Finite difference modelling in structurally complex anisotropic medium

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ABSTRACT

Modelling seismic data is a very important aspect in exploration seismology. In this paper, we derive a wave equation for a tilted TI medium and apply it to a thrust sheet model. We also compare the data generated to the physical modelling data. It is also shown that the traveltimes of both physical modelling data and the numerical data match.

INTRODUCTION

The velocity structure of the earth is fundamentally anisotropic, i.e. the velocity varies with the direction of propagating of energy. Modelling algorithms which are used to model seismic data need to take into account the velocity anisotropy. Seismic modelling plays a very important role in exploration seismology. It is used in planning and designing seismic surveys, processing of data acquired, and in the interpretation of the data.

Anisotropy is an area of active research as shown by the number of publications over the last few decades. Helbig (e.g. Helbig, 1980), Thomsen (e.g. Thomsen, 1986), Alkhalifah (e.g. Alkhalifah et al., 1996), and Tsvankin (e.g. Tsvankin and Thomsen, 1994) have published many papers on the topic of anisotropy. Alkhalifah (2000) derived a wave equation for acoustic medium in the paper titled "Acoustic wave equation for VTI medium." Later, he proposed a scheme for numerically modelling seismic data in orthorhombic medium (Alkhalifah, 2003). Zhang et al. (2002) extended Alkhalifah's VTI formulation to TTI medium.

The most common symmetry observed in the context of exploration seismology, is Vertical Transverse Isotropy (VTI). VTI symmetry, as the name implies, is only valid when the symmetry axis is vertical. The approximation of VTI symmetry is not valid when the axis of symmetry is tilted, such as in structurally complex areas. The case in which the axis of symmetry is not vertical is termed 'tilted transverse isotropy.' Zhang et al. (2002) proposed a seismic modelling scheme in a 2-D TTI medium. Most of the commercial seismic modelling programs that are available, like *NORSAR* and *GX II*, simulate VTI medium, but can't handle TTI medium.

In this paper we develop a modelling algorithm which can be used to model 2-D seismic data in TTI medium. This modelling technique will be implemented using a Finite Difference technique.

WAVE EQUATION IN TTI MEDIA

In order to derive the wave equation in TTI media, we start with the phase velocity formulation of Daley et al. (1999). The equation for phase velocity in an anisotropic medium can be written as Equation 1. Phase velocity (V_{ph})in 2-D VTI can be written as

(Daley et al., 1999)

$$V_{ph}^2\theta = V_e^2(\theta) + \frac{A_D^2 \sin^2(\theta)\cos^2(\theta)}{V_e^2(\theta)}, \tag{1}$$

where for acoustic case A_D can be written as

$$A_D{}^2 = A_{13}^2 - (A_{11} * A_{33}), \tag{2}$$

where for acoustic case V_e can be written as

$$V_e{}^2 = A_{11}\sin^2(\theta) + A_{33}\cos^2(\theta), \tag{3}$$

A_{11} can be written as

$$A_{11} = A_{33} * (1 + 2\epsilon). \tag{4}$$

2-D WAVE EQUATION IN 2-D MEDIA

Equation 1 is now used to derive the wave equation in 2-D VTI medium. Dividing both sides by $V_{ph}{}^2$ and multiplying by $i\omega^2$ we get the following equation

$$(i\omega)^2 = (i\omega)^2 A_{11}p^2 + A_{33}q^2 + \frac{A_{D_{13}}^2 p^2 q^2}{A_{11}p^2 + A_{33}q^2}, \tag{5}$$

where p, q are the slownesses in the x and z directions respectively. They can be written as

$$-i\omega p = \frac{\partial}{\partial x}, \tag{6}$$

and

$$-i\omega q = \frac{\partial}{\partial z}. \tag{7}$$

Substituting Equations 6 and 7 into 5 we get

$$(i\omega)^2 = A_{11}\frac{\partial^2}{\partial x^2} + A_{33}\frac{\partial^2}{\partial z^2} + A_{D_{13}}^2\frac{\partial^4}{\partial x^2 \partial z^2}. \tag{8}$$

Using

$$-i\omega = \frac{\partial}{\partial \tau}, \tag{9}$$

Equation 8 can be written as

$$\frac{\partial}{\partial t^2} = A_{11}\frac{\partial^2}{\partial x^2} + A_{33}\frac{\partial^2}{\partial z^2} + A_{\delta}\frac{\partial^4}{\partial x^2 \partial z^2}, \tag{10}$$

where,

$$A_{\delta} = 2 * A_{33} * (\delta - \epsilon). \tag{11}$$

Equation 11 reduces to zero when both anisotropy parameters are equal and the symmetry of the medium reduces to the degenerate case of elliptical anisotropy. Equation 10 can now be solved using a finite-difference scheme. This equation is valid in VTI medium. If the media were TTI, we need to transform this wave equation to be valid in TTI medium. The following method is used to achieve this transformation.

AXES ROTATION IN 2-D TTI MEDIA

Up to this point we have defined the velocities, A_{11}, A_{33}, $A_{D_{13}}$, with respect to the principal crystallographic axes. Therefore, for structurally deformed medium, we need to define an angle of rotation θ, from unprimed model coordinates to primed model coordinates (Daley et al., 1999). The following recipe is used to rotate unprimed model coordinates to primed coordinates.

$$\begin{bmatrix} x \\ z \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -sin(\theta) \\ \sin(\theta) & cos(\theta) \end{bmatrix} \begin{bmatrix} x\prime \\ z\prime \end{bmatrix},$$

where the primed coordinates are the unrotated coordinates and unprimed are the rotated coordinates. Using the above orthogonal matrix, the unrotated directional space derivatives can be written in rotated coordinates as follows:

$$\frac{\partial}{\partial x} = \cos\theta \frac{\partial}{\partial x\prime} - \sin\theta \frac{\partial}{\partial z\prime} \tag{12}$$

$$\frac{\partial}{\partial z} = \sin\theta \frac{\partial}{\partial x\prime} + \cos\theta \frac{\partial}{\partial z\prime}. \tag{13}$$

Equations 12 and 13 are substituted in the wave equation derived for TTI medium (Equation 10). This equation is then numerically solved using the finite-difference technique.

TESTING

The equation is tested on different models.

Model 1

The model 1 parameters are

- Velocity=3755 m/s
- $\epsilon = 0.2$
- $\delta = 0.2$

A snapshot of the wave propagating is shown in Figure 1. The shot is located at the centre of the model.

Model 2

The model 2 parameters are

- Velocity=3755 m/s

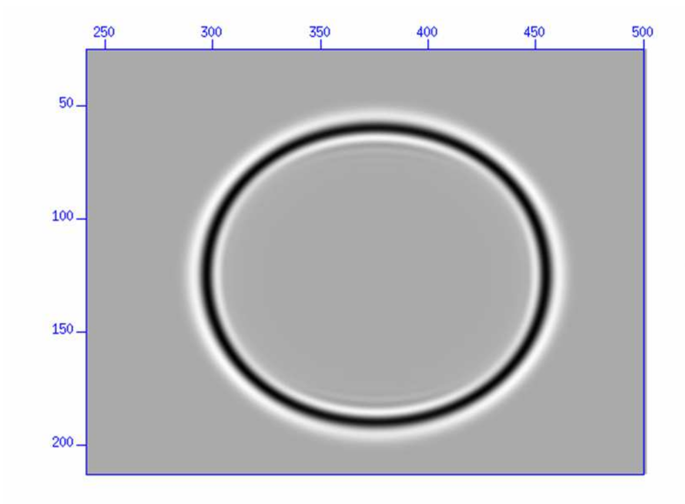

FIG. 1. Snapshot of the wavefront in medium with $\epsilon = 0.2$ and $\delta = 0.2$.

- $\epsilon = 0.2$
- $\delta = 0.1$

A snapshot of the wave propagating is shown in Figure 2. The shot is located at the centre of the model.

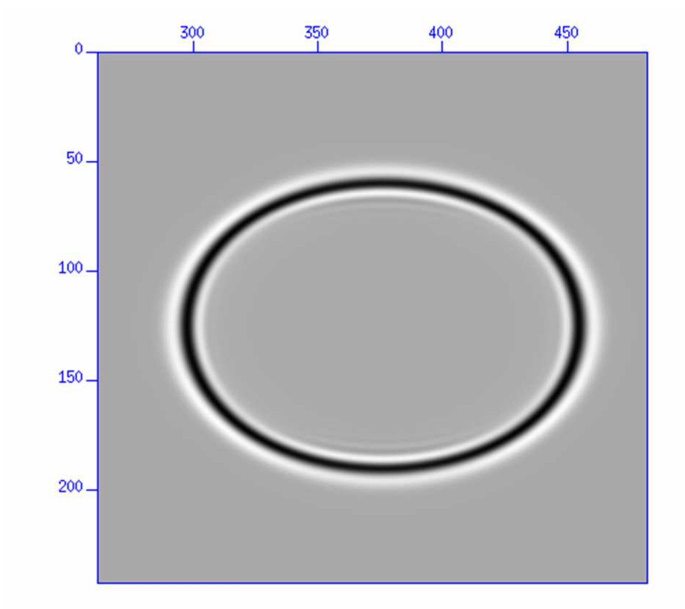

FIG. 2. Snapshot of the wavefront in medium with $\epsilon = 0.2$ and $\delta = 0.1$.

Model 3

The model 3 parameters are

- Velocity=3755 m/s
- ϵ=0.2
- δ=-0.2

A snapshot of the wave propagating is shown in Figure 3. The shot is located at the centre of the model.

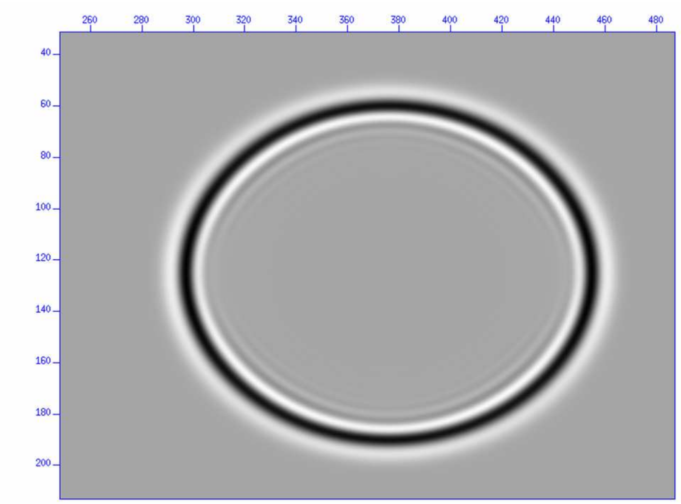

FIG. 3. Snapshot of the wavefront in medium with dip of $\epsilon = 0.2$ and $\delta = -0.2$.

Model 4

The model 4 parameters are

- Velocity=3755 m/s
- dip=$15°$
- $\epsilon = 0.2$
- $\delta = 0.2$

A snapshot of the wave propagating is shown in Figure 4. The shot is located at the centre of the model.

Model 5

The model 5 parameters are

- Velocity=3755 m/s

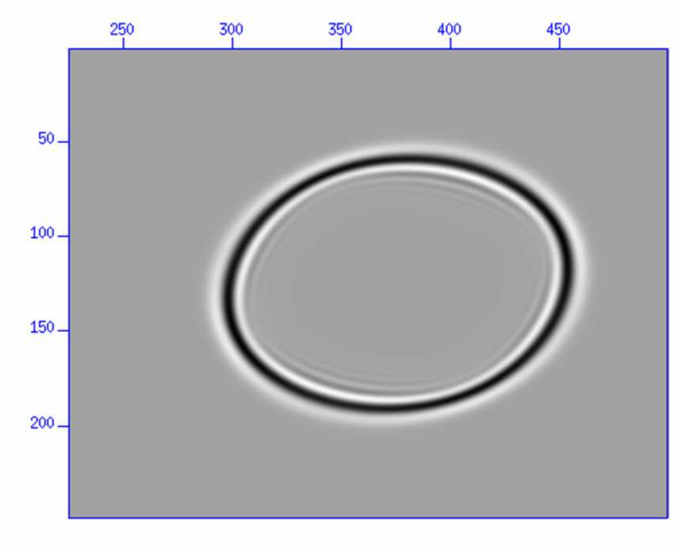

FIG. 4. Snapshot of the wavefront in medium with $\epsilon = 0.2$ and $\delta = -0.2$.

- dip=30°
- $\epsilon = 0.2$
- $\delta = 0.2$

A snapshot of the wave propagating is shown in Figure 5. The shot is located at the centre of the model.

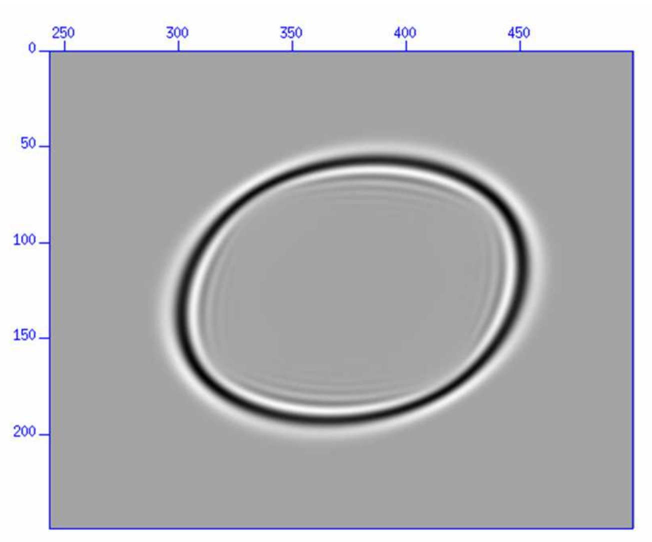

FIG. 5. Snapshot of the wavefront in medium with dip of $30°$.

Model 6

The model 6 parameters are

- Velocity=3755 m/s
- dip=45°
- $\epsilon = 0.2$
- $\delta = 0.2$

A snapshot of the wave propagating is shown in Figure 6. The shot is located at the centre of the model.

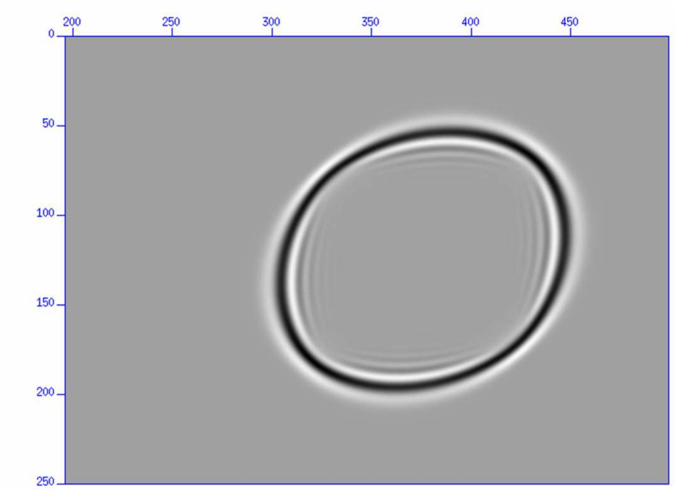

FIG. 6. Snapshot of the wavefront in medium with dip of $45°$.

Model 7

The model 7 parameters are

- Velocity=3755 m/s
- dip=45°
- $\epsilon = 0.2$
- $\delta = 0.2$

A snapshot of the wave propagating is shown in Figure 7. The shot is located at the centre of the model.

THRUST SHEET MODEL

Leslie and Lawton (2001) acquired seismic data over a physical model of an anisotropic thrust sheet. The physical model is illustrated in Figure 8. The algorithm described above is now tested on this model.

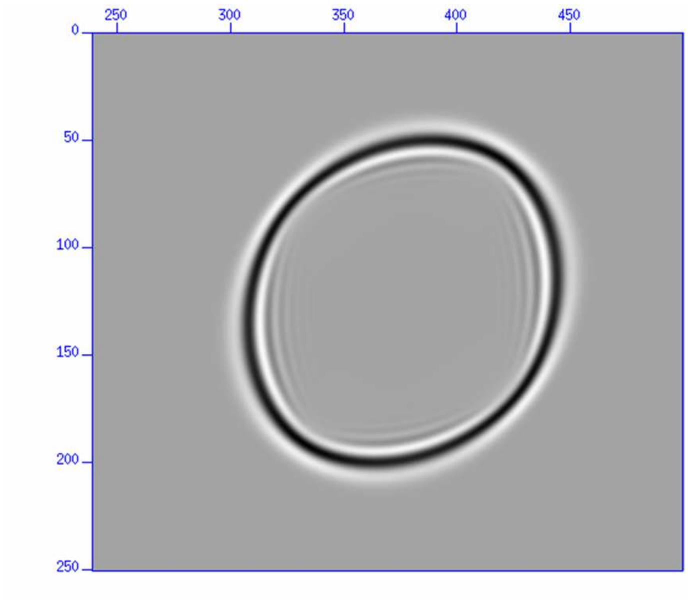

FIG. 7. Snapshot of the wavefront in medium with dip of $60°$.

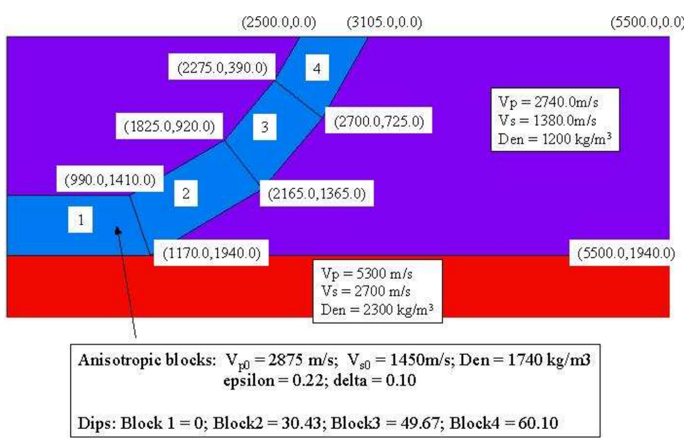

FIG. 8. Thrust sheet model (Courtesy Don Lawton).

Comparison between numerical and physical modelling data

Figure 9 allows for the comparison at the shot records acquired over the physical modelling data and the numerical modelling data. It can be seen that the traveltimes in both

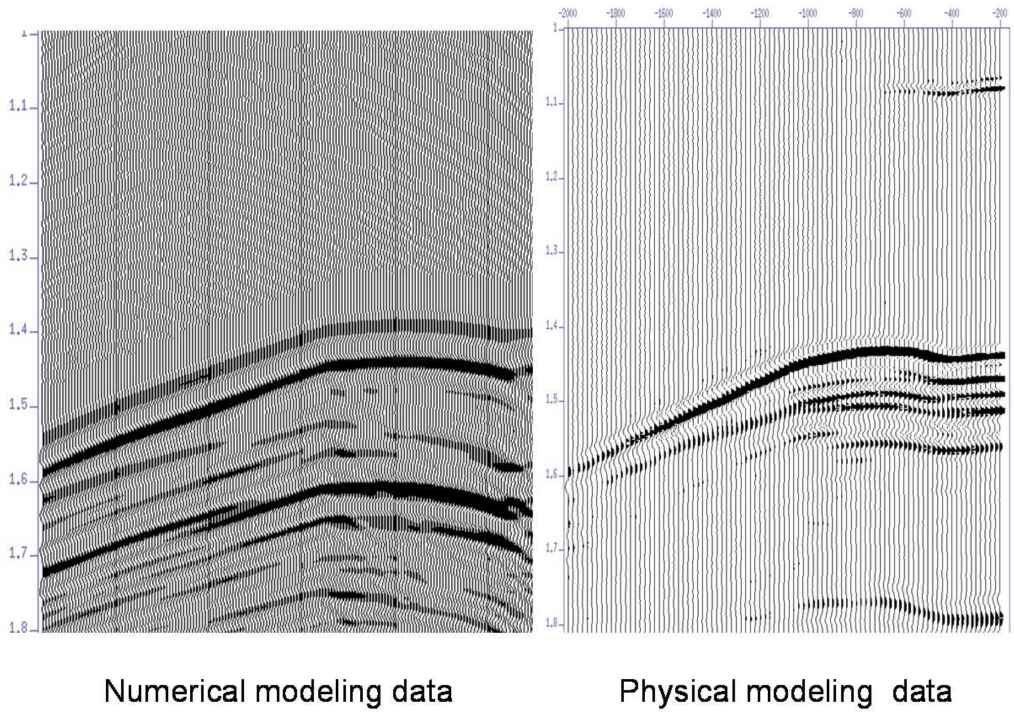

FIG. 9. Comparison between numerical modelling data and physical modelling data.

sections match with each other (Figure 9). The interesting section in the data is where the dipping anisotropic section meets the surface. As the anisotropic layer's fast direction is oriented upwards towards the surface, a pull up in traveltime is expected. We see a pull up in traveltime of the same magnitude in both the physical model data and the numerical model data.

The energy below the main reflection in the numerical modelling data is due to edge reflections.

CONCLUSIONS

In this paper we derived a wave equation for a TTI medium. We tested this equation on various simple models and displayed snapshots. The algorithm is then applied to a numerical version of a thrust sheet model. The data is then compared to the data acquired on a physical version of the same model. In future, we plan to extend this method to 3-D medium, and to improve the boundary condition implementation in the modelling code.

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