Comparison of two P-S conversion-point mapping approaches for Vertical Transversely Isotropic (VTI) media

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ABSTRACT

Determination of the conversion point is an important step in P-SV converted-wave survey design and in data processing. Unlike the midpoint determination in P-P wave exploration that is determined geometrically, the conversion point in P-S exploration is determined by physical properties of the formations. In practical processing, it is obtained by calculation followed by depth-variant sorting. The depth-dependent conversion-point position is often approximated by asymptotic conversion point, which is at a constant offset to the source. The influence of anisotropy on the P-S conversion point has to be considered together with the effect of reflector depth, offset, as well as layering. We developed a general raytracing algorithm for multi-layered VTI modelling using exact velocity equations for weak anisotropy to map the raypath and the conversion point location. The conversion point can also be determined by using the γ_{eff} method, where γ_{eff} refers to effective velocity ratio in anisotropic media. Both methods were applied for a one-layer model and a multi-layered model. In a single-layer model, the relative error is shown to be less than 8% for short-to-intermediate offsets. The γ_{eff} method can be applied to obtain the conversion point for offset to depth ratio less than 1.5. In the multi-layered model, the relative error increases with the increasing offset and can reach 14% at offsets of twice the depth.

INTRODUCTION

Converted-wave exploration is receiving considerable attention in oil and gas exploration conducted both on land and the ocean-bottom for it can provide higher resolution than the traditional P-P method (Stewart et al., 2002).

The incident P-wave converts part of its energy to S-waves at interface. The upgoing S-wave travels more steeply than the downgoing P ray, because of Snell's law and the fact that $V_P/V_S > 1$. The offset of the conversion-point from the source is based on physical properties of the media, not simple geometry.

For a single, homogeneous, isotropic layer, the exact conversion-point displacement can be expressed as (Thomsen, 1999):

$$\frac{x_C}{x} = \frac{V_P t_P(x)\sin\theta_P}{V_P t_P(x)\sin\theta_P + V_S t_S(x)\sin\theta_S} = \frac{1}{1+\dfrac{t_S(x)}{\gamma^2 t_P(x)}} \quad , \tag{1}$$

where t_P is the one-way, oblique traveltime through the layer for the P-wave, t_S is the corresponding one-way S-wave time, V_P is P-wave velocity, and V_S is the shear-wave velocity.

At the limit when $z/x \to \infty$, which means the P- and S-wave raypaths are almost vertical, the ratio of traveltimes becomes $t_S/t_P \to t_{S0}/t_{P0} = V_P/V_S = \gamma$. So equation (1) is reduced to the Asymptotic Conversion Point (ACP) (Tessmer and Behle, 1988):

$$x_{C0} = x\frac{\gamma}{1+\gamma} \ . \tag{2}$$

A Common Conversion Point (CCP) gather should be obtained by computation instead of by sorting. However, it is common to bin the traces with a range of offsets from 0 to $X_{\max}$ with a common ACP. Thomsen (1999) proved that the smearing of the true conversion point couldn't be neglected. It is clear that the actual conversion point at finite z/x differs significantly from the shallow reflectors when $z/x \geq 1$, where considerable exploration interest for converted-waves lies.

TWO CONVERSION-POINT MAPPING METHODS IN VTI MEDIA

From the introduction, we have already known that the ACP deviates from the true location of P-S conversion point, even in the homogeneous, isotropic layer. Numerous investigations have shown that the anisotropy may affect the basic processing and interpretation steps for converted waves. The most commonly considered type of anisotropy is Vertical Transversely Isotropic (VTI). So anisotropy has to be taken into consideration for realistic problems. Two numerical methods are undertaken to map the conversion point in the VTI model: the γ_{eff} method and the exact equation method.

Thomsen's anisotropic equations

According to Thomsen (1986), three parameters, ε, δ, and γ, define anisotropy properties. When we do forward raytracing, we assume that the rock anisotropic properties are already known to us. In this paper, an algorithm will be developed based on this assumption, with exact ε and δ values for weak anisotropy.

The P- and S- wave group velocities in anisotropic media are angle-dependent. The phase velocity is the velocity of wavefront, the value of which may be different from the group velocity which is the speed of energy transportation. Consequently, the phase angle, θ, will differ from the group angle ϕ, which is the direction of energy transport from the source. The following equations show how P- and S- wave velocities vary with angles, and the connection between the ray angle and phase angle.

$$v_P^2(\theta) = \alpha_0^2\left[1 + \varepsilon\sin^2\theta + D^*(\theta)\right] \tag{3}$$

$$v_{SV}^2(\theta) = \beta_0^2\left[1 + \frac{\alpha_0^2}{\beta_0^2}\varepsilon\sin^2\theta - \frac{\alpha_0^2}{\beta_0^2}D^*(\theta)\right] \tag{4}$$

$$D^*(\theta) = \frac{1}{2}\left(1 - \frac{\beta_0^2}{\alpha_0^2}\right)\left\{\left[1 + \frac{4(2\delta\text{-}\varepsilon)}{(1-\beta_0^2/\alpha_0^2)}\sin^2\theta\cos^2\theta + \frac{4(1-\beta_0^2/\alpha_0^2+\varepsilon)\varepsilon}{(1-\beta_0^2/\alpha_0^2)^2}\sin^4\theta\right]^{1/2} - 1\right\} \tag{5}$$

$$\tan\left[\phi(\theta)\right] = \frac{\left(\tan(\theta) + \frac{1}{v}\frac{dv}{d\theta}\right)}{\left(1 - \frac{\tan(\theta)}{v}\frac{dv}{d\theta}\right)} \tag{6}$$

Here, α_0 is the vertical P- wave velocity, β_0 is the vertical S-wave velocity, θ is the phase angle, and ϕ is the group angle.

We designed an algorithm for a multi-layer model to calculate the conversion-point position at each reflector using these exact equations. The basic principle in P-S wave raytracing in VTI media is that the phase angles and the phase velocities obey Snell's law, which is expressed as:

$$\frac{v_P}{\sin\theta_P} = \frac{v_S}{\sin\theta_S} \quad . \tag{7}$$

However, ray angles and ray velocities do not obey Snell's law.

The γ_{eff} method

A Taylor expansion form as a function of x/z, was derived in order to compute the conversion point more efficiently. It is asymptotically correct at both limits ($x/z \to 0$ and $x/z \to \infty$) and varies smoothly in between them. This is expressed as (Thomsen, 1999):

$$\frac{x_c}{x} \approx C_0 + C_2\frac{\left(x/z\right)^2}{\left(1 + C_3\left(x/z\right)^2\right)} \quad , \tag{8}$$

where the coefficients are $C_0 = \frac{\gamma}{1+\gamma}$; $C_2(\gamma) = \frac{\gamma}{2}\frac{(\gamma-1)}{(\gamma+1)^3}$; and $C_3 = \frac{C_2}{1-C_0}$.

The anisotropy effect is attributed to a parameter defined as effective velocity ratio. In a single-layer case, it can be expressed as:

$$\gamma_{eff} = \frac{\gamma_2^2}{\gamma_0} = \gamma_0 \frac{(1+2\delta)}{(1+2\sigma)} \,, \tag{9}$$

where σ is the anisotropy parameter defined by (Tsvankin and Thomsen, 1994) as:

$$\sigma = \left(\frac{V_{p0}^2}{V_{s0}^2}\right)(\varepsilon - \delta) \ . \tag{10}$$

In a multi-layered anisotropic model, the conversion-point location is not only affected by the anisotropy, but also by the layering effect. The converted-wave moveout velocity, at every vertical time t_{C0}, is (Thomsen, 1999):

$$V_{C2}^2(t_{C0}) = \frac{V_{P2}^2}{1+\gamma_0} + \frac{V_{S2}^2}{1+\gamma_0} = \frac{V_{P2}^2}{1+\gamma_0}\left(1+\frac{1}{\gamma_{eff}}\right); \tag{14}$$

$$\gamma_{eff} = \frac{1}{(1+\gamma_0)V_{C2}^2/V_{P2}^2 - 1} \,. \tag{15}$$

Here γ_0 is the vertical velocity ratio, and V_{P2} is the short-spread P-wave moveout velocity and V_{S2} is the S-wave equivalent.

Yang and Lawton (2001) mapped the conversion point in VTI with different anisotropic parameters for a single layer case. The conversion-point location is dependent on the relationship of ε and δ. Here, we computed the conversion-point displacement in VTI relative to its location in isotropic media by γ_{eff} method. From equation (9), we can see that if $\delta = \sigma$, so $\gamma_{eff} = \frac{\gamma_2^2}{\gamma_0} = \gamma_0 \frac{(1+2\delta)}{(1+2\sigma)} = \gamma_0$. In this case,

$\sigma = \left(\frac{V_{P0}^2}{V_{S0}^2}\right)(\varepsilon - \delta) = \delta$, so

$$\delta = \frac{V_{P0}^2 / V_S^2}{1 + V_{P0}^2 / V_S^2}\varepsilon = \frac{\gamma_0^2}{1+\gamma_0^2}\varepsilon \,. \tag{16}$$

When $\gamma_0 = 2$ and $\delta = 0.8\varepsilon$, then $\gamma_{eff} = \gamma_0$. Thus, in this situation and for a single-layer case, the conversion point is located equivalently to that in isotropic case.

EXAMPLES AND DISCUSSION

We tested the conversion-point mapping methods on a single-layer VTI model and a three-layer VTI model.

Single layer case

First, we applied Thomsen's equation methods on a single-layer VTI model defined with properties in Figure 1. In this raytracing experiment, the offset ranges from zero to twice the depth. Figure 2 shows the raypaths generated on this model.

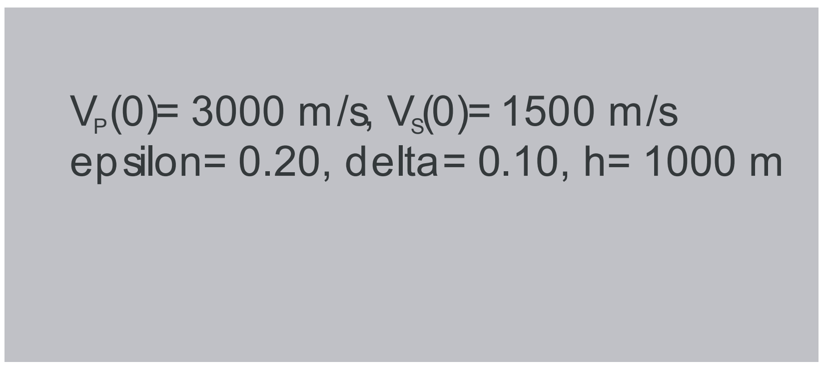


FIG. 1. The one-layer VTI model with properties defined as shown.

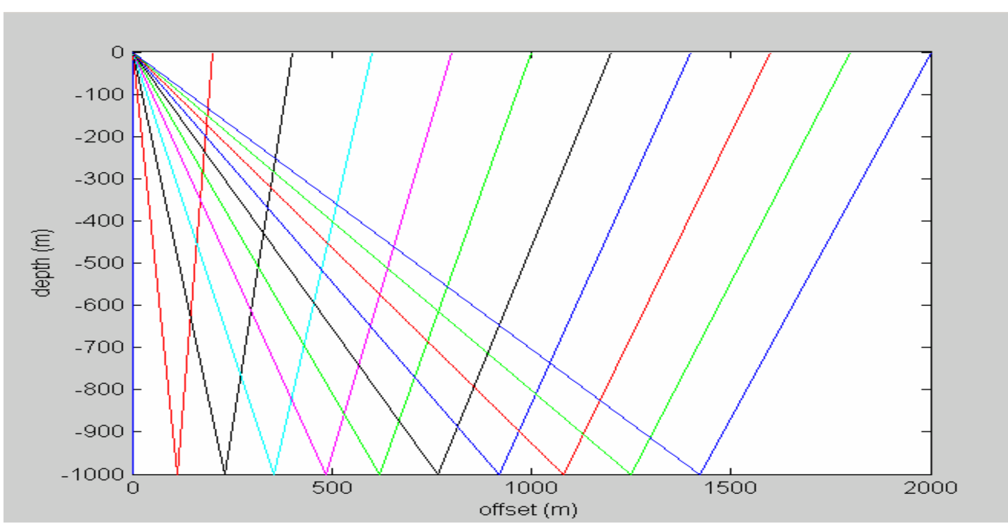


FIG. 2. The raypaths of the P-S converted wave, for a series of offsets, generated from the one layer VTI model shown in Figure 1, using the exact equations.

The $x^2 - t^2$ relationship obtained from raytracing is shown in Figure 3, and also a straight line is plotted for comparison. We can see that the $x - t$ curve is nonhyperbolic.

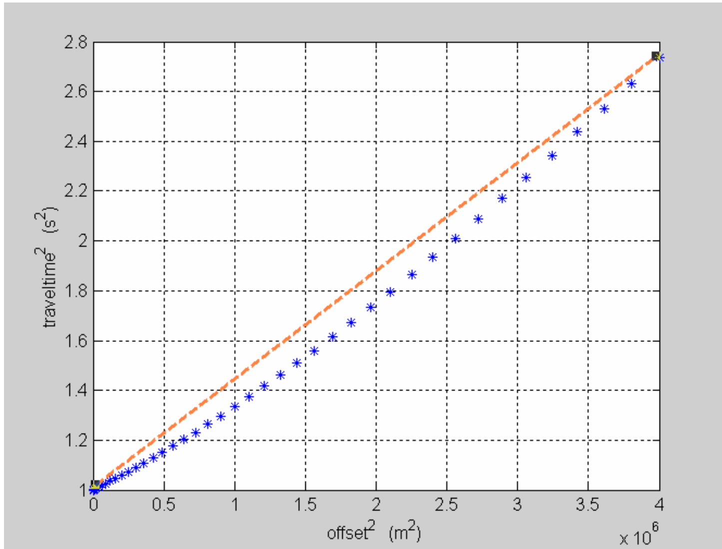


FIG. 3. The $t^2 - x^2$ curve shows nonhyperbolic moveout.

Equation (9) was applied to calculate γ_{eff}, the effective velocity ratio of this VTI layer. Then γ in equation (8) was replaced by γ_{eff} to calculate the conversion-point location. Figure 4 shows the relative location of the conversion point obtained from these two methods. For this example, $\gamma_0 = 2.0$ and $\gamma_{eff} = 1.333$, and $\gamma_0 < \gamma_{eff}$, so the conversion point is located towards the receiver relative to the isotropic case.

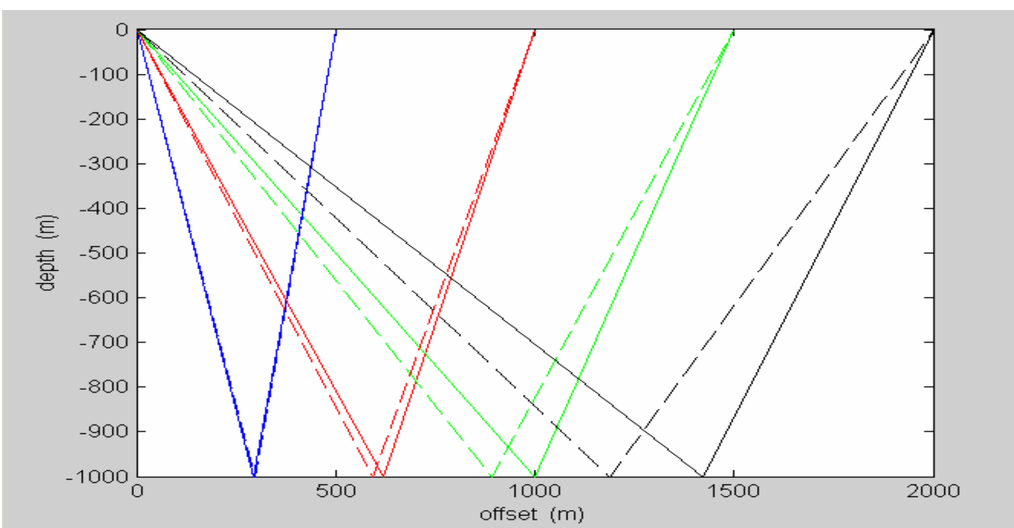


FIG. 4. This figure displays the raypaths and the corresponding conversion point location generated from two methods. The solid lines represent the conversion point location obtained from Thomsen's exact equations, where the dashed line represents the position obtained from γ_{eff} method.

In order to analyze the efficiency of γ_{eff} method, relative error is defined as:

$$error = \frac{x_C^{exact} - x_C^{eff}}{offset}, \tag{17}$$

and was plotted verses offset in Figure 5. From the display of the relative error, we can see that the γ_{eff} method is sufficient for short-to-intermediate offsets. In a single-layer case for long offset, such as offset-to-depth ratio equals to 2, the relative error reaches 11%.

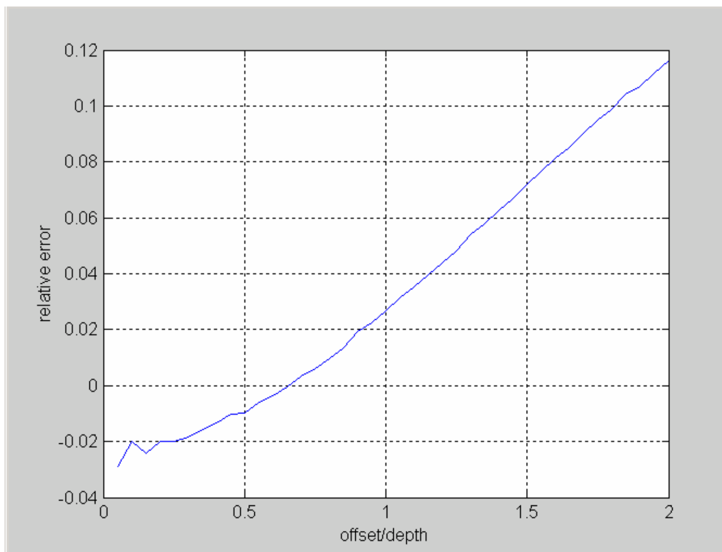
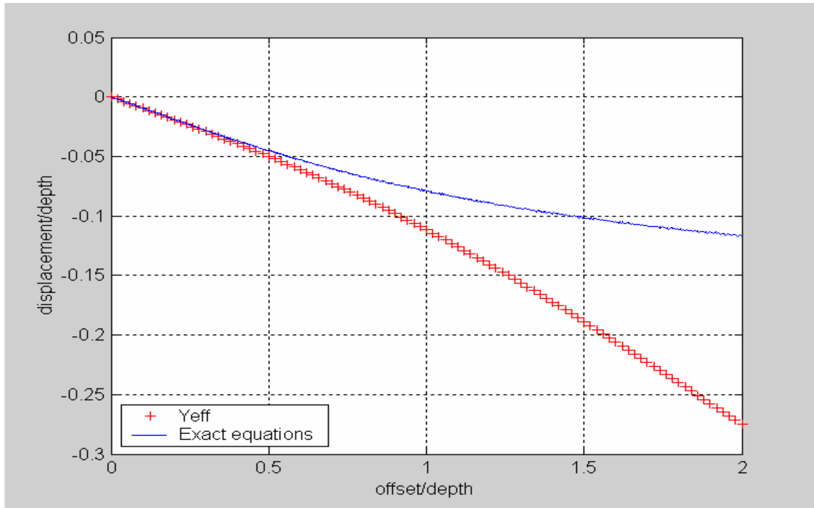
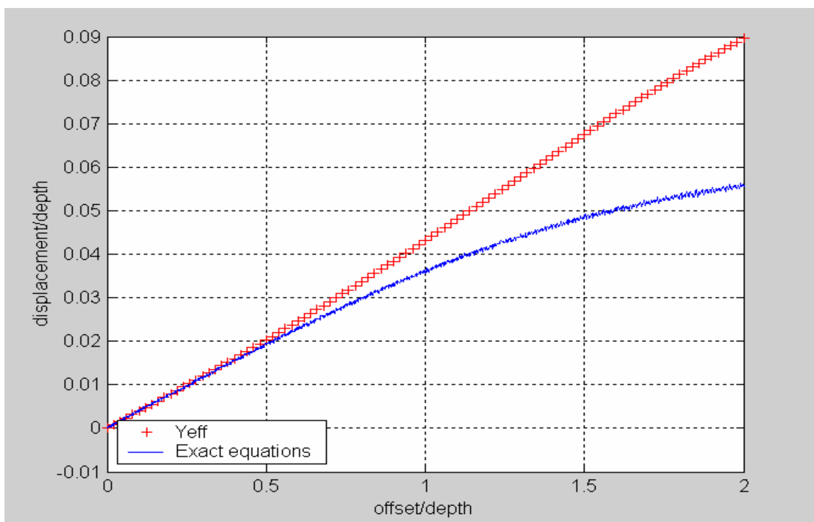


FIG.5. This figure shows the relative error of the conversion point obtained from the γ_{eff} method varying with the offset/depth ratio.

In the VTI model, the relationship of ε and δ determines that conversion point is displaced towards the source or towards the receiver. For the single-layer VTI model, we applied both methods to calculate the displacement of the conversion point from the isotropic case for two different types of anisotropy. Figure 6 shows a comparison between the displacements of the conversion-point obtained from these two methods. When $\varepsilon > \delta$, the displacement of the conversion point is negative, which means that the conversion point is displaced toward the source relative to the isotropic case (Figure 6 (a)). When $\varepsilon <= \delta$, the displacement of the conversion point is a positive meaning the conversion point is displaced towards the receiver compared to the isotropic case (Figure 6 (b)).



(a)



(b)

FIG. 6. Variation of the conversion-point displacement relative to the isotropic conversion-point as a function of offset/depth, obtained from two methods, at the case of: (a) epsilon=0.20, delta=0.10; (b) epsilon=0.10, delta=0.10.

Multi-layered model

We now study more realistic case, a general multi-layered VTI model. A Matlab program was developed for raytracing where the user can define the properties of each layer for modelling. An example for a three-layer model with properties defined for each layer is shown in Figure 7.

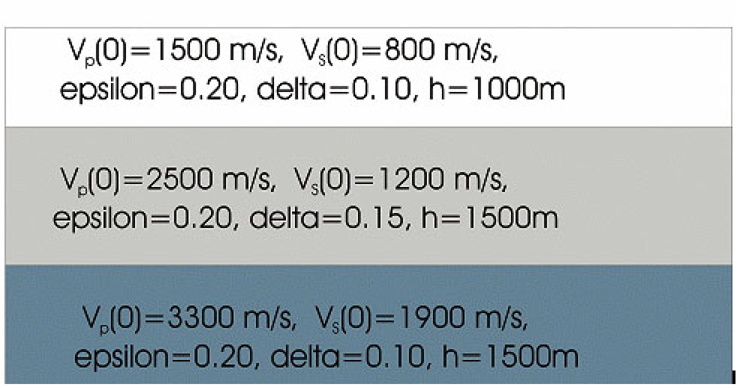


FIG. 7. A three-layer VTI model in with properties defined as shown.

First, we calculated (by raytracing) the P-S conversion-point position at the base of this model using the exact equations for offsets ranging from zero to twice the depth. The raytracing results obtained are shown in Figure 8.

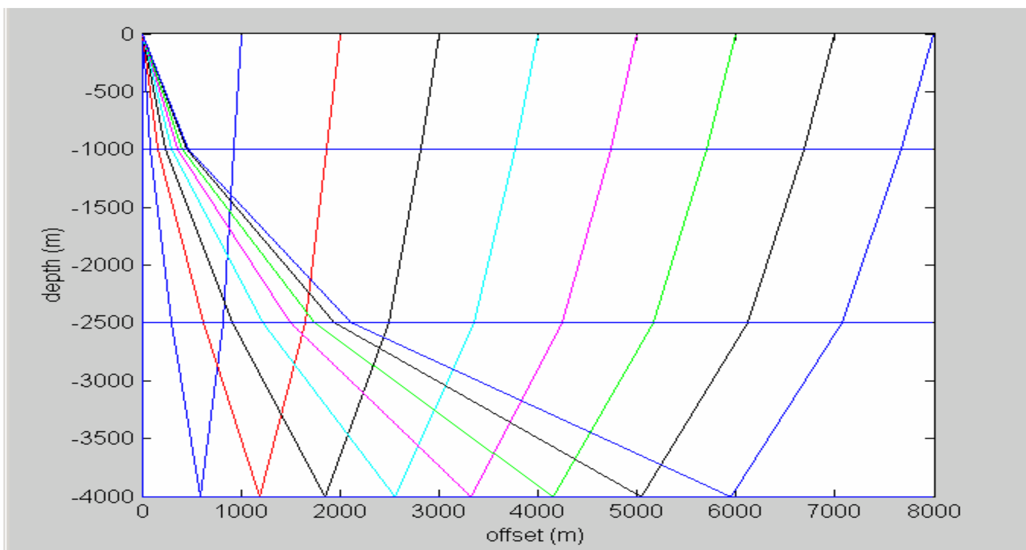


FIG. 8. The P-S wave raypaths generated from the three-layered VTI model using Thomsen's exact equations.

The $x^2 - t^2$ curve and its best-fit straight line are shown in Figure 9. From this figure, we can see that the $x-t$ curve is nonhyperbolic. Calculating the slope of the best-fit straight line, we could obtain the converted NMO velocity V_C to flatten the $x-t$ curve with a value of 2090 m/s. Then we shoot P-P rays on this model using the same survey. A similar method is used to compute a P-wave NMO velocity. The average vertical velocity ratio is calculated, and shows $\gamma_0 = 1.91$. Equation (15) was used to compute the effective velocity ratio $\gamma_{eff} = 1.384$. Since $\varepsilon > \delta$ in this model, $\gamma_{eff} < \gamma_0$ and the conversion point moves towards the source compared to the isotropic case. By replacing

γ by γ_{eff} in equation (8), we calculate the conversion point at the bottom of this model and compare it with the exact location, with results shown in Figure 10.

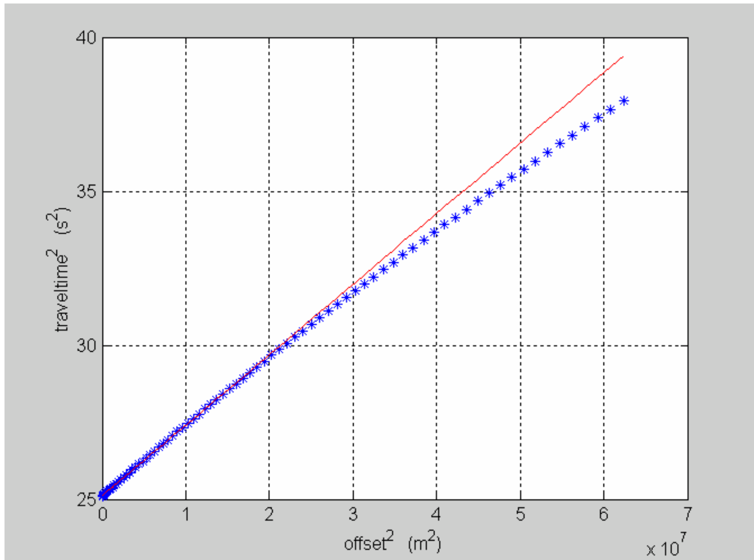


FIG. 9. The $t^2 - x^2$ curve shows nonhyperbolic moveout in multi-layered VTI media. The straight line is used to obtain the short-spread converted NMO velocity, $V_C = 2134\, m/s$.

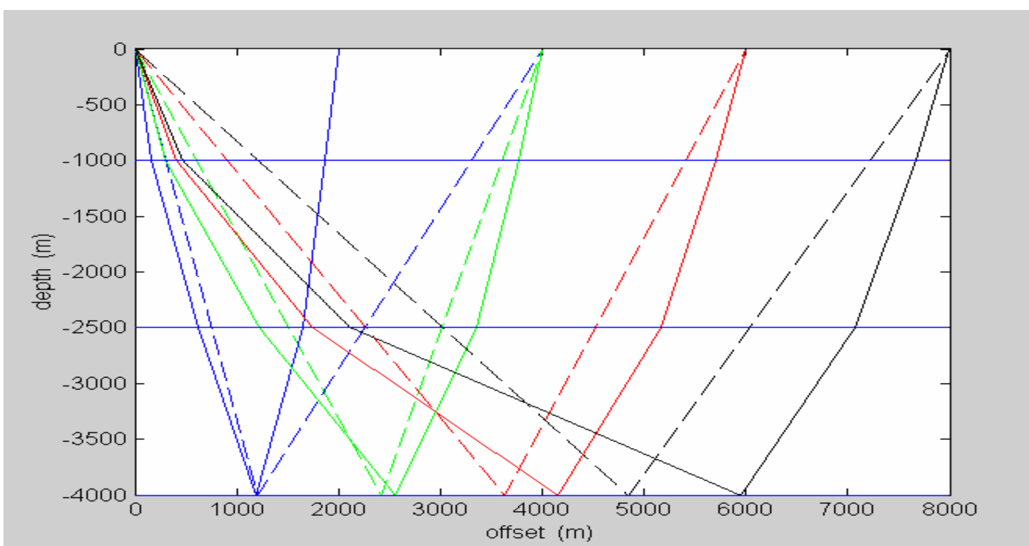


FIG. 10. The P-S conversion-point position obtained from two methods being applied on the three-layer VTI model. The solid lines show the ray path for certain offsets generated from Thomsen's exact equations and the dashed lines show the conversion-point position by effective velocity with γ_{eff} = 1.384.

The deviation was calculated using equation (17) and the relative error is plotted verses the offset-to-depth ratio in Figure 11. We can see that when offset-to-depth ratio equals to 2, the deviation reaches 14% of the offset.

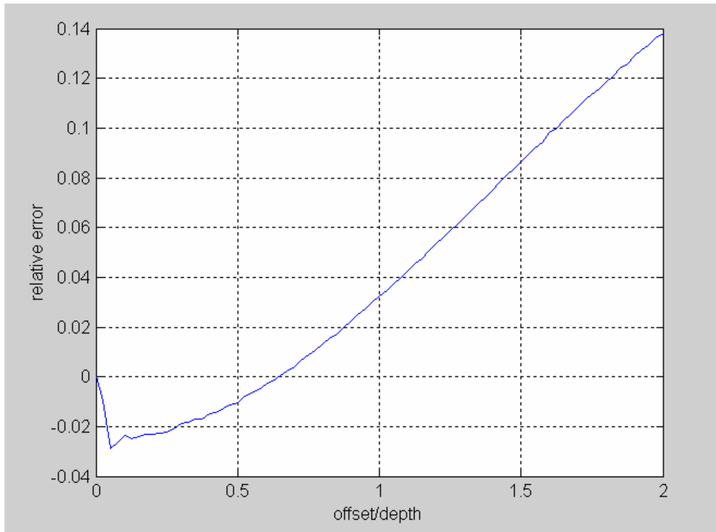


FIG. 11. The relative error of the conversion point at the base of 3-layer model obtained from the effective velocity ratio method.

CONCLUSION

The P-S conversion point was calculated by using two methods: the exact anisotropy equations method and the γ_{eff} method, on a single-layer model and on a multi-layered model. The relative error of using γ_{eff} method was also calculated for both models to study the efficiency of this method. In the single-layer model and multi-layered model, the relative error is less than 8% for short-to-intermediate offsets (offset-to-depth ratio less than 1.5). The relative error increases with the increasing offset in both models. For long offset, which is offset-to-depth ratio greater than 2, the γ_{eff} method is an insufficient approximation for mapping P-S conversion-point.

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