Estimating seismic shear reflectivity from converted waves

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ABSTRACT

A review is given of two theories relating shear-wave reflectivity to converted-wave reflectivity. Two new theories are also given for predicting shear reflectivity from converted-waves, which are found in Equations (5) and (7) of this paper. The four theories are compared through extensive calculation, which show that Equation (7) is the most accurate for data in which there are no known relationships between earth parameter contrasts. Further calculations show that other theories are more accurate if the data is assumed to satisfy some other condition, such as Gardner's relationship. An expression is also put forward that expresses shear-wave reflectivity in terms of an AVO intercept and gradient from converted waves.

INTRODUCTION

Shear-wave reflectivity varies with offset angle as does P-wave reflectivity. The S-wave reflectivity, $R_{SS}(0)$, is especially valuable when evaluated at zero offset, as we can obtain the shear impedance of earth layers using the same inversion techniques as for P-impedance. One potential source of $R_{SS}(0)$ is from shear-wave seismic experiments. However, these are costly, and have not become standard in oil exploration. On the other hand, converted waves are becoming more commonly measured, and it is believed that information on R_{SS} can be extracted from the R_{PS} of converted wave data. This would provide explorationists with a valuable tool to help in delineating rock properties.

In some of the first work on this idea, Stewart and Bland (1997) showed that

$$R_{PS}(\theta) \approx 4\frac{\beta}{\alpha}\sin(\theta)R_{SS}(0) \quad (1)$$

This expression is accurate at small angles, and thus extrapolation of $R_{PS}(\theta)$ to $\theta = 0$ could be used to estimate $R_{SS}(0)$. Stewart and Bland (1997) recommended fitting a curve to $R_{PS}(\theta)(\alpha/[4\beta])$ to estimate $R_{SS}(0)$. Goodway (2001) developed another expression (see following section) which could be used with a simple stacking procedure to estimate $R_{SS}(0)$. In this study, we develop additional expressions for $R_{SS}(0)$ and develop a more rigorous means of assessing their accuracy.

THEORY

Goodway (2001) presented a derivation of the following expression:

$$R_{SS}(0) \approx \frac{-R_{PS}(\theta)}{4\sin\varphi(\tan\varphi\sin\theta - \cos\theta)} \tag{2}$$

Here θ is the average of P-wave reflection and transmission angles, and φ is the analogous average for S-waves. The derivation of Eq. (2) assumes that the linear Aki-Richards approximation (Aki and Richards, 1980) is accurate, and also that the density contrast, $\Delta\rho/\rho = 2(\rho_2 - \rho_1) / (\rho_2 + \rho_1)$, is small.

To derive our first alternate expression, we also begin with the Aki-Richards approximation for R_{PS}:

$$R_{PS} = -\frac{\sin\theta}{\cos\varphi}\left[\left(1 - 2\sin^2\varphi + 2\frac{\beta}{\alpha}\cos\theta\cos\varphi\right)\frac{\Delta\rho}{\rho} - \left(4\sin^2\varphi - 4\frac{\beta}{\alpha}\cos\theta\cos\varphi\right)\frac{\Delta\beta}{\beta}\right] \tag{3}$$

This can be rearranged to

$$R_{PS} = -\frac{\sin\theta}{\cos\varphi}\left[\left(1 + 2\sin^2\varphi - 2\frac{\beta}{\alpha}\cos\theta\cos\varphi\right)\frac{\Delta\rho}{\rho} - \left(4\sin^2\varphi - 4\frac{\beta}{\alpha}\cos\theta\cos\varphi\right)\left(\frac{\Delta\rho}{\rho} + \frac{\Delta\beta}{\beta}\right)\right] \tag{4}$$

Noting that $\Delta\rho/\rho + \Delta\beta/\beta = -2\,R_{SS}(0)$, we could drop the remaining $\Delta\rho/\rho$ term as a simple way of obtaining Equation (2). Instead, we elect to approximate this term. To do this we employ results of studies that have shown that it is feasible to express Gardner's relation (Gardner et al., 1974) in terms of shear velocity instead of compressional velocity (Dey and Stewart, 1997; Potter and Stewart 1998; Potter 1999; Wang, 2000; Ursenbach, 2001). In one of these studies (Ursenbach, 2001) it was shown that for a lithology-independent expression, the exponent does not vary much from the 0.25 value of the original Gardner equation. This implies that $\Delta\rho/\rho \approx (1/4)\,\Delta\beta/\beta$ and allows us to replace $\Delta\rho/\rho$ as follows:

$$\frac{\Delta\rho}{\rho} = \frac{1}{5}\left(4\frac{\Delta\rho}{\rho} + \frac{\Delta\rho}{\rho}\right) \approx \frac{1}{5}\left(\frac{\Delta\beta}{\beta} + \frac{\Delta\rho}{\rho}\right) = -\frac{2}{5}R_{SS}(0)$$

This is similar to a manipulation carried out by Larsen (Larsen, 1999, Equation 2.6). Making this substitution yields

$$R_{PS} \approx -\frac{\sin\theta}{5\cos\varphi}\left(1 - 18\sin^2\varphi + 18\frac{\beta}{\alpha}\cos\theta\cos\varphi\right)R_{SS}(0) \tag{5}$$

This constitutes our first result.

To derive our second result we again begin from Eq. (3), and note that it can be rearranged to the following result:

$$R_{PS} = -\frac{\sin\theta}{4\cos\varphi}\left[\left(1+2\sin^2\varphi-2\frac{\beta}{\alpha}\cos\theta\cos\varphi\right)\left(\frac{\Delta\rho}{\rho}-\frac{\Delta\beta}{\beta}\right)+\left(1-6\sin^2\varphi+6\frac{\beta}{\alpha}\cos\theta\cos\varphi\right)\left(\frac{\Delta\rho}{\rho}+\frac{\Delta\beta}{\beta}\right)\right] \quad (6)$$

The quantity $\Delta\rho/\rho - \Delta\beta/\beta = \Delta\ln(\rho/\beta)$ will be small if $\Delta\rho/\rho$ and $\Delta\beta/\beta$ are comparable. Furthermore, its coefficient is small for small angles and for β/α near ½. We thus neglect this term and obtain our final result:

$$R_{PS} \approx \frac{\sin\theta}{2\cos\varphi}\left(1-6\sin^2\varphi+6\frac{\beta}{\alpha}\cos\theta\cos\varphi\right)R_{SS}(0) \quad (7)$$

Finally, we include a description of one more method that is somewhat different from those above. It also begins with the Aki-Richards expression for R_{PS}, but expands R_{PS} in powers of sinθ and obtains coefficients for the linear and cubic terms. [See for instance Eqs (4a) and (4b) of Ramos and Castagna (2001). Their Eq. (4c) is incorrect but is not needed here.] These coefficients correspond to the intercept and gradient that would be obtained if one plotted $R_{PS}(\theta)/\sin\theta$ against $\sin^2\theta$. Both the intercept (I) and gradient (G) are linear combinations of $\Delta\rho/\rho$ and $\Delta\beta/\beta$ so that it is possible to combine them in such a way as to yield $R_{SS}(0)$. We have obtained this result as

$$R_{SS}(0) = \frac{1+\frac{5}{2}\frac{\beta}{\alpha}}{2\left(1+\frac{\beta}{\alpha}\right)^2} I - \frac{1-2\frac{\beta}{\alpha}}{2\frac{\beta}{\alpha}\left(1+\frac{\beta}{\alpha}\right)^2} G . \quad (8)$$

This is derived on the basis of the Aki-Richards approximation, according to which

$$I = -\left(\frac{1}{2}+\frac{\beta}{\alpha}\right)\frac{\Delta\rho}{\rho} - 2\frac{\beta}{\alpha}\frac{\Delta\beta}{\beta},$$

$$G = \frac{\beta}{\alpha}\left(\frac{1}{2}+\frac{3}{4}\frac{\beta}{\alpha}\right)\frac{\Delta\rho}{\rho} + \frac{\beta}{\alpha}\left(1+2\frac{\beta}{\alpha}\right)\frac{\Delta\beta}{\beta}.$$

(This corrects typographical errors in a previous version of this expression [Ursenbach & Stewart, 2002].) This is a valuable result in its own right as it involves no approximations other than the Aki-Richards approximation. It involves a more involved converted-wave AVO analysis than simple stacking, so it will not be compared here to the other methods above. Our preliminary studies though have indicated that its accuracy is similar to that of the Shuey method for P-wave AVO.

CALCULATIONS AND COMPARISONS

The first issue of interest in performing calculations is to ascertain whether expressing $R_{SS}(0)$ as a function of $R_{PS}(\theta)$, which seems plausible from the Aki-Richards expressions, is in fact reasonable in reality. To do this we crossplot these two quantities for several different values of θ. In each of these plots in Figure 1 approximately 1000 sets of ($\Delta\alpha/\alpha$, $\Delta\beta/\beta$, $\Delta\rho/\rho$) values have been used, with each relative contrast value being sampled from a grid of values in the range [-0.2,0.2]. The value of β/α is fixed at 0.5.

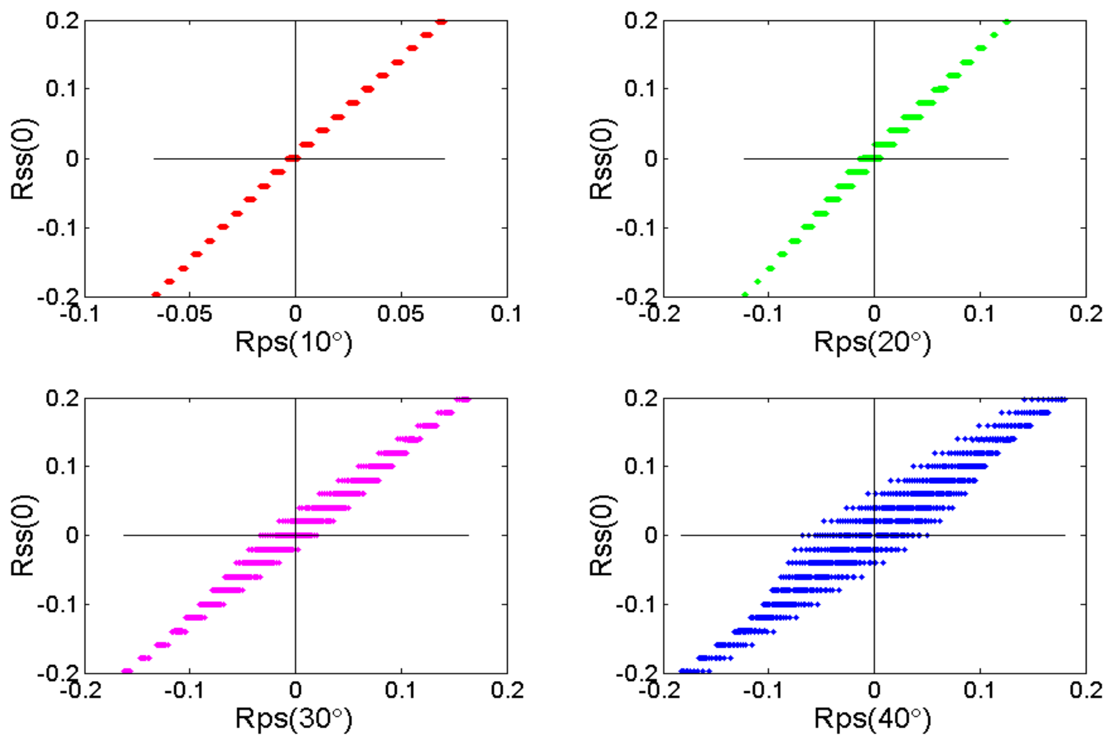

Figure 1: Crossplots of $R_{SS}(0)$ and $R_{PS}(\theta)$ for four different values of θ. In each plot, a large number of values of earth property contrasts have been sampled in the range [-0.2,0.2]. The value of β/α is fixed at ½. The exact Zoeppritz values of $R_{SS}(0)$ and $R_{PS}(\theta)$ have been used, not the Aki-Richards approximate values.

From the above we see a generally linear relationship between $R_{SS}(0)$ and $R_{PS}(\theta)$ at each angle, but the functional relationship is tightest at small offsets. We also see that the slope changes between the plots as the angle changes. To a fair degree of approximation then we may assume that the ratio between $R_{SS}(0)$ and $R_{PS}(\theta)$ is a constant which is dependent on θ but which is roughly independent of $\Delta\alpha/\alpha$, $\Delta\beta/\beta$, and $\Delta\rho/\rho$.

Next we test the above stacking expressions by crossplotting the true $R_{SS}(0)$ for a given ($\Delta\alpha/\alpha$, $\Delta\beta/\beta$, $\Delta\rho/\rho$) against the various predictions. The accuracy in Figures 2-5 appears slightly different for the various approximations, but all follow the general trend dictated by Figure 1, which essentially sets the maximum accuracy obtainable by any approximation.

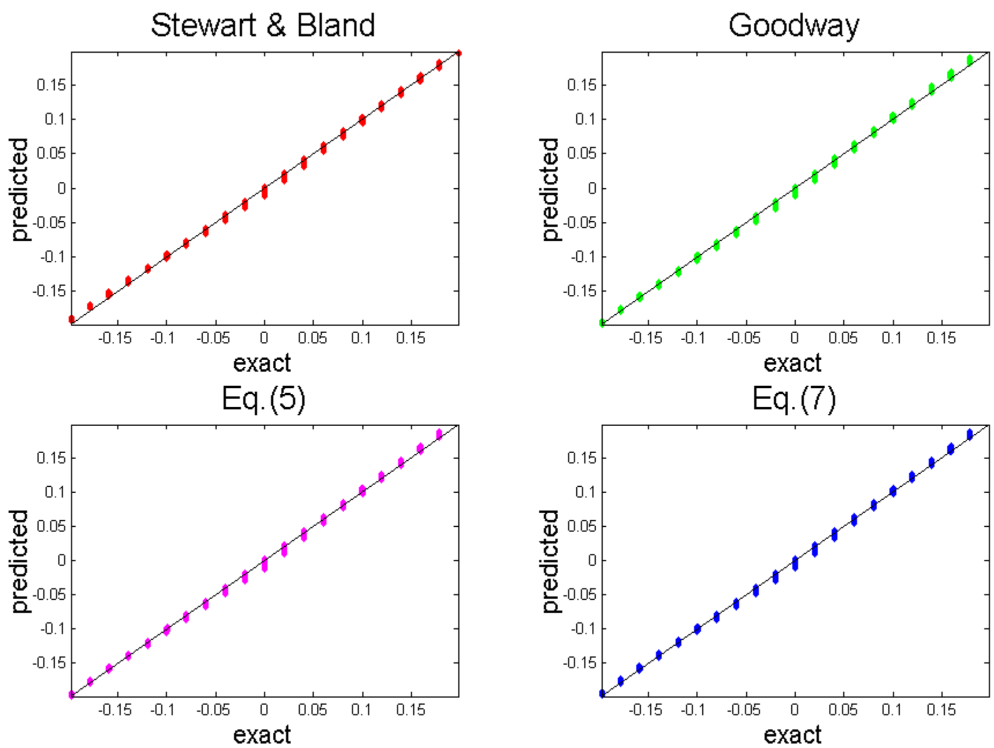

Figure 2: Crossplots of the exact $R_{SS}(0)$ for given earth parameters and the values that would predicted by four different approximations using the same earth parameters and $R_{PS}(10°)$.

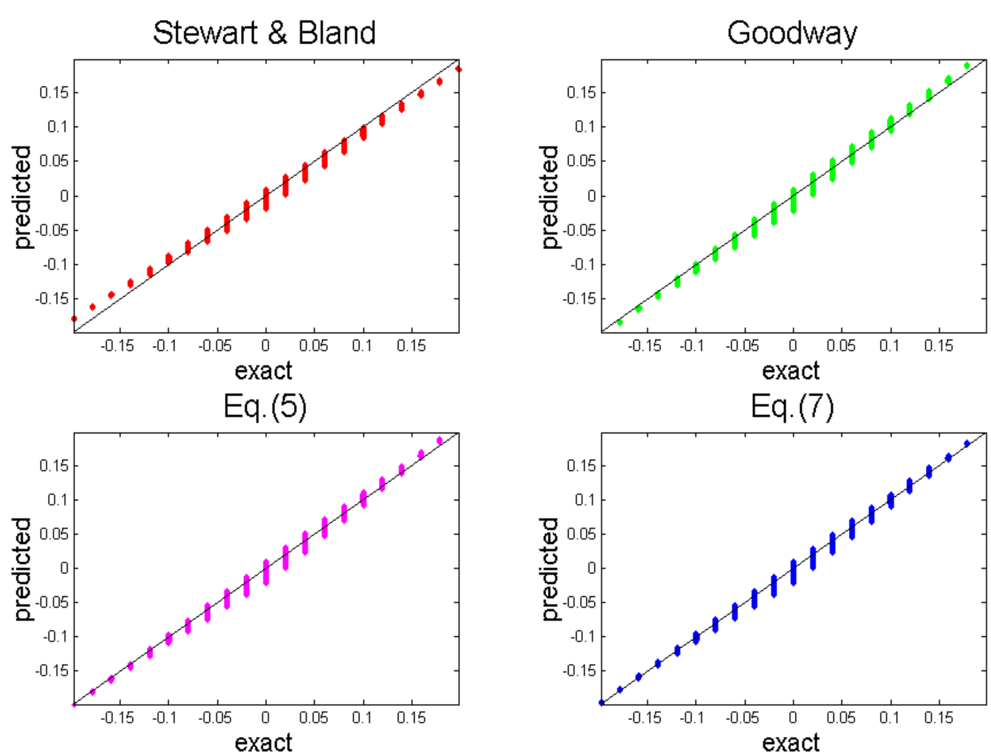

Figure 3: The same as Figure 2 but using predictions from $R_{PS}(20°)$.

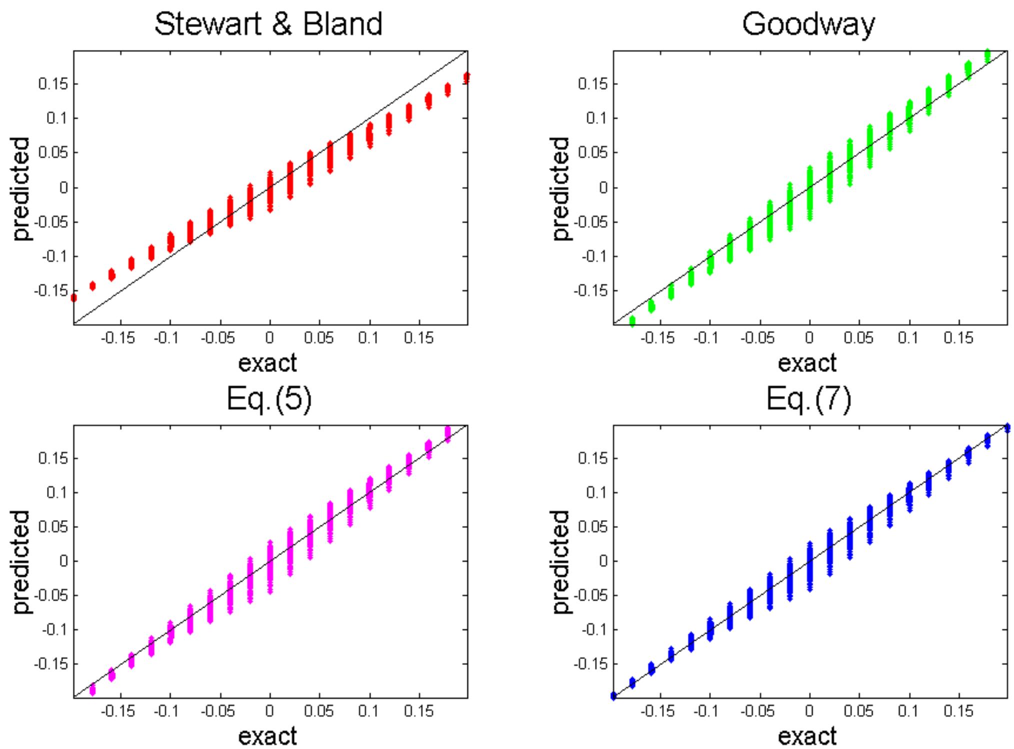

Figure 4: The same as Figure 2 but using predictions from $R_{PS}(30°)$.

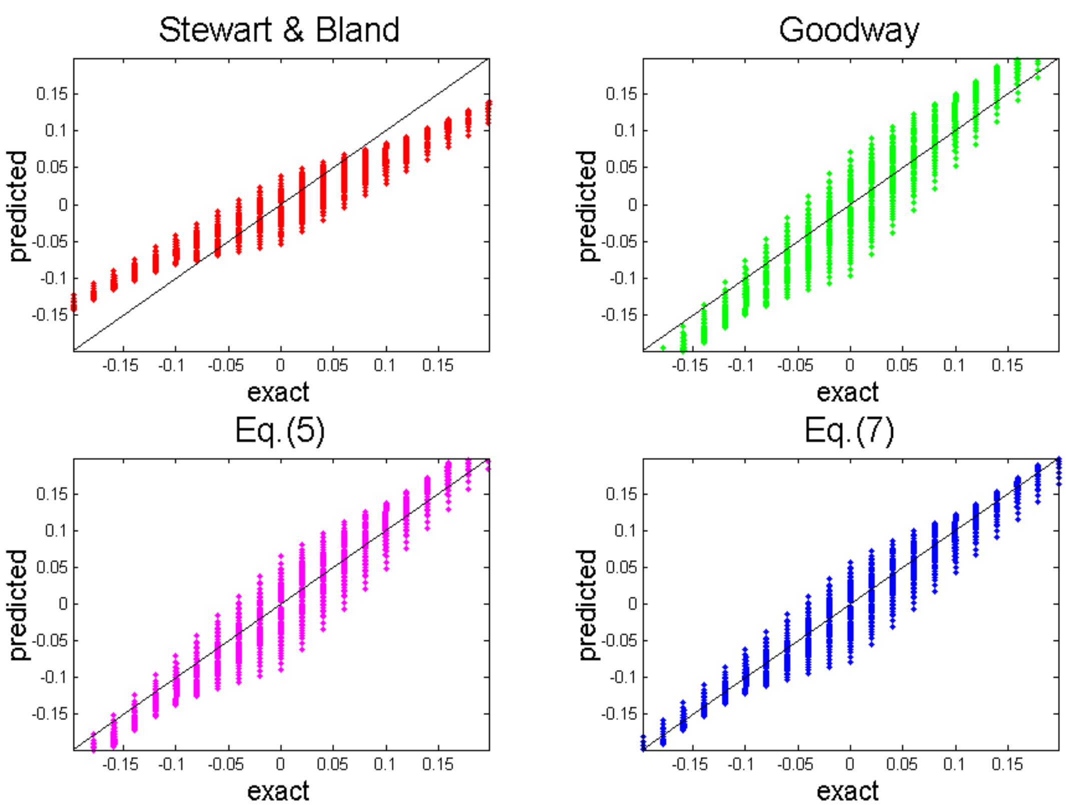

Figure 5: The same as Figure 2 but using predictions from $R_{PS}(40°)$.

From Figures 2-5 we see that the Stewart and Bland approximation is best for positive reflectivities at very small angles but the other three approaches are superior at large angles, as expected.

To distinguish among these three methods we use the exact Zoeppritz equations to generate the ratio $R_{SS}(0)$ / [$R_{PS}(\theta)/\sin\theta$] for approximately 1000 sets of earth parameters, sampled from a grid of points in the range [-0.1,0.1]. From these SS/PS functions, we calculate an average ratio. A number of different averages were tested (arithmetic, harmonic, geometric). In the end the only average which appeared to be centered in the main concentration of lines was a weighted average, in which each ratio was weighted by $R_{SS}(0)^2$. A set of calculated ratios, and their weighted average, is shown below in Figure 6.

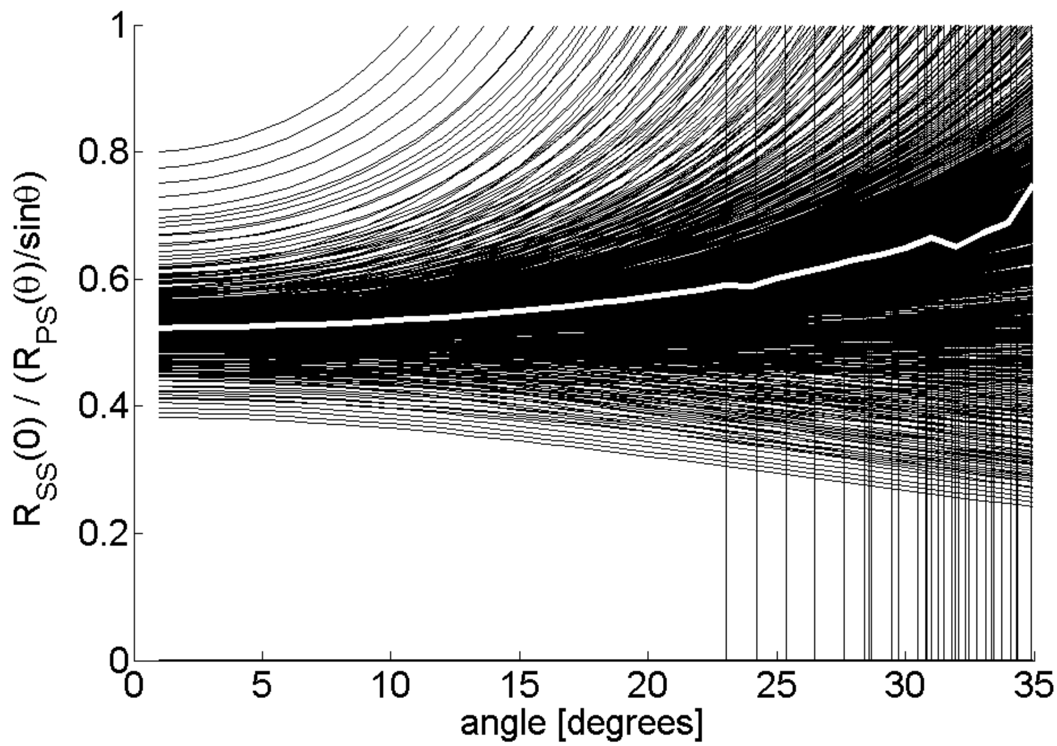

Figure 6. A display of the ratio of $R_{ss}(0)$ to $R_{PS}(\theta)/\sin\theta$. $R_{ss}(0)$ to $R_{PS}(\theta)$ have been calculated from the exact Zoeppritz equations. Each of the dark lines represents this SS/PS value calculated for a different set of earth parameters. The light line in the center is the weighted average, where $R_{ss}(0)^2$ has been used as the weighting factor. The velocity ratio β/α was constrained to a value of 0.47 for all lines in this figure, while $\Delta\alpha/\alpha$, $\Delta\beta/\beta$, and $\Delta\rho/\rho$ all varied between –0.1 and 0.1.

In Figure 6, the velocity ratio β/α was constrained to a value of 0.47, while Δα/α, Δβ/β, and Δρ/ρ were sampled independently from a range of –0.1 to 0.1. Most of the ratio values are within about 0.05 of the weighted average line. Inspection of individual lines and their associated $R_{SS}(0)$ values and $R_{PS}(\theta)/\sin\theta$ functions (not shown) indicate that the outlying lines - including the unruly vertical lines - generally correspond to very small $R_{SS}(0)$ values. This is consistent with the weighted average being superior to the other averages. In practical terms, it is more important to estimate $R_{SS}(0)$ accurately when it is large than when it is very small.

Next, we compare this weighted average to the three approximations given above. These results are displayed in Figures 7 and 8 below.

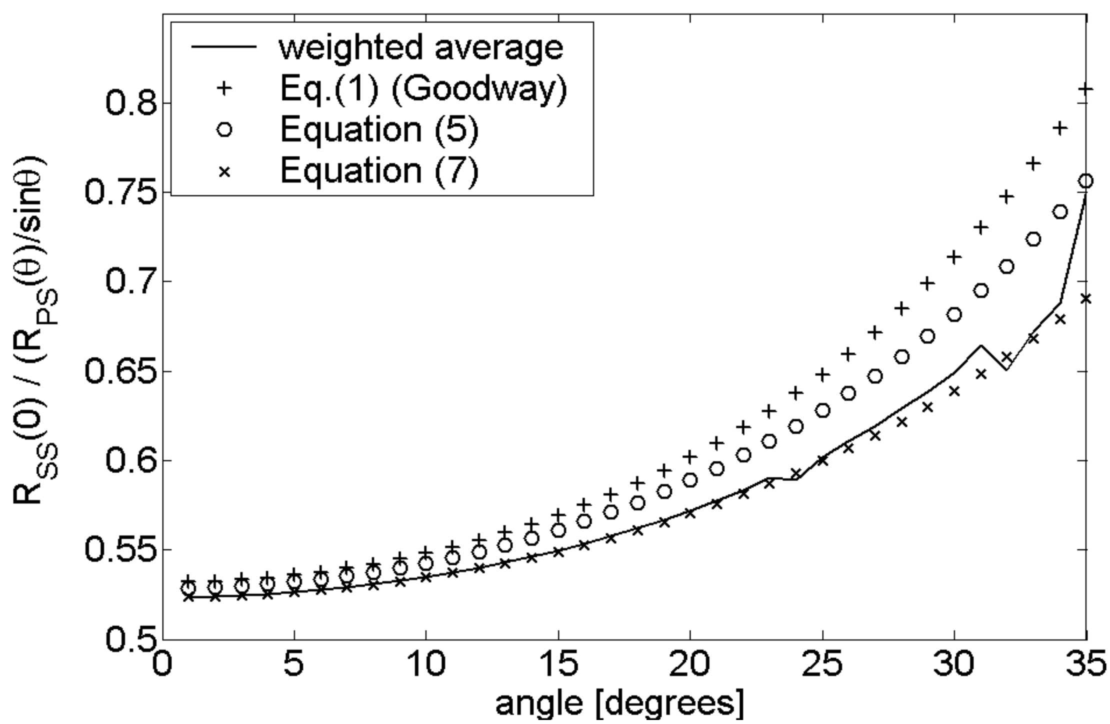

Figure 7. Comparison of the weighted average from Figure 6 with three approximate theoretical expressions defined in the text. Eq. (7) appears to represent the average quite accurately.

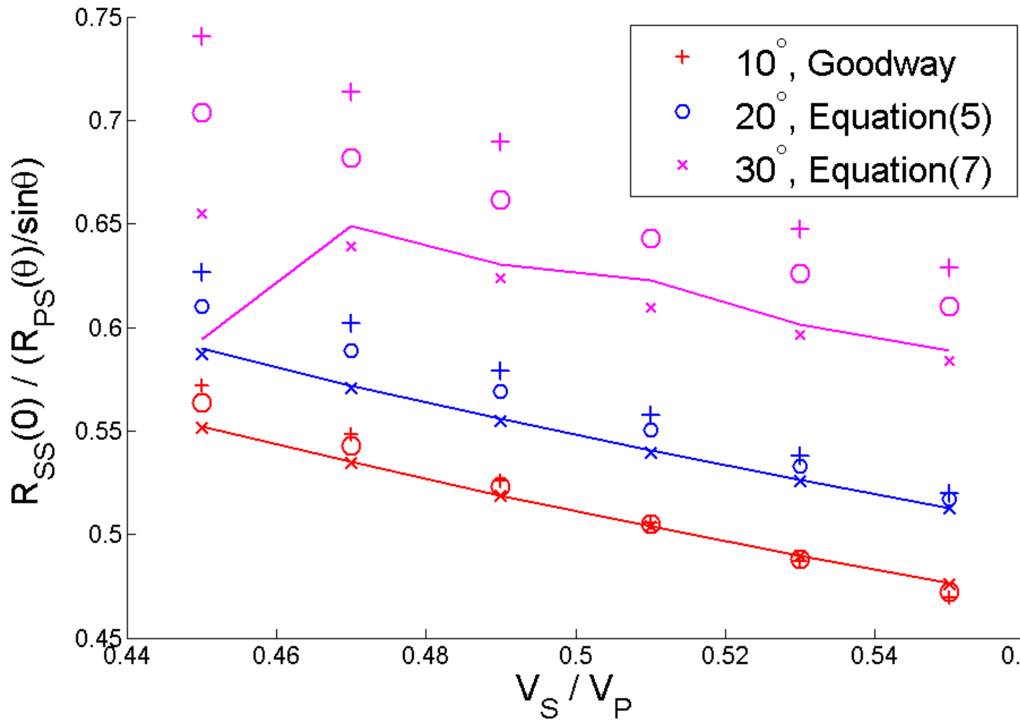

Figure 8. The description of this figure is similar to that of Figure 7 except that values are plotted against the β/α ratio with θ = 10, 20, 30 degrees for the three lines.

From Figures 7 and 8 we would conclude that Eq. (7) is the most accurate theoretical expression. However, another issue concerns the choice of sample earth parameters. Suppose that we felt that Δρ/ρ would on average be more representative of the real earth if it were related to velocity contrast through a Gardner-type relation. This possibility is explored in Figure 9 below. Figures 9a and 9b illustrate the case when Δρ/ρ = ¼ Δα/α, as per the original Gardner relation. Δα/α and Δβ/β are still sampled independently from the interval [-.1,.1], and β/α and θ are still fixed to the same values as in Figures 7 and 8. Figures 9c and 9d are similar but with Δρ/ρ = 0.235 Δβ/β, as derived by Ursenbach (2001). In Figures 9a and 9b, Goodway's Eq. (1) is extremely accurate, while in Figures 9c and 9d Eq. (5) is most representative. This latter result is reasonable since it was derived using a shear-wave Gardner relation, similar to the one imposed on the averaged SS/PS values in Figure 9c and 9d. Calculations were also carried out using a generalized Gardner relation (Ursenbach, 2001) that sets Δρ/ρ = 0.080 Δα /α + 0.164 Δβ /β. These results are not shown, but display behavior intermediate between 9a,b and 9c,d, as one would expect. Thus, depending on what sort of understanding one has regarding the earth layers under consideration, one theory may be found to be more suitable than another.

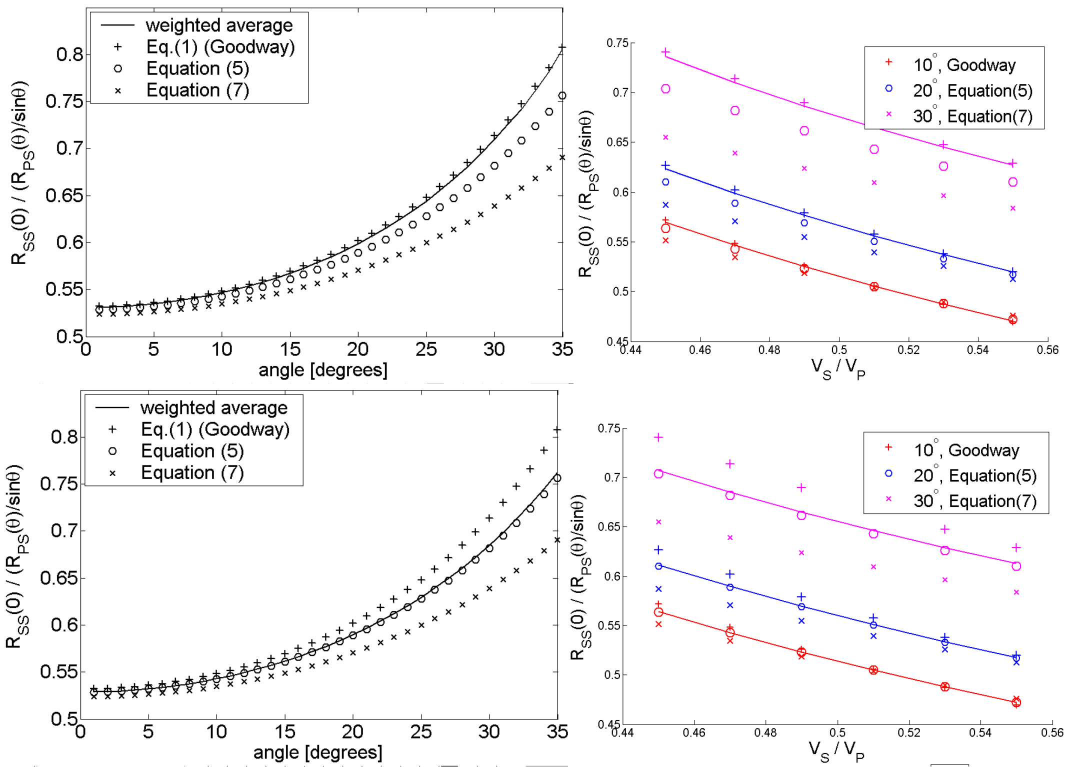

Figure 9. Weighted average SS/PS values with constrained Δρ/ρ values, compared with three approximate theoretical expressions defined in the text. In the upper plots Δρ/ρ = ¼ Δα/α, while in the lower plots Δρ/ρ = 0.235 Δβ/β. The former is best described by Equation (1) of Goodway (2001), while the latter is best described by Equation (5) of this paper.

A USEFUL EMPIRICAL EXPRESSION

Many of the calculations to this point have used a typical but arbitrary velocity ratio of β/α = 0.47. In Figure 10 we illustrate in a different way the effect on the weighted average ratios when β/α varies. Δρ/ρ is again allowed to vary independently of Δα/α and Δβ/β, as in Figure 7. The various theoretical expressions are not shown, but their behavior at other values of β/α is comparable to that shown in Figures 7-9. Also shown in Figure 10 is the fitting of an empirical function to the weighted averages. The fitting is only attempted within the range of 0 to 20 degrees. We note from Figures 1-6 that the ratios tend to vary more widely at higher angles, and this is reflected in unruly behavior above 20 degrees in Figures 6 and 10. Although the theoretical expressions are quite accurate, as shown above, the empirical expressions are simple in form and may be convenient for some applications. When a value of β/α is not available, for instance, a reasonable course is to assume that β/α = ½, resulting in the empirical expression $R_{SS}(0) = [½ + ½ \sin^2 (0.9\ \theta)]\ R_{SS}(\theta)/\sin\theta$, suitable for stacking of traces with θ < 20°.

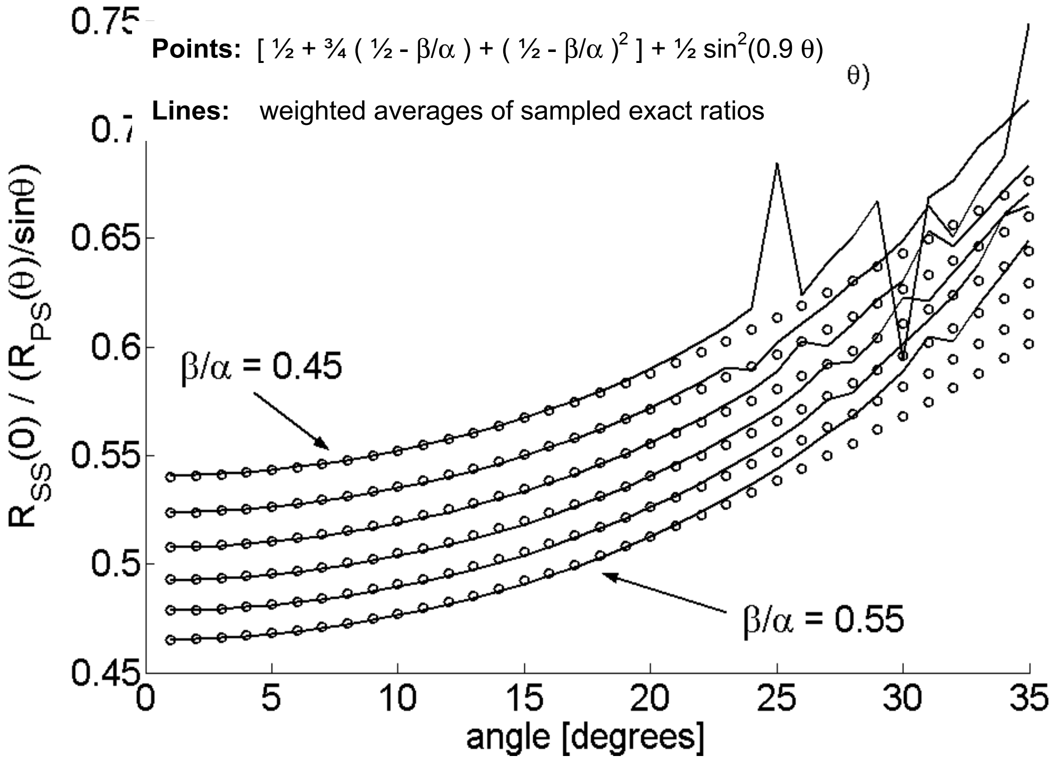

Figure 10. The weighted averages of SS/PS values for several values of β/α are shown as solid lines. This figure illustrates that the functional form of the SS/PS average is largely independent of velocity ratio, but is shifted upwards as β/α decreases. Empirical fits are also shown as points on the graph. The simple form of these may be useful for some applications.

CONCLUSIONS

We have derived two new theoretical expressions relating the zero-offset shear reflectivity to converted-wave reflectivities as a function of offset. These, along with an earlier expression by Goodway (2001), and also some simple empirical functions, have been shown to have encouraging potential for the stacking of converted-wave traces to obtain shear reflectivity. We have also presented a new expression, subject only to the Aki-Richards approximation, which could be used in an AVO procedure to obtain the zero-offset shear reflectivity.

A number of specific observations arise from the calculations shown in this paper. It is more accurate to estimate $R_{SS}(0)$ at small angles than at large angles, and the method of Stewart and Bland can be most accurate at small angles. The other approximations are more accurate at large angles. The estimations are more accurate when the target $R_{SS}(0)$ value is, itself, large rather than small. Another useful observation is that from the spread in plots one could perhaps estimate an uncertainty in the estimated $R_{SS}(0)$ value. Finally, we note that the actual values of the earth parameters will influence which of the theories is most likely to be accurate in real data.

REFERENCES

Aki, K., and Richards, P.G., 1980. *Quantitative Seismology: Theory and Methods*, W.H. Freeman and Co., San Francisco.

Dey, A.K., and Stewart, R.R., 1997. Predicting density using V_S and Gardner's relationship, in *CREWES 1997 Research Report*, Vol. 9. (Available at www.crewes.org)

Gardner, G.H.F., Gardner, L.W., and Gregory, A.R., 1974. Formation velocity and density – The diagnostic basics for stratigraphic traps. *Geophys.*, 39, 770-780.

Goodway, W., 2001. Elastic-wave AVO for lithology description and fluid detection, M.Sc.Thesis, University of Calgary.

Larsen, J. A., 1999. AVO Inversion by Simultaneous P-P and P-S Inversion, M.Sc. Thesis, University of Calgary.

Potter, C.C., 1999. Relating density and elastic velocities in clastics: an observation, in *CREWES 1999 Research Report*, Vol. 11.

Potter, C.C., and Stewart, R.R., 1998. Density predictions using V_P and V_S sonic logs, in *CREWES 1998 Research Report*, Vol. 10.

Stewart, R.R. and Bland, H., 1997. An approximate relationship between R^{ps} and R^{ss}, in *CREWES 1997 Research Report*, Vol. 9.

Ursenbach, C.P., 2001. A generalized Gardner relation, in *CREWES 2001 Research Report*, Vol. 13.

Ursenbach, C.P. and Stewart, R.R., 2002. Estimating Pure Shear Reflectivity from Converted Wave AVO, in *CSEG 2002 Extended Abstracts*.

Wang, Z., 2000. Velocity-Density Relationships in Sedimentary Rocks, in *Seismic and acoustic velocities in reservoir rocks*, Vol. 3, Recent developments. *Soc. Expl. Geophys.*, Geophysics Reprint Series.