Suppression of multiples from complex sea-floor by a wave-equation approach

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ABSTRACT

We present a new efficient wave-equation scheme for prediction and subtraction of water-layer multiples and peg-legs from an arbitrary 2D irregular sea-floor. The scheme accounts for both multiple reflections and diffractions. The method requires approximate knowledge of the sea-floor geometry. The predicted multiples are split into three terms, where each term contains multiple events which require the same amplitude correction. Therefore the method has much fewer problems with the adaptive subtraction of predicted multiples than the iterative SRME approach, especially for data from areas with a shallow sea-floor. In our scheme all source-side and receiver-side multiples of all orders are suppressed simultaneously in one consistent step (in one or a few time windows) from the Radon-transformed CMP or common shot gathers. The prediction of multiples both from receiver-side and source-side starts with data in the same domain, therefore no additional sorting or additional Radon transformations are required.

INTRODUCTION

Removal of water-layer multiples and peg-legs from hard and/or irregular sea-floor is still one of the major processing problems in offshore exploration. For data from such areas, methods based on velocity discrimination are often inefficient owing to complicated moveout of multiple events. The surface-related multiple elimination approach (SRME) which predicts all surface multiples from the data themselves, should work at least for data from areas with deep water. Only one iteration of SRME is required for prediction of multiples and fewer problems occur with different amplitude corrections for interfering multiples of different order. However, in practice this method almost never leads to the best results. Probably this is related to sampling issues, 3D effects, the fact that the designature operator (which is used for adaptive subtraction of multiples) should be angle/offset dependent, and because the designature operator is less compact and involves more unknowns than the reflectivity series required by wave-equation (WE) approaches. WE methods do not target all free-surface multiples, but do deal with those that are most often troublesome, i.e. water-layer multiples and peg-legs. In contrast to SRME, these methods require approximate knowledge of the sea-floor geometry. The method described in this paper belongs to the class of WE methods (see references).

The main features of our scheme are the following: 1) The adaptive subtraction of the predicted multiples is performed in the tau-p domain, therefore we take into account the angle-dependency of the reflection coefficients from the water-bottom; 2) Multiples from source side and receiver side are suppressed simultaneously; 3) The prediction procedure starts from the Radon transformed input CMP or common shot gathers and results in the Radon transformed CMP or common shot gathers of the predicted multiples; 4) The prediction of multiples is performed in the same domain as used for multiple suppression. Therefore no additional sorting or additional Radon transformations are required; 5) Both multiple reflections and diffractions are predicted.

SUPPRESSION OF WATER-LAYER MULTIPLES AND PEG-LEGS BY THE WAVE-EQUATION APPROACH

The rigorous derivation of our wave-equation multiple suppression operator is given in Lokshtanov (1999b). Here we will follow a more intuitive approach. Denote by D the pre-stack data (in whatever domain) along the profile. Denote by P_g the operator for receiver-side extrapolation of the input data through the water-layer. P_g includes the propagation of the recorded wavefield down to the sea-floor, reflection from the sea-floor (multiplied by the reflection coefficient of the free-surface) and propagation up to the free-surface. As a result of such extrapolation, the primary water-bottom reflection is 'transferred' to the first-order multiple; the first-order multiple is 'transferred' to the second-order multiple; each primary reflection from below the water-bottom is 'transferred' to the first-order receiver-side peg-leg; each first-order receiver-side peg-leg is 'transferred' to the second-order peg-leg and so on. Therefore the operator $\left(I + P_g\right)$ applied to the data D removes all 'pure' water-layer multiples and water-layer receiver-side peg-legs: the result $F_1 = \left(I + P_g\right)D$ is free from all 'pure' water-layer multiples and receiver-side peg-legs. Suppose now that we exchange the positions of sources and receivers using the reciprocity principle (neglecting the difference in source and receiver directivity patterns). The source-side peg-legs for the original geometry become the receiver-side peg-legs for the new geometry. Similarly to the previous step, these peg-legs can be removed from data F_1 by the operator $\left(I + P_s\right)$ where P_s is the source-side extrapolation operator. Note that the result F_1 contains the primary reflection from the water-bottom. Consequently the operator P_s applied to F_1 creates the first-order 'pure' water-layer multiple which is already removed from F_1. Therefore the data F without all 'pure' water-layer multiples and peg-legs can be obtained as follows:

$$F = \left(I + P_s\right)\left(I + P_g\right)D - P_s D_w, \tag{1}$$

where D_w is the primary reflection from the water-bottom. This reflection is easily separated from the rest of the data in the tau-p domain. The formula (1) can be rewritten as:

$$F = D + D_g + D_s + D_{sg} \ , \tag{2}$$

where $D_g = P_g D$, $D_s = P_s(D - D_w)$, $D_{sg} = P_s P_g D$. So far we have assumed that the extrapolation operators include the reflection coefficients of the water-bottom. In practice, we do not know these coefficients. Therefore we calculate the results

D_g, D_s, D_{sg} (see next section) assuming that the reflection coefficients are equal to one for all angles of incidence and all reflection points along the sea-floor. Such extrapolation predicts correctly the kinematics of multiples, but not their amplitudes. Consequently, all extrapolation results in (2) should be properly scaled. The 'scaled version' of (2) is applied trace by trace to the Radon transformed CMP or common shot gathers. For each *p*-trace the operator has the form:

$$f(\tau) = d(\tau) + r_g(\tau) * d_g(\tau) + r_s(\tau) * d_s(\tau) + r_{sg}(\tau) * d_{sg}(\tau), \tag{3}$$

where $d(\tau), d_g(\tau), d_s(\tau), d_{sg}(\tau)$ are *p*-traces for the input data and the results of extrapolation from the receiver-side, source-side (of muted input data) and source-side after receiver-side respectively. The filters $r_g(\tau), r_s(\tau), r_{sg}(\tau)$ account for angle-dependent reflection coefficients from the water-bottom and small phase-shifts due to imperfect knowledge of the water-bottom geometry. The filters are estimated from the criterion of minimum energy of *f*. Note that for a 'locally' 1D structure, the operator (3) is transformed into our single-channel data-consistent deconvolution operator *Remul* (Lokshtanov, 1999a).

EXTRAPOLATION OF THE RADON-TRANSFORMED CMP GATHERS

The multiple suppression operator (3) requires the Radon-transformed CMP or common shot (CS) gathers of input data and of extrapolation results. The general case of irregular sea-floor is considered in the following sections. Here we describe an efficient procedure for calculation of the required Radon-transformed CMP gathers assuming 'locally' horizontal water-bottom and arbitrary 2D structure below it. Note that in contrast to the conventional phase-shift extrapolation our procedure does not require separate FK or Radon transforms (of common shots and common receiver gathers) for source-side and receiver-side extrapolations. Assume that CMP coordinate *y* increases in the shot direction. Then the input Radon transformed CMP gathers $D(p, y)$ can be represented as follows (Lokshtanov, 1999b):

$$D(p, y) = \frac{\omega}{2\pi} \int R(p, p_d) \exp\{i\omega\, p_d y\} dp_d, \tag{4}$$

where $R(p, p_d)$ is the complex (frequency dependent) amplitude of the reflected plane wave with slowness p_g due to the incident plane wave with slowness p_s; $p_g = p - p_d/2$, $p_s = p + p_d/2$. With these notations the results of receiver-side extrapolation $D_g(p, y)$ can be obtained as:

$$D_g(p,y) = \frac{\omega}{2\pi}\int R(p,p_d)\exp\{i\omega(p_d y + 2q_g h)\}dp_d =$$

$$\frac{\omega}{2\pi}\int D(p,x)\left\{\int \exp\{i\omega[p_d(y-x)+2q_g h]\}dp_d\right\}dx, \tag{5}$$

where h is the 'local' water-bottom depth, while $q_g = (1/c^2 - p_g^2)^{1/2}$ is the vertical slowness; c is water velocity. The integral in the curly brackets can be calculated by the stationary phase approximation. For each pair x, y the stationary point p_d^{st} corresponds to a simple relation (figure 1): $x - y = h\ \mathrm{tg}\alpha_g$ where $p_g = c\sin\alpha_g$ and $p_g = p - p_d^{st}/2$. The extrapolation from the source-side is performed in a similar way.

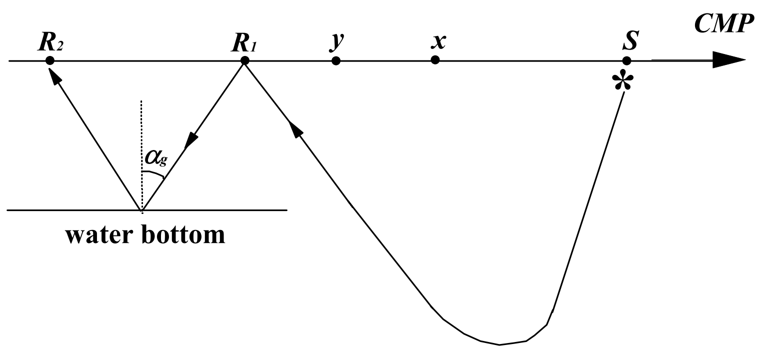

FIG. 1. Illustration of the stationary phase result: *x* and *y* are CMP positions of the primary and multiple events respectively. They are related by: $x - y = 0.5(R_1 - R_2) =$ h tg α_g.

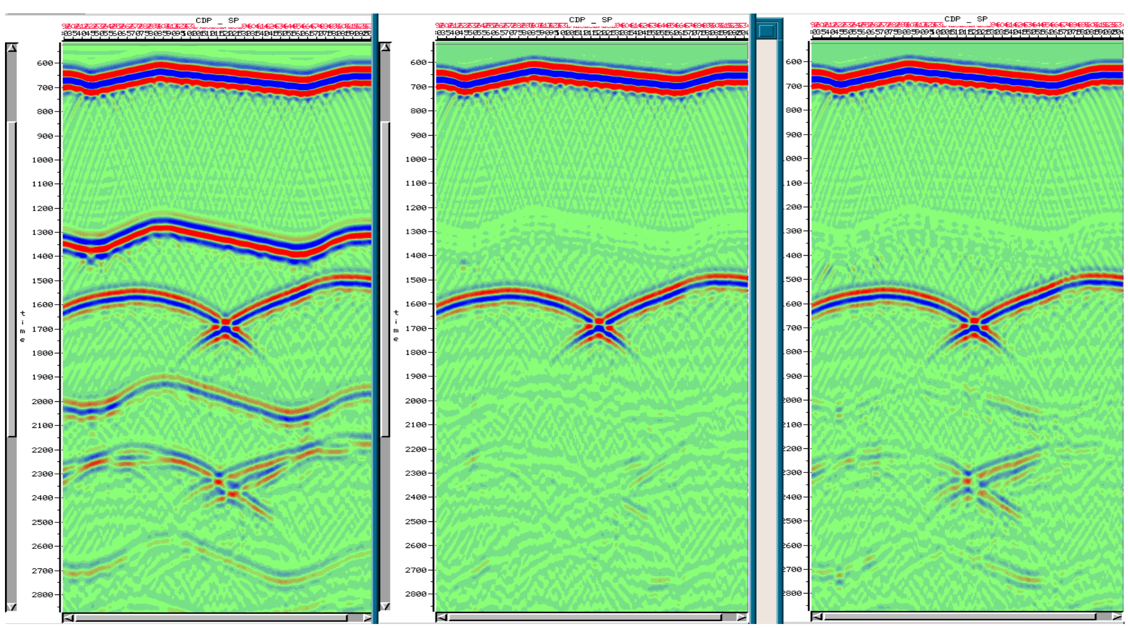

FIG. 2. Constant *P* –section before multiple suppression (left), after multiple suppression by the new wave-equation (WE) approach (centre) and after *Remul* (right).

We generated synthetic finite-difference data with all multiples for a model with an irregular water-bottom and a single complex interface below the water-bottom. The data were CMP-sorted and Radon-transformed. Figure 2 shows constant *P* sections (the same *p*-trace for all CMPs) for the input data (left), for data after our new wave-equation (WE) multiple suppression (centre) and after *Remul* (right). The result after WE is almost perfect, while after *Remul* we still have residuals of water-layer peg-legs from the second reflector. Note the difference in the positioning of the source-side and the receiver-side peg-legs in the input data. The kinematics of these peg-legs is correctly predicted by the WE approach, but not by *Remul*. The difference in the results of these two approaches increases with increasing slowness *p* (or increasing offsets or incident angle).

Figure 3 shows real data stacks before and after WE multiple suppression. Both stacks were created with the same velocity and mute libraries. The results on pre-stack level are given in Figure 4. It shows constant *P* sections (the same *p-trace* for all CMPs) before multiple suppression, after WE multiple suppression and the difference. Note how the weak dipping primaries are extracted from below strong water-layer peg-legs from Top and Base Cretaceous. Other synthetic and real data examples can be seen in Lokshtanov (2000).

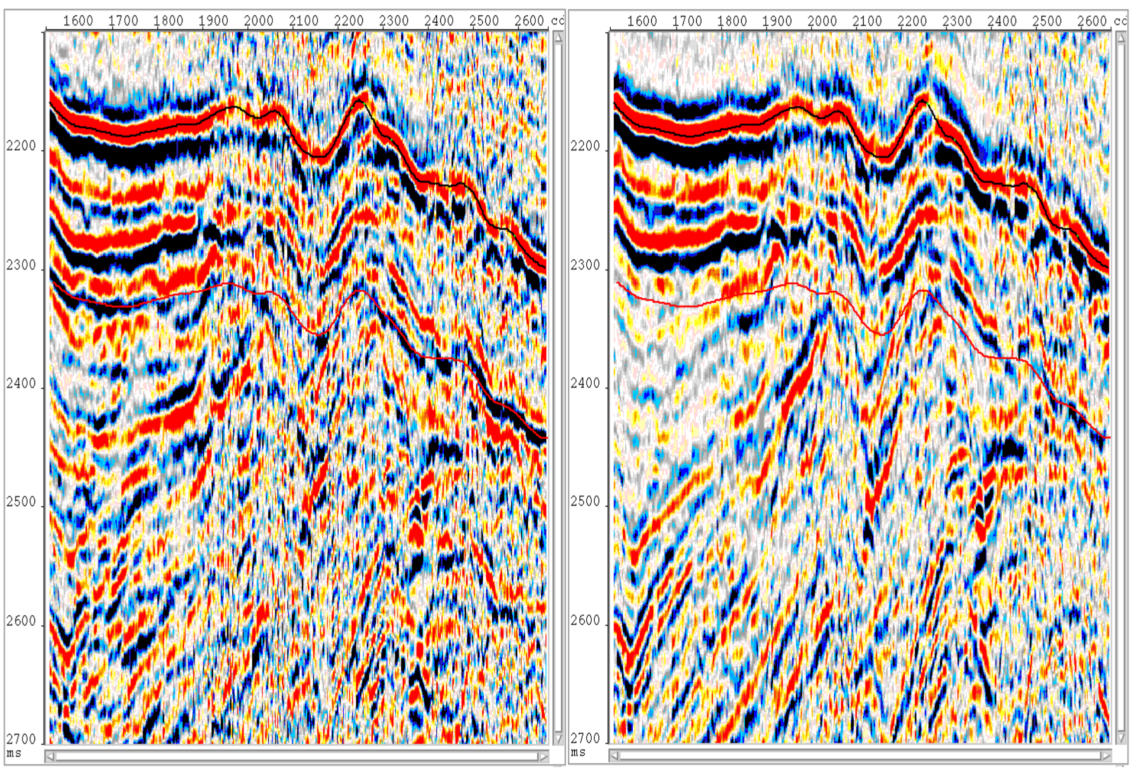

FIG. 3. Stack before multiple suppression (left) and after WE multiple suppression (right). The pink line shows the expected position of the first-order water-layer peg-leg (expected blue event) from the Top Cretaceous (black line). The multiple period is about 140 ms.

EXTRAPOLATION OF THE RADON-TRANSFORMED CS GATHERS

In the current section we consider the prediction of multiples for a general case of an irregular 2D sea-floor and an arbitrary 2D structure below it. Multiples from both receiver- and source-side are predicted from Radon-transformed common shot (CS) gathers. Denote by $D(p_r, x_s, \omega)$ the Radon-transformed CS gathers, where p_r is the receiver-side ray parameter, x_s is the source coordinate and ω is the frequency. For each frequency the recorded CS wavefield can be extrapolated down to the sea-floor:

$$W\big(x, z(x), x_s\big) = \frac{\omega}{2\pi} \int D(p_r, x_s) \exp\{i\omega[p_r(x - x_s) + q_r z(x)]\} dp_r, \tag{6}$$

where $z(x)$ is the depth of the water-bottom at lateral position x; $q_r = (1/c^2 - p_r^2)^{1/2}$ is the vertical slowness of the plane wave with horizontal slowness p_r; c is water velocity. In the prediction procedure we assume that the reflection coefficients are equal to one for all angles of incidence and all reflection points along the sea-floor. Therefore the result (6) defines the reflected wavefield along the water-bottom. This wavefield is constructed by superposition of phase-shifted recorded plane waves, therefore the effects of multiscattering along the sea-floor are not accounted for. The next step is a decomposition of reflected/scattered wavefield into plane-wave contributions. According to Wenzel et al. (1990), the amplitude $D_g(p_{sc}, x_s)$ of the reflected/scattered plane-wave with slowness p_{sc} is defined by the following expression:

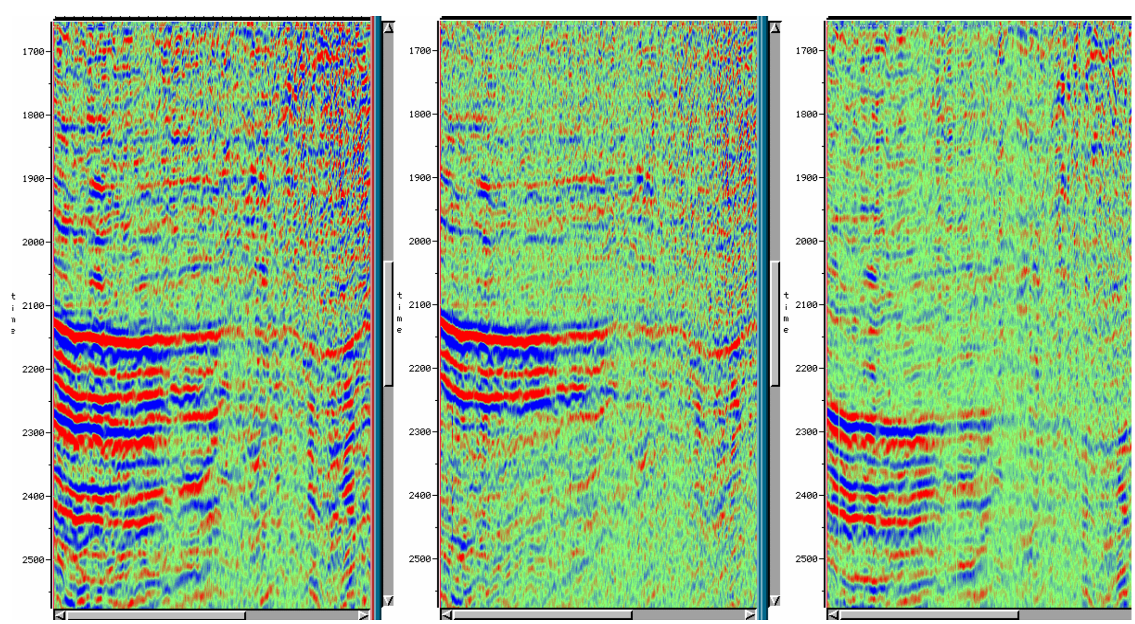

FIG. 4. Constant *P* sections for input data (left), after WE multiple suppression (centre) and the difference (right). The corresponding vertical angle of wave propagation at the surface is about 8°. Note how the weak dipping primaries are extracted from below strong multiples.

$$D_g(p_{sc},x_s)=\int W(x,z(x),x_s)\cdot\left[1+\frac{dz}{dx}\cdot\frac{p_{sc}}{q_{sc}}\right]\exp\{i\omega[-p_{sc}(x-x_s)+q_{sc}z(x)]\}dx, \qquad (7)$$

where $q_{sc}=(1/c^2-p_{sc}^2)^{1/2}$. According to (7), each point of the boundary acts as a secondary plane-wave source with the amplitude $W(x,z(x))\cdot\left[1+\frac{dz}{dx}\cdot\frac{p_{sc}}{q_{sc}}\right]$. Formula (7) also does not account for effects of multiscattering and is derived assuming that the reflected/scattered wavefield in the water layer consists of up-going waves only. Formulas (6) and (7) define the shot-by-shot procedure for the prediction of multiples from the receiver side.

Consider now the extrapolation of input data $D(p_r,x_s,\omega)$ from the source side. First we use FFT to decompose the input data into contributions with different propagation angles (wavenumbers) from the source side:

$$R(p_r,k,\omega)=\int D(p_r,x_s,\omega)\exp\{-ikx_s\}dx_s, \qquad (8)$$

where the source side wavenumber k_s is defined as: $k_s=\omega p_r+k$. Then we extrapolate the results of decomposition down to the sea-floor:

$$W(x,z(x),p_r)=\frac{1}{2\pi}\int R(p_r,k)\exp\{i[k_s x+k_{sz}z(x)]\}dk, \qquad (9)$$

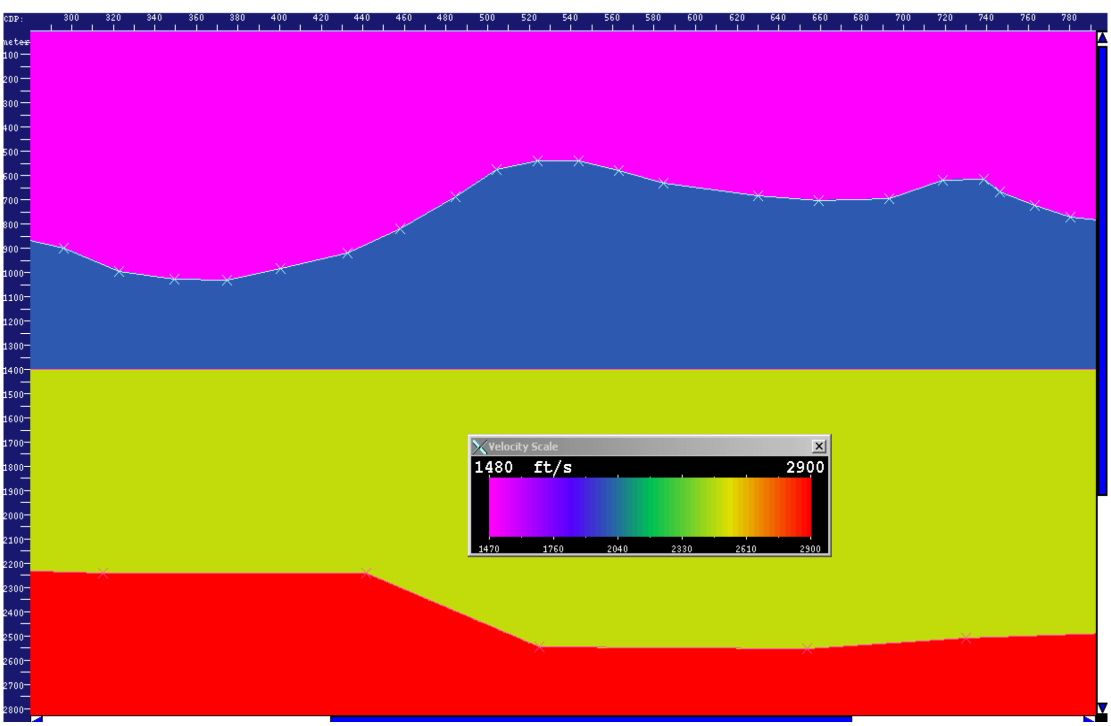

FIG. 5. Velocity model for FD modelling (with ProMax). Colour velocity scale is in m/s.

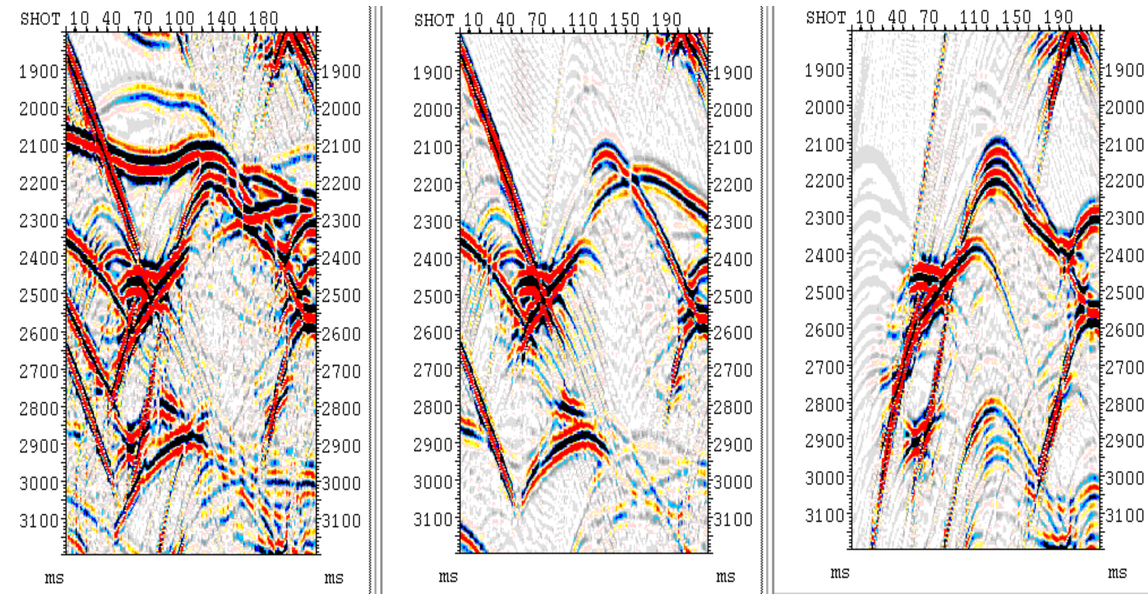

FIG. 6. Constant p_r sections (angle at the surface is about 20 degrees). Multiple events in the input (left) can be identified in the result of receiver-side prediction (centre) or in the result of source-side prediction (right). Note that a strong event at about 2070 ms at the left side of the input section is a primary reflection from the third boundary.

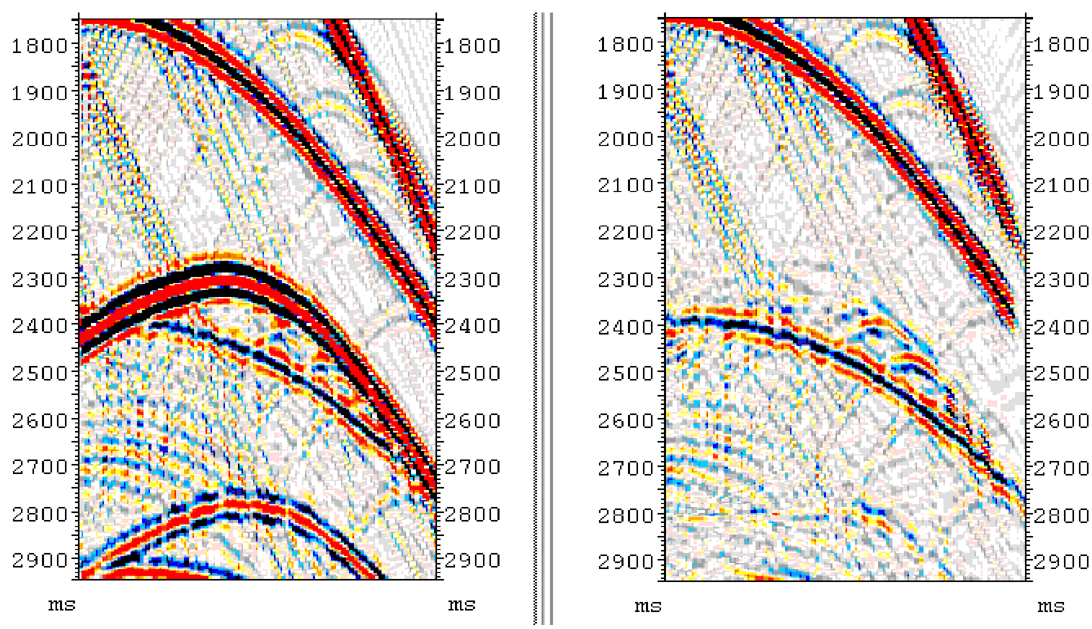

FIG. 7. Input shot gather (left) and shot gather after multiple prediction, subtraction and inverse Radon transform. A primary reflection from the third interface (at about 2400 ms at minimum offset) is extracted from below a strong multiple.

where $k_{sz} = (\omega^2/c^2 - k_s^2)^{1/2}, \;\; \mathrm{Im}(k_{sz}) \geq 0$. Finally, we use (7) to define reflected/scattered responses for each k and then the inverse of (8) to calculate Radon-transformed CS gathers after source-side extrapolation. Note that all these steps starting from (8) are performed in a double loop over p_r and over ω. Also, note that in the definition of the sea-floor geometry $z(x)$ and the sign convention used, the positive direction of x coincides with the shooting direction for source-side extrapolation and has the opposite direction for receiver-side extrapolation. Figures 6 and 7 show the prediction and subtraction results from finite-difference synthetic data for a model with a strongly irregular sea-floor, Figure 5. The multiples are well predicted (Figure 6) and strongly suppressed (Figure 7).

CONCLUSIONS

If structural variations in the crossline direction are not severe and the main free-surface multiples are water-layer multiples and peg-legs, our wave-equation (WE) approach performs well and is computationally efficient. Both multiple reflections and multiple diffractions are accounted for. All predicted multiples of all orders are suppressed simultaneously in one consistent step in one or a few time windows.

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