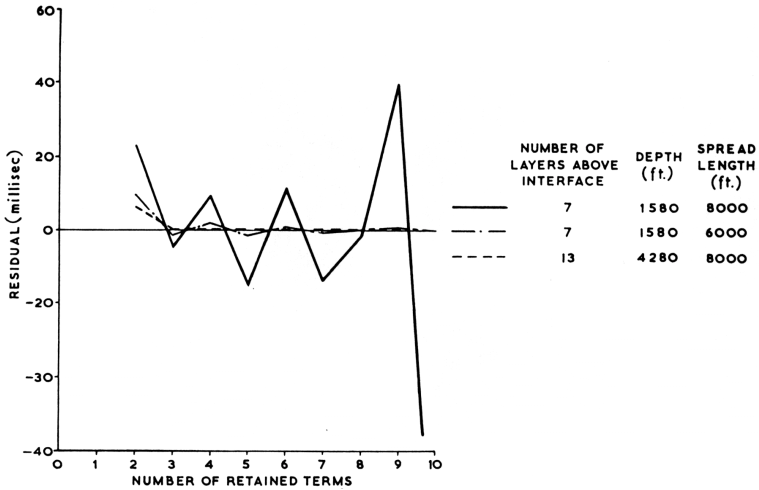

FIG. 1. Residuals versus the number of terms included in the series of equation 1 (from Al-Chalabi, 1973)

At far offsets he shows how the approximated traveltimes do not improve, as more terms are included (Figure 2). The residue values from the plot on Figure 2 are defined as the difference between the exact and the approximated traveltimes.

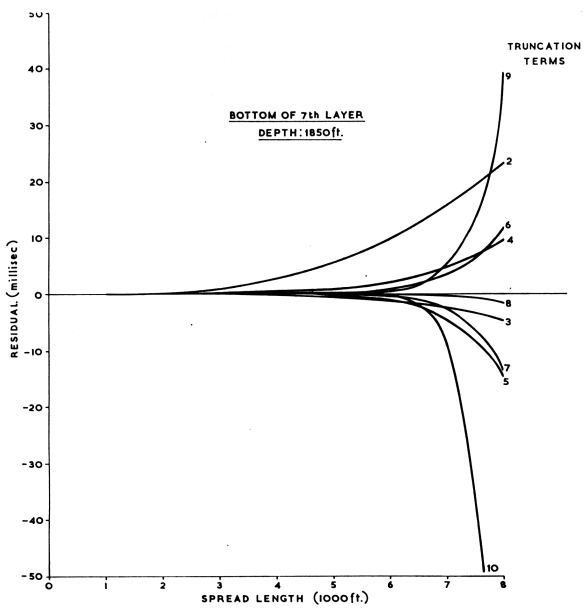

FIG. 2. Residuals vs offset for different truncation series (from Al-Chalabi, 1973)

Tessmer and Behle (1988) developed a similar expression to the Taner and Koehler (1969) traveltime series, but for the PS or SP case:

$$t_{PS_n}^{2}(x_{PS}) = c_1^{PS} + c_2^{PS} x_{PS}^2 + c_3^{PS} x_{PS}^4 + \dots \tag{11}$$

And also gave a formula to calculate the coefficients:

$$a_m^{(PS)} = \sum_{k=1}^{n} h_k \left(V_{P_k}^{2m-3} + V_{S_k}^{2m-3} \right) \tag{12}$$

In their work, they showed some of the results obtained when truncating the series on different terms, Figure 3.

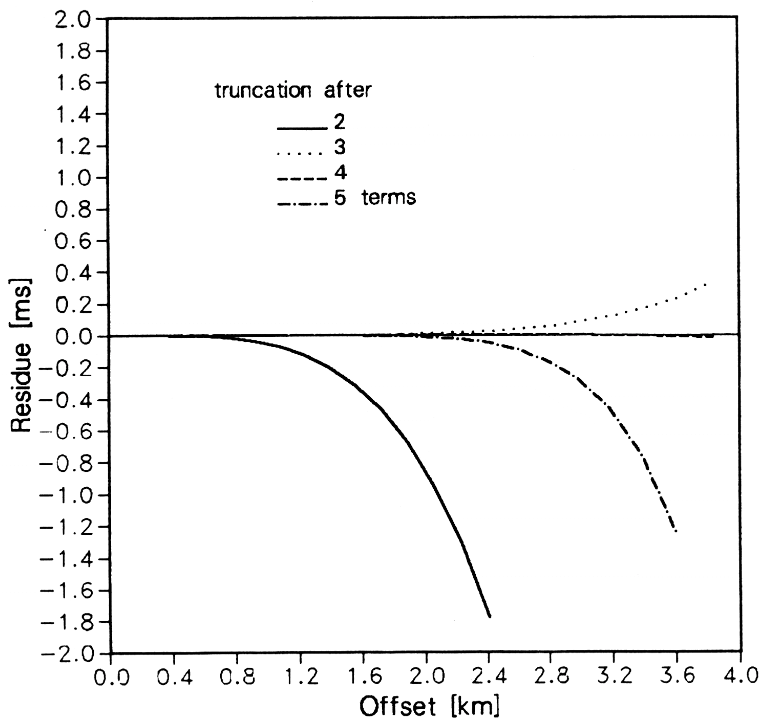

FIG. 3. Time residues between different truncated series and exact traveltimes for PS waves (from Tessmer and Behle, 1988)

An important result from this work is that this series does not converge as rapidly as Taner and Koehler PP travel-time expression. Nonetheless, they found that truncating the first two terms of the expansion (which represents a hyperbola), could be used to give a reasonable approximate for PS moveout correction.

Figure 3 shows the residue time differences (which is the difference between the exact and approximated traveltime) for a PS reflection from an interface at 4 km depth. It can be observed that for an offset/depth ratio of ½, the three term's series give a much better approximation than that from using the two terms.

In their work they state that the relation between the coefficients C_n^{PS} and a_n^{PS} is the same as Taner and Koehler series (TK series), i.e. they use the same relation between C_n and a_n. The first coefficient for TK series represents the *two way zero offset PP wave traveltime*, equation 13, while for TB series is the *two way zero offset traveltime for a PS reflection*, equation 14.

$$c_1 = (a_1)^2 \qquad \therefore\ c_1 = \left(t_{0_n}^{PP}\right)^2 = \left(2\sum_{k=1}^{n}\frac{h_k}{V_{P_k}}\right)^2 \tag{13}$$

$$c_1^{PS} = \left(a_1^{(PS)}\right)^2 \qquad \therefore\ c_1^{PS} = \left(t_{0_n}^{(PS)}\right)^2 = \left[\sum_{k=1}^{n} h_k\left(\frac{1}{V_{P_k}} + \frac{1}{V_{S_k}}\right)\right]^2 \tag{14}$$

The second coefficient for TK series represents *the PP wave RMS velocity*, equation 15, while for TB series is the same RMS velocity but for *a PS wave*, equation 16.

$$c_2 = \frac{a_1}{a_2} \qquad \therefore\ c_2 = \frac{1}{\tilde{V}_{PP_n}^2} = \frac{\sum_{k=1}^{n}\frac{h_k}{V_{P_k}}}{\sum_{k=1}^{n} h_k V_{P_k}} \tag{15}$$

$$c_2^{PS} = \frac{a_1^{(PS)}}{a_2^{(PS)}} \qquad \therefore\ c_2^{PS} = \frac{1}{\tilde{V}_{PS_n}^2} = \frac{\sum_{k=1}^{n} h_k\left(\frac{1}{V_{P_k}} + \frac{1}{V_{S_k}}\right)}{\sum_{k=1}^{n} h_k\left(V_{P_k} + V_{S_k}\right)} \tag{16}$$

The third coefficient for TK and TB series is defined in equation 17. It does not have any physical interpretation. For TB series this coefficient is defined as equation 18.

$$c_3 = \frac{(a_2)^2 - a_1 a_3}{4(a_2)^4} \qquad \therefore\ c_3^{PS} = \frac{\left(a_2^{(PS)}\right)^2 - a_1^{(PS)} a_3^{(PS)}}{4\left(a_2^{(PS)}\right)^4} \tag{17}$$

$$c_3^{PS} = \frac{\left(\sum_{k=1}^{n} h_k\left(V_{P_k} + V_{S_k}\right)\right)^2 - \sum_{k=1}^{n} h_k\left(\frac{1}{V_{P_k}} + \frac{1}{V_{S_k}}\right)\sum_{k=1}^{n} h_k\left(V_{P_k}^3 + V_{S_k}^3\right)}{4\left(\sum_{k=1}^{n} h_k\left(V_{P_k} + V_{S_k}\right)\right)^4} \tag{18}$$

In general, the TK series can be obtained from the TB series by assuming that all S-wave velocities equal P-wave velocities, i.e. $V_{S_k} = V_{P_k}$.

The definition of the ray parameter p, equation 19, involves angles of incidence θ_n at different interfaces in a horizontally layered medium, as well as the interval velocities V_{P_n}, (Taner et.al, 1969).

$$p = \frac{\sin(\theta_n)}{V_{Pn}} = \frac{d}{dx}\left(t_{PS_n}(x)\right) \tag{19}$$

$$\sin(\theta_n) = V_{Pn}\frac{d}{dx}\left(\sqrt{c_1^{PS} + c_2^{PS}x_{PS}^2 + c_3^{PS}x_{PS}^4}\right) \tag{20}$$

The relation between angles of incidence and traveltimes, equation 20, is used in this work to estimate the angles from a velocity model.

The derivative of TB series, equation 11, is substituted into the ray parameter definition, equation 19, obtaining the equation used in this work to calculate angles of incidence, equation 21.

$$\sin(\theta_n) = \frac{V_{Pn}\left(c_2^{PS}x_{PS} + 2c_3^{PS}x_{PS}^3\right)}{\sqrt{c_1^{PS} + c_2^{PS}x_{PS}^2 + c_3^{PS}x_{PS}^4}} \tag{21}$$

In 2000, Todorov developed a relation to estimate angles of incidence for PS data, equation 22. This equation is obtained from the TK series truncated at the second term.

$$\sin(\theta_n) = \frac{2gx_{PS}V_{Pn}}{\tilde{V}_{P_n}^2\sqrt{\left(\frac{2\bar{V}_{S_n}}{\bar{V}_{P_n} + \bar{V}_{Sn}}\right)^2 t_{PS_0}^2 + \left(\frac{2gx_{PS}}{\tilde{V}_{P_n}}\right)^2}} \tag{22}$$

where

$$g = \frac{1}{1 + \frac{\bar{V}_{P_n}}{\bar{V}_{S_n}}\frac{\tilde{V}_{S_n}^2}{\tilde{V}_{P_n}^2}}$$

$$x_{PP} = 2gx_{PS}$$

$$\tilde{V} = \text{RMS velocity}$$

$$\bar{V} = \text{average velocity}$$

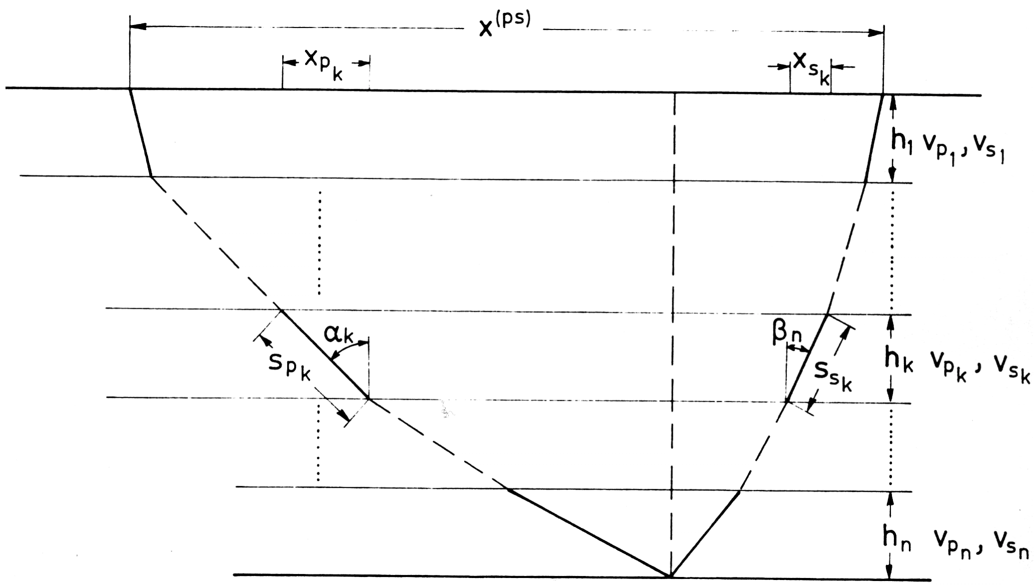

FIG. 4. The raypath of a PS-converted wave in a horizontally layered medium (from Tessmer and Behle, 1988)

Figure 4 explains in a graphical way the main problem of this work: obtain a better approximation to the calculation of angles of incidence by including the third term of the traveltime series.

MODEL USED

Using the sonic log from 08-08 well in the Blackfoot field, Alberta, a velocity model was defined, Figure 5.

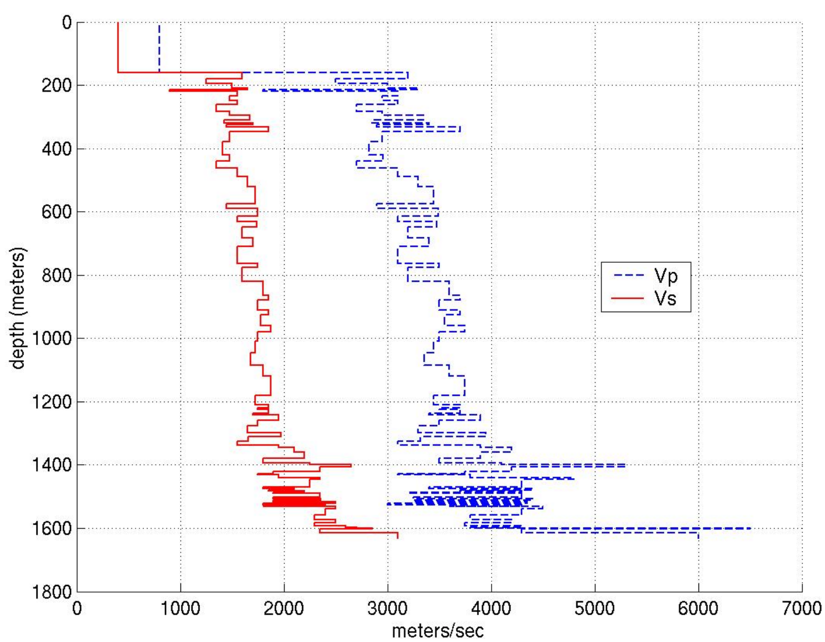

FIG. 5. P-wave and S-wave Velocity model for the Blackfoot field, Alberta.

Because the S sonic log was obtained only on the approximated interval from 1300 m to 1600 m, a V_P/V_S ratio of 2 was defined on the missing intervals.

CALCULATING ANGLES OF INCIDENCE

Using equations 13 and 14 a Matlab script was developed to estimate the angles of incidence at the following offsets: 500, 1000, 1500 m.

To compare with the approximations, the exact angles of incidence were obtained by ray tracing the model. A function defined as *traceray_ps* from the CREWES toolbox was used to obtain a vector containing the ray parameters for each interface of the velocity model. Using the definition of ray parameter, equation 15, the angles of incidence were obtained.

$$\theta_n = \arcsin\left(p \cdot V_{P_n}\right) \tag{15}$$

ANGLES OF INCIDENCE OBTAINED

The results obtained for each offset value: 500, 1000 and 1500 m, are presented in Figures 6, 7 and 8, respectively.

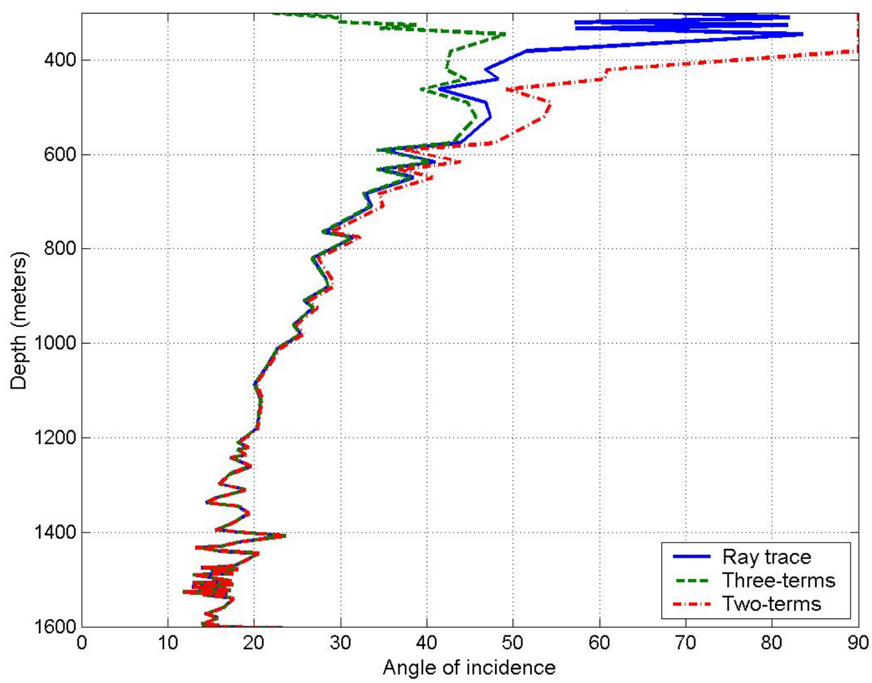

FIG. 6a. Angles of incidence for P-S reflection for source offset of 500 m and reflector depths as indicated

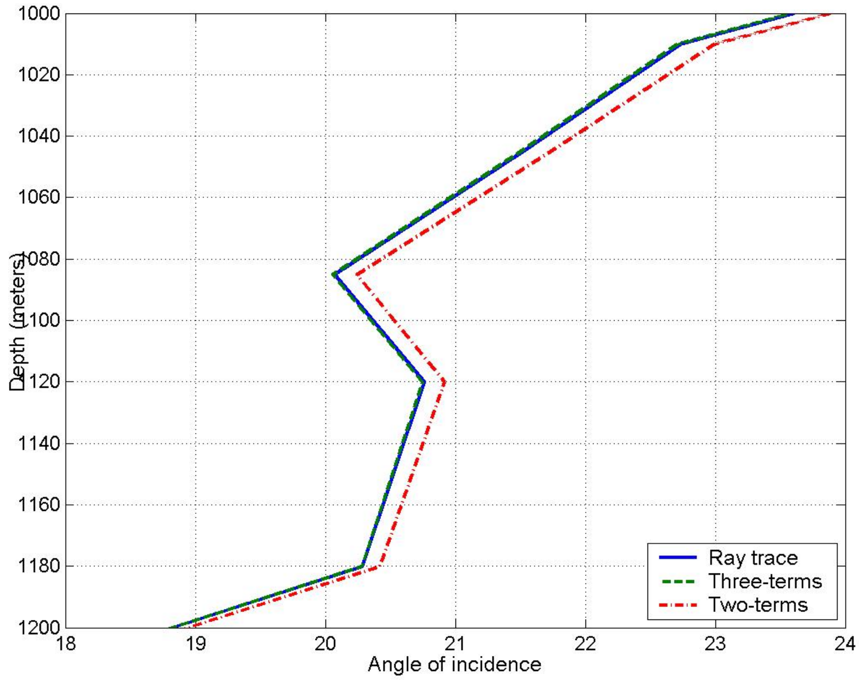

FIG. 6b. Zoom of Figure 6a

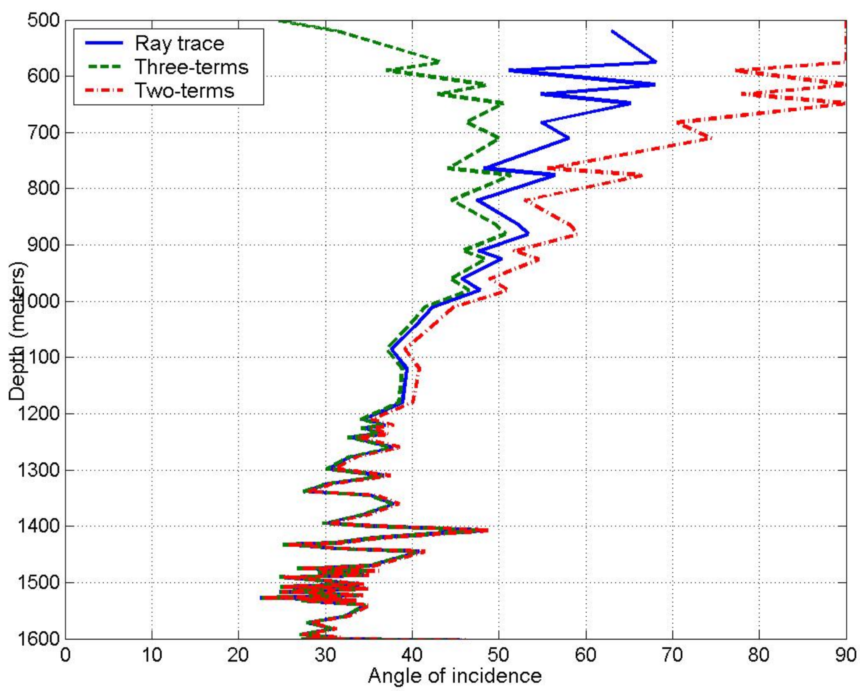

FIG. 7a. Angles of incidence for P-S reflection for source offset of 1000 m and reflector depths as indicated

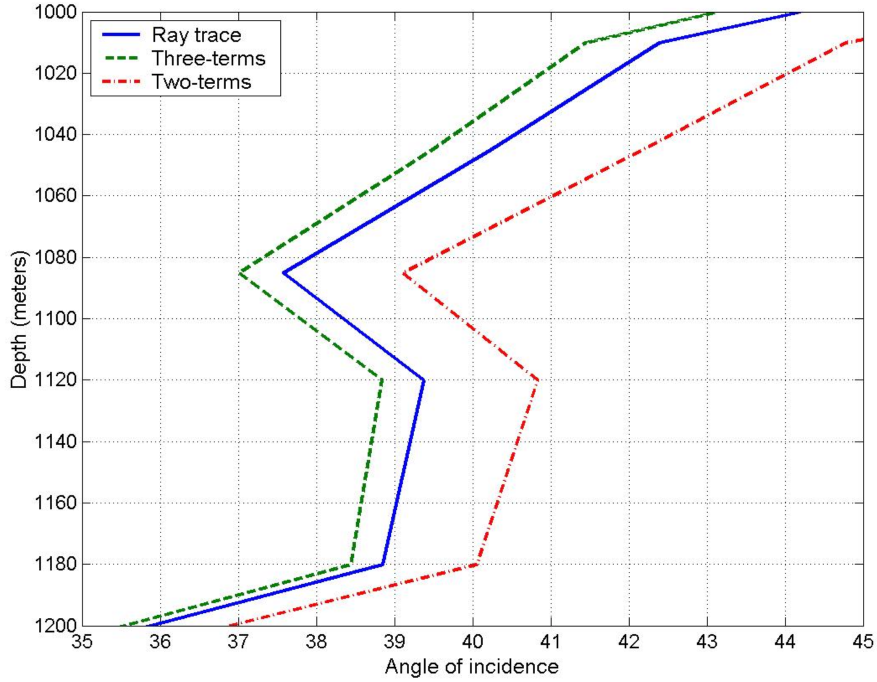

FIG. 7b. Zoom of Figure 7a

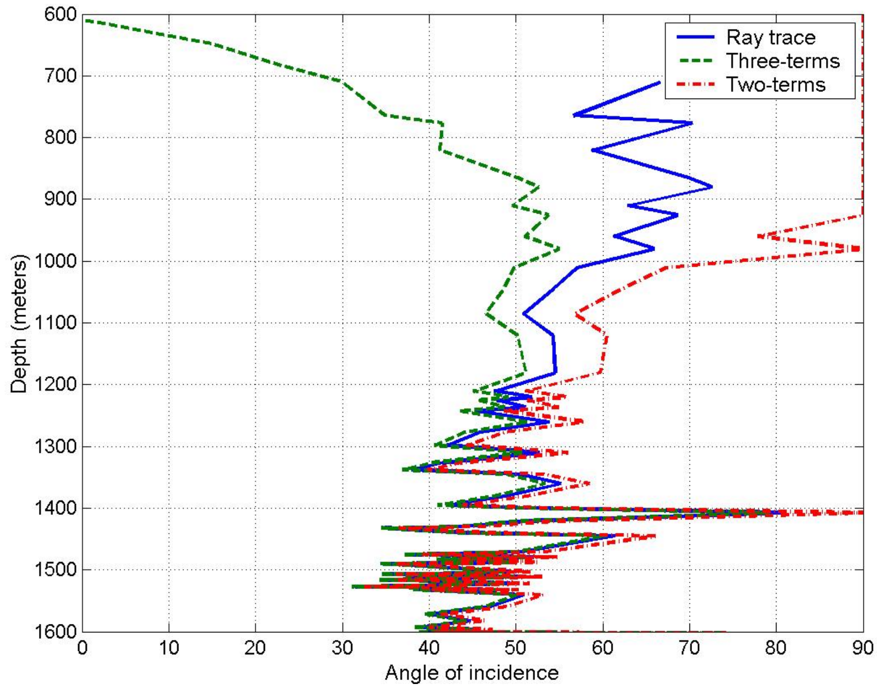

FIG. 8a. Angles of incidence for P-S reflection for source offset of 1500 m and reflector depths as indicated

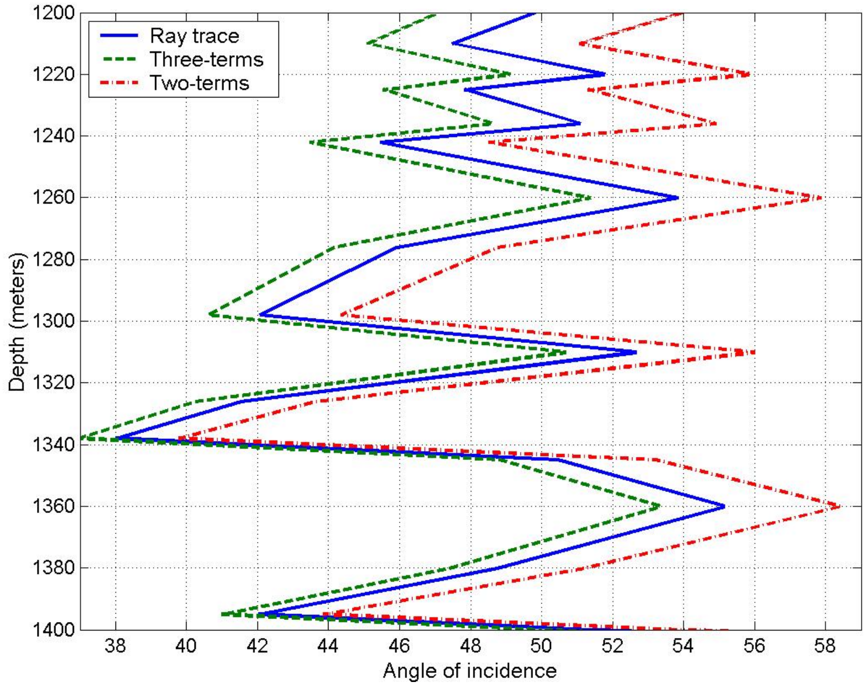

FIG. 8b. Zoom of Figure 8a

From the graphs, we see that:

i) For shallow depths both approximations are bad

ii) The two-term approximation always overestimates the angle of incidence.

iii) For thin layers, the three-term approximation is more accurate than the two-term approximation.

To observe the accuracy of both approximations, equations 13 and 14, a plot of the difference between each approximation and the exact angle of incidence against depth are presented in Figures 9, 10 and 11, for each offset.

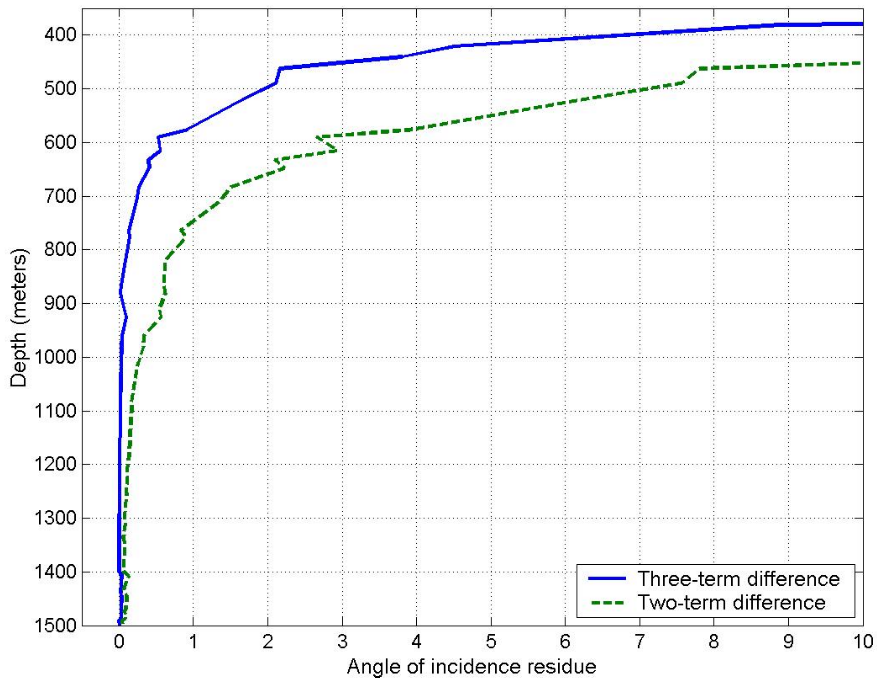

FIG. 9. Accuracy of the angles of incidence approximations (source offset of 500 m)

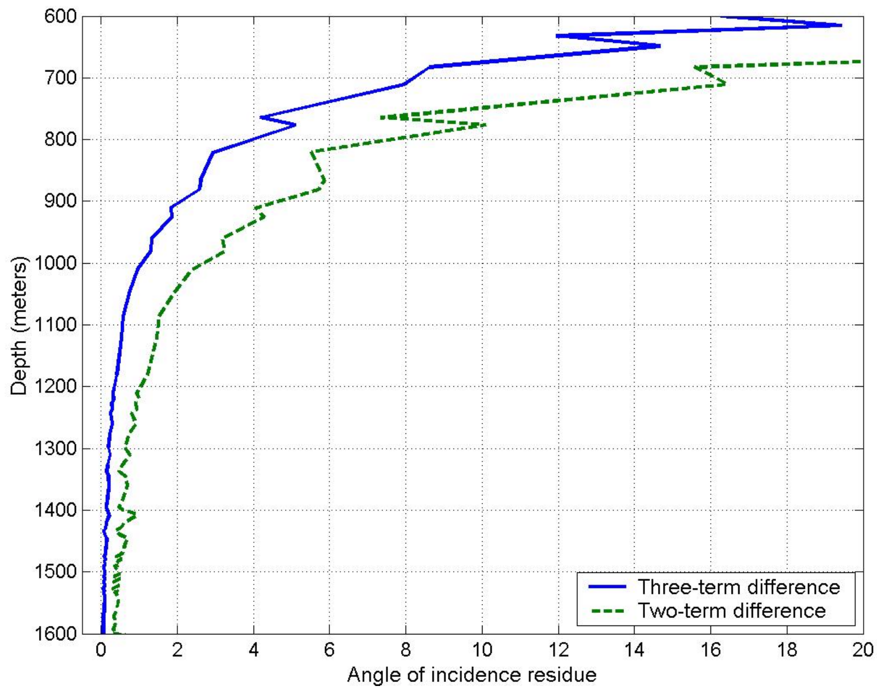

FIG. 10. Accuracy of the angles of incidence approximations (source offset of 1000 m)

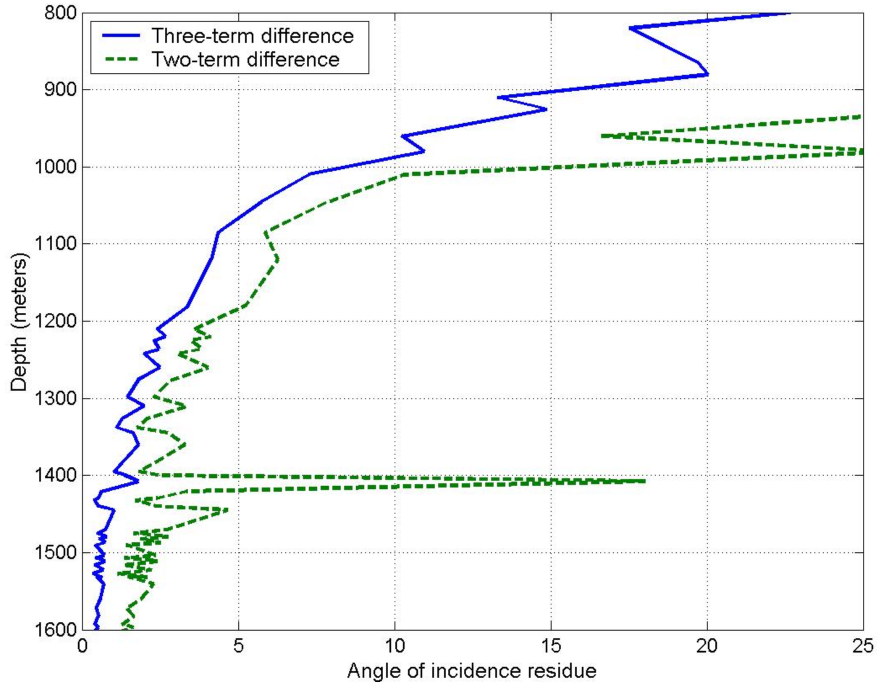

FIG. 11. Accuracy of the angles of incidence approximations (source offset of 1500 m)

From these Figures, we can state that:

i) Both approximations are poor when offset to depth ratios are greater than 1.2.

ii) The two-term series is a reasonable approximation of the exact angles of incidence.

iii) The three-term approximation is a considerably better approximation.

iv) The three-term approximation has the 5-degree limit error at 1050 m depth approximately, while the two-term has it at 1200 m.

v) For thin layers with a velocity inversion, the three-term series is more accurate than the two-term series.

A plot of offset/depth ratio versus depth including all three offsets is given in Figure 12. From the angles of incidence plots (Figures 6a, 7a and 8a) the depth value where the three-term approximation is close to the exact solution is taken and marked on the corresponding offset/depth ratio curve, Figure 12.

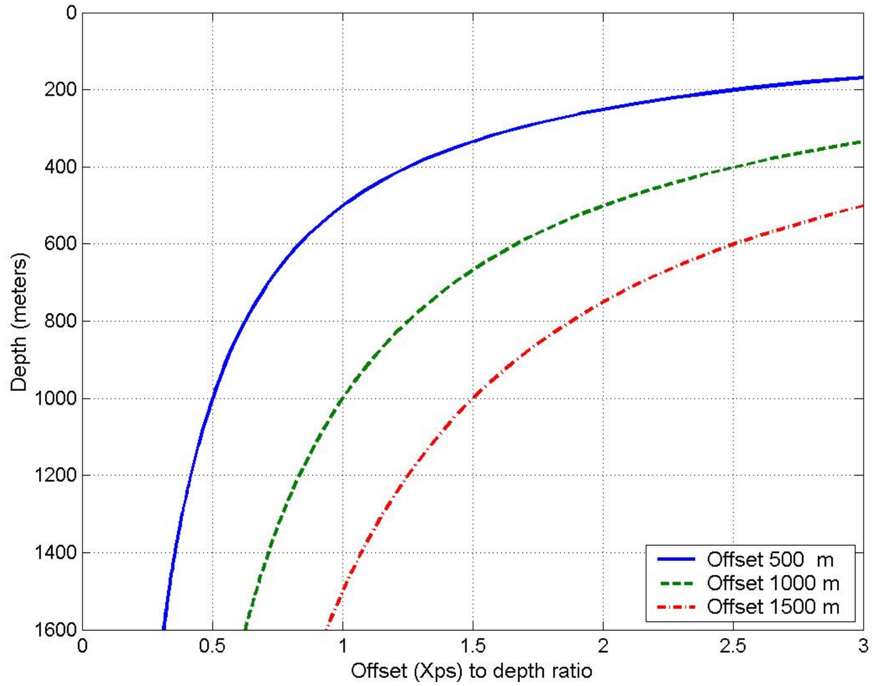

FIG. 12. Offset/depth ratio curves for each offset analyzed

Now it can be assured that the offset/depth ratio value that maintains the estimate under a 5-degree error (using the three-term approximation) is 1.5 for a 1500 m offset.

CONCLUSIONS

We concluded that the two-term series is a good approximation to the exact angles of incidence for P-S reflection obtained by ray tracing, but the three-term is better.

By including the third term in the approximation of the traveltime equation, more accurate calculations of angles of incidence can be obtained. For thin layers presenting velocity inversion the three-term gives more accurate angle of incidence estimation.

FUTURE WORK

i) Obtain analytically the conditions where the three-term approximation is not valid.

ii) Investigate the accuracy needed in AVA analyses and inversion processes.

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