The FOCI method versus other wavefield extrapolation methods

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ABSTRACT

Recursive wavefield extrapolation methods are more powerful than ray theory based methods because of their great ability to handle strong lateral velocity variations. There are different methods to calculate frequency-space convolution operators for wavefield extrapolation. Wavefield extrapolation methods have two major problems (1) the extrapolator instability and (2) they are computationally expensive.

The *forward operator and conjugate inverse* (FOCI) method is an appropriate method for designing accurate and efficient extrapolation operators that remain stable in a recursive algorithm. The FOCI's results are comparable with other results obtained with other known methods such as Hale's and the weighted least square (WLSQ) extrapolation methods. Further, the FOCI method is computationally more efficient than the other methods.

The amplitude and phase spectra of the FOCI's, Hale's, and WLSQ's extrapolators are shown to compare their stabilities and accuracies. The impulse responses of these extrapolators are also shown to further compare their accuracies. The Marmousi dataset is used to illustrate the quality of the three extrapolators in a prestack depth migration in the presence of strong lateral velocity variations and steeply dipping events.

INTRODUCTION

Recursive wavefield extrapolation methods in the space-frequency domain are becoming increasingly popular because of their great ability to handle strong lateral velocity variations. However, these methods have two major problems (1) they are computationally more expensive than other methods such as the Kirchhoff methods and (2) the extrapolators are often unstable. Unstable spatial convolution operators mean that the wavefield is amplified at each depth step. Despite their computational cost, images obtained by recursive wave field extrapolation methods are often superior to those obtained from ray theory based methods.

There are different ways to design spatial convolution operators for recursive wavefield extrapolation. The most common approach is to design an operator that approximates the exact phase-shift operator in the frequency-wavenumber domain then transform it to the spatial domain. Some methods use nonlinear least-square algorithms to design the extrapolator like Holberg (1988) for 2-D media and Blacquiere (1989) for 3-D media. However, these methods are expensive and sometimes yield unstable extrapolators. Other methods smooth the phase of the phase-shift operator in the frequency-wavenumber domain then transform it to the spatial domain (Blacquiere, 1989). However, the spatial operator has to be relatively long to be stable, which reduces the efficiencies of these methods.

Hale (1991) introduced a method to calculate a stable explicit extrapolator. This method is based on the Taylor expansion of the exact phase-shift operator in the frequency-wavenumber domain and the use of basis functions. Hale's method can design short stable operators but can not handle high angles of propagation. Further, it is computationally expensive and requires the use of both symbolic and numerical mathematical software packages. Thorbecke et al. (2004) have introduced a weighted least-squares method (WLSQ), which is not perfectly stable but has a controlled instability. The problem with the WLSQ method is that it is not stable for short operators and coarse sampling.

Margrave et al. (2004) introduced a new method for designing spatial operators called the FOCI method. "FOCI" is an acronym for *forward operator and conjugate inverse*, which suggests the key concept in operator stabilization by Wiener filtering. However, there are three key innovations in the method with the other two being: (2) the use of dual operator tables to reduce evanescent filtering, and (3) spatial resampling of the lower frequencies to increase operator accuracy and decrease run times.

In this paper, comparisons of the FOCI method with other extrapolation methods are shown. The other methods are Hale's and WLSQ's methods. The objective of these comparisons is to investigate the stability, accuracy, ability to handle high angles of propagation, and efficiency of the FOCI method versus other industry standard methods. A brief review of theory of each method is shown to have some insight into these methods. The prestack depth migrations of the Marmousi dataset using these extrapolators will be shown to compare these extrapolators in the presence of strong lateral velocity variations and steeply dipping events.

THEORY OF HALE'S EXTRAPOLATOR

To derive Hale's extrapolator, we start with the 2-D scalar isotropic wave equation

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial z^2}=\frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2}. \tag{1}$$

After taking 2-D Fourier transformation, equation (1) becomes

$$\frac{\partial^2\bar{\bar{\psi}}}{\partial z^2}=-k_z{}^2\bar{\bar{\psi}} \tag{2}$$

where

$$k_z{}^2=\frac{\omega^2}{v^2}-k_x{}^2. \tag{3}$$

Equation 2 is just a 1D Helmholtz equation whose solution, for upgoing or downgoing waves, is

$$\bar{\bar{\psi}}\left(k_x,z,\omega\right)=\bar{\bar{\psi}}\left(k_x,z=0,\omega\right)e^{ik_z z}. \tag{4}$$

Note that ψ is the wavefield representing pressure, $\bar{\bar{\psi}}$ represents its 2-D Fourier transform, *t* is the two-way travel time, and *x* and *z* are the spatial and depth coordinates. So, the wavefield at some depth *z*, $\bar{\bar{\psi}}\left(k_x,z,\omega\right)$, can be obtained by multiplying the recoded wavefield at the surface, $\bar{\tilde{\psi}}\left(k_x, z=0, \omega\right)$, by a phase shift operator, $e^{ik_z z}$, in a homogeneous medium. Let's denote the exact phase-shift operator by

$$D(k) = e^{ik_z \Delta z}, \tag{5}$$

where

$$k_z = \left(\frac{1}{\Delta x}\right)\left[\left(\frac{\omega \Delta x}{v}\right)^2 - k^2\right]^{1/2}, \tag{6}$$

and

$$k = \Delta x k_x. \tag{7}$$

We can rewrite Equation 5 as

$$D(k) = \ i\left(\frac{\Delta z}{\Delta x}\right)\left[\left(\frac{\omega \Delta x}{v}\right)^2 - k^2\right]^{1/2}. \tag{8}$$

Note that the quantities $\omega\Delta x / v$ and k have been normalized. The normalization of $\Delta z / \Delta x$ and $\omega\Delta x / v$, uniquely determine the exact phase-shift operator, $D(k)$. The exact-phase operator $D(k)$ is the same term applied in the phase-shift migration and can only handle velocity varying with depth (Gazdag, 1978). For a general inhomogeneous medium with significant lateral velocity variations, downward continuation can be carried out conveniently in the $\omega - x$ domain as a $x -$dependant convolution (Holberg, 1988).

The symmetry of the exact-phase, $D(k)$, with respect to k implies that the complex extrapolation filter coefficients, w_n (the actual extrapolator in the $\omega - x$ domain), should be even. Specifically, we expect: $w_{-n} = w_n$. Therefore, the number of coefficients N should be odd, with the coefficient index n bounded by $-\left(N-1\right)/2 \le n \le \left(N-1\right)/2$. Due to the symmetry of w_n, the filter can be specified by *(N+1)/2* complex coefficients, where *N* denotes the number of complex coefficients needed to define the operator. The Fourier transform of the operator is defined by

$$\tilde{W}(k) = \sum_{n=-(N-1)/2}^{(N-1)/2} w_n e^{-ikn}. \tag{9}$$

Because of the symmetry of w_n,

$$\tilde{W}(k) = \sum_{n=0}^{(N-1)/2} \left(2-\delta_{n0}\right) w_n \cos\left(kn\right), \tag{10}$$

where δ_{n0} is the Kronecker delta function defined by

$$\delta_{n0} = \begin{cases} 1, & if \;\; n=0 \\ 0, & otherwise \end{cases}. \tag{11}$$

In Hale's method, the coefficients of the filter are represented as a sum of *M* weighted basis functions:

$$w_n = \sum_{m=0}^{M-1} c_m b_{mn}, \tag{12}$$

where Hale's choice for the basis functions is

$$b_{mn} = \left(2-\delta_{m0}\right)\cos\left(\frac{2\pi mn}{N}\right). \tag{13}$$

In Hale's method, instead of determining $\left(N+1\right)/2$ complex filter coefficients, only *M* complex weights c_m are determined. To ensure stability, the number, *M*, of weights must be less than the number $\left(N+1\right)/2$ complex filter coefficients. Therefore, only the first *M* even derivatives of the exact phase and the actual Fourier transforms are matched and using the remaining $(N+1)/2-M$ degrees of freedom to ensure stability. To determine, c_m, we begin with Fourier transform of the extrapolation filter

$$\tilde{W}\left(k\right) = \sum_{m=0}^{M-1} c_m \left(2-\delta_{m0}\right) \sum_{n=0}^{(N-1)/2} \left(2-\delta_{n0}\right)\cos\left(\frac{2\pi mn}{N}\right)\cos\left(kn\right) = \sum_{m=0}^{M-1} c_m B_m\left(k\right), \tag{14}$$

where

$$B_m\left(k\right) = \left(2-\delta_{m0}\right) \sum_{n=0}^{(N-1)/2} \left(2-\delta_{n0}\right)\cos\left(\frac{2\pi mn}{N}\right)\cos\left(kn\right) \tag{15}$$

are the Fourier transformed basis functions. By matching the l^{th} even derivative at $k=0$, we obtain the linear equation:

$$\sum_{m=0}^{M-1} c_m B_m^{(2l)}\left(0\right) = D^{(2l)}\left(0\right), \tag{16}$$

which is a system of linear equations that can be solved to determine c_m. Then c_m is used in Equation 14 to obtain $\tilde{W}\left(k\right)$, which can be transformed to w_n by applying an inverse Fourier transform.

THEORY OF WLSQ'S EXTRAPOLATOR

A brief review of the weighted least square (WLSQ) approach to design explicit extrapolation operators is shown (Thorbecke et al, 2004). The spatial Fourier transform of the convolution operator, $W(x,\omega,\Delta z)$, can be written as

$$\tilde{W}(k_x,\omega,\Delta z) = \int_{x_1}^{x_2} \exp(ik_x x) W(x,\omega,\Delta z)\, dx \tag{17}$$

for $k_{x,1} \le k_x \le k_{x,2}$. The discrete representation of Equation 17 can be written as

$$\tilde{W}(n\Delta k_x) = \Delta x \sum_{m=-M}^{m=M} \exp(in\Delta k_x m\Delta x) W(m\Delta x) \tag{18}$$

for $-N \le n \le N$. The discrete spatial Fourier transform of $W(x,\omega,\Delta z)$ can be also represented in matrix notation such as

$$\tilde{w} = \Gamma w, \tag{19}$$

where *w* represents the short operator and $\tilde{w}$ is its spatial Fourier transform, which gives an approximation to the exact phase-shift operator. Further, *m* and *n* represent the samples of the short operator and its Fourier transform, respectively. Equation 19 has more equations than unknowns and to solve this problem, the weighted prediction error function (Menke, 1989) is used

$$\varepsilon = \tilde{e}^H \tilde{\Lambda} \tilde{e}, \tag{20}$$

where *H* superscript denotes a complex-conjugate transpose and

$$\tilde{e} = \Gamma \langle w \rangle - \tilde{w}. \tag{21}$$

The least square solution of Equation 19 is given by

$$\langle w \rangle = \left[\Gamma^H \tilde{\Lambda} \Gamma \right]^{-1} \Gamma^H \tilde{\Lambda} \tilde{w}, \tag{22}$$

where $\tilde{\Lambda}$ is a diagonal matrix containing a weighting function and its components are given by

$$\Lambda_{nm} = w(n\Delta k_x)\,\delta_{nm}, \tag{23}$$

where w is a box-shaped weighting function and the components of Γ are given by

$$\Gamma_{nm} = \exp(in\Delta k_x m\Delta x). \tag{24}$$

To design the so-called smooth phase-shift operator such as, the exact phase operator $\tilde{w}$ is equal to the phase-shift operator for the propagating waves. In the evanescent region (outside the band of interest) the amplitude and phase are defined by a cubic spline which goes smoothly to zero. The spectrum of the $\tilde{w}$ be defined by

$$\left\|\tilde{W}\left(k_x,\omega,\Delta z,\alpha_{\max}\right)\right\| = \begin{cases} 1.0 & \left|k_x\right| \le k\sin\left(\alpha_{\max}\right) \\ \text{spline} & \left|k_x\right| > k\sin\left(\alpha_{\max}\right), \\ 0 & \left|k_x\right| = \dfrac{pi}{\Delta x} \end{cases} \tag{25}$$

where $\alpha_{\max}$ is the maximum propagation angle of interest. Then $\tilde{w}$ is used in Equation 22 to obtain *w* or the final extrapolator.

THEORY OF FOCI EXTRAPOLATOR

Operator stability is enhanced by the design of a forward operator for one-half the intended depth step and its conjugate inverse (hence the FOCI acronym), which are then convolved to form the final operator. The forward operator is created by simply applying a spatial localizing window to the theoretical operator. Then a band-limited inverse to the forward operator is designed as a Wiener filter. The wavenumber band of this inverse design is restricted to the non-evanescent wavenumbers. The resulting Wiener filter has an amplitude spectrum that stabilizes the forward operator and a phase spectrum that is the negative of the forward operator. The sign of the phase is reversed (complex conjugation) and the forward operator is convolved with its designed conjugate inverse. The result is a much more stable operator that has the correct phase for a full depth step (see Margrave et al., 2004 for the full mathematical derivation).

Achieving great computational efficiency usually means trying to find a short, stable operator with good phase accuracy. The standard approach of using an operator whose length (in points) is fixed and independent of frequency, usually means that some range of low frequencies will be handled poorly. This is because the evanescent boundary moves to lower wavenumbers with decreasing frequency. Generally, for a sufficiently low frequency, with *n* equally spaced samples across the wavenumber spectrum, most of the designed wavenumber samples will be in the evanescent region. Since only the non-evanescent wavenumbers have useful phase information, the resulting operator will be very inaccurate and relatively unstable. This issue is addressed in FOCI method by breaking the seismic dataset into “frequency chunks” and then spatially resampling each chunk (Margrave et al., 2004). The purpose of the spatial resampling is to place the Nyquist wavenumber just above the highest non-evanescent wavenumber thereby forcing most of the n design wavenumbers to fall in the wavelike portion of the spectrum. The spatial resampling is implemented with a boxcar low-pass filter in the wavenumber domain. This eliminates any possible aliasing and also serves as a kind of evanescent filter. There are a number of benefits to spatial resampling and these include: (1) better operator phase control (more accuracy), (2) increased stability, (3) approximate evanescent filtering, (4) dramatic reduction of the data volume. The net effect is that accuracy and stability is improved with a large increase in computational efficiency.

COMPARISON OF AMPLITUDE AND PHASE SEPCTRA OF HALE ,WLSQ, AND FOCI EXTRAPOLATORS

Hale's extrapolator has some problems

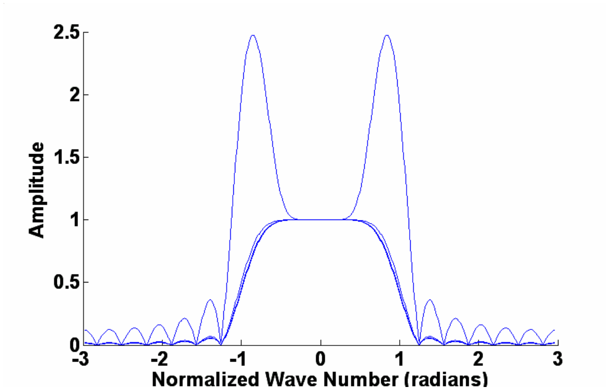
- there is no direct formula for choosing the number of terms, M, to match the truncated Taylor's series of the exact phase transform,
- there is a relatively large phase error,
- it is expensive to calculate the extrapolator due to computing the high order derivatives of the exact phase transform in order to solve for the weight function,
- it cannot handle high angles of propagation with short operators.

Consequently, choosing a constant M value will cause some normalized frequencies to be unstable (Figure 1.a). To make this filter stable for all normalized frequencies, different normalized frequencies should have different M values. By breaking the normalized frequencies into small ranges and assigning a different M value for each range, stability can be achieved for all normalized frequencies. This process is done subjectively (Figure 1.b), requiring the inspection of each operator for each frequency to ensure that the correct M value has been chosen. In addition, the phase error associated with Hale's extrapolator is a direct result of matching fewer terms to the truncated Taylor series so that the remaining degrees of freedom are used to ensure stability. Furthermore, for longer extrapolators, it can be computationally expensive to design the extrapolator because of taking the higher order derivatives of the exact phase-shift operator.

The stability of the WLSQ extrapolator is less than Hale's but has a controllable instability. A stable extrapolator as defined by Thorbecke et al (2004) must have amplitudes that are much less than 1.001 for all wavenumbers. Our analysis shows that Thorbecke has two major problems

- the extrapolator is sensitive to the stability factor that is added to the weighting function,
- the condition of the operator amplitudes to being much less than 1.001 is obtainable only for small depth steps and long operators.

(a)



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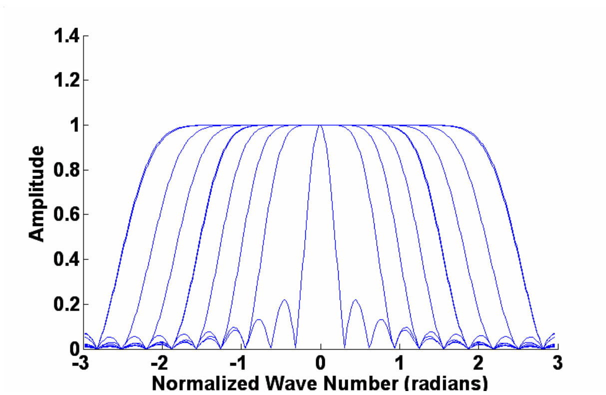


FIG 1. Amplitude spectra of Hale's extrapolators for different normalized frequencies in the frequency-wavenumber domain for (a) constant *M* and (b) varying *M.* The frequencies are from 10-100 Hz incrementing by 10 Hz, v=2000 m/s, $\Delta x = \Delta z = 10$ m, and M=1-8 depending on the value of $2 * pi * f * \Delta x / v$.

Consequently, the WLSQ extrapolator can not yield stable results for larger depth steps and short operators, which reduces its efficiency. Figure 2a shows that the 25-point extrapolator is stable or its amplitude is much less than 1.001 only for small depth step. When using a shorter operator and fixing the other parameters, even for small depth step size, the extrapolator becomes less stable (Figure 2b). Figure 2c shows that changing the depth step from 2 to 10 meters reduces the stability of the 25-point extrapolator. For large depth steps, longer operators have much better stability (Figure 2d). The operator stability is a function of the depth step size and its length. On other hand, once correct parameters have been chosen, the WLSQ extrapolator can very well handle lateral velocity variations and high angles of propagation as we shall see. Further, the extrapolation table can be calculated is computationally much cheaper than Hale's.

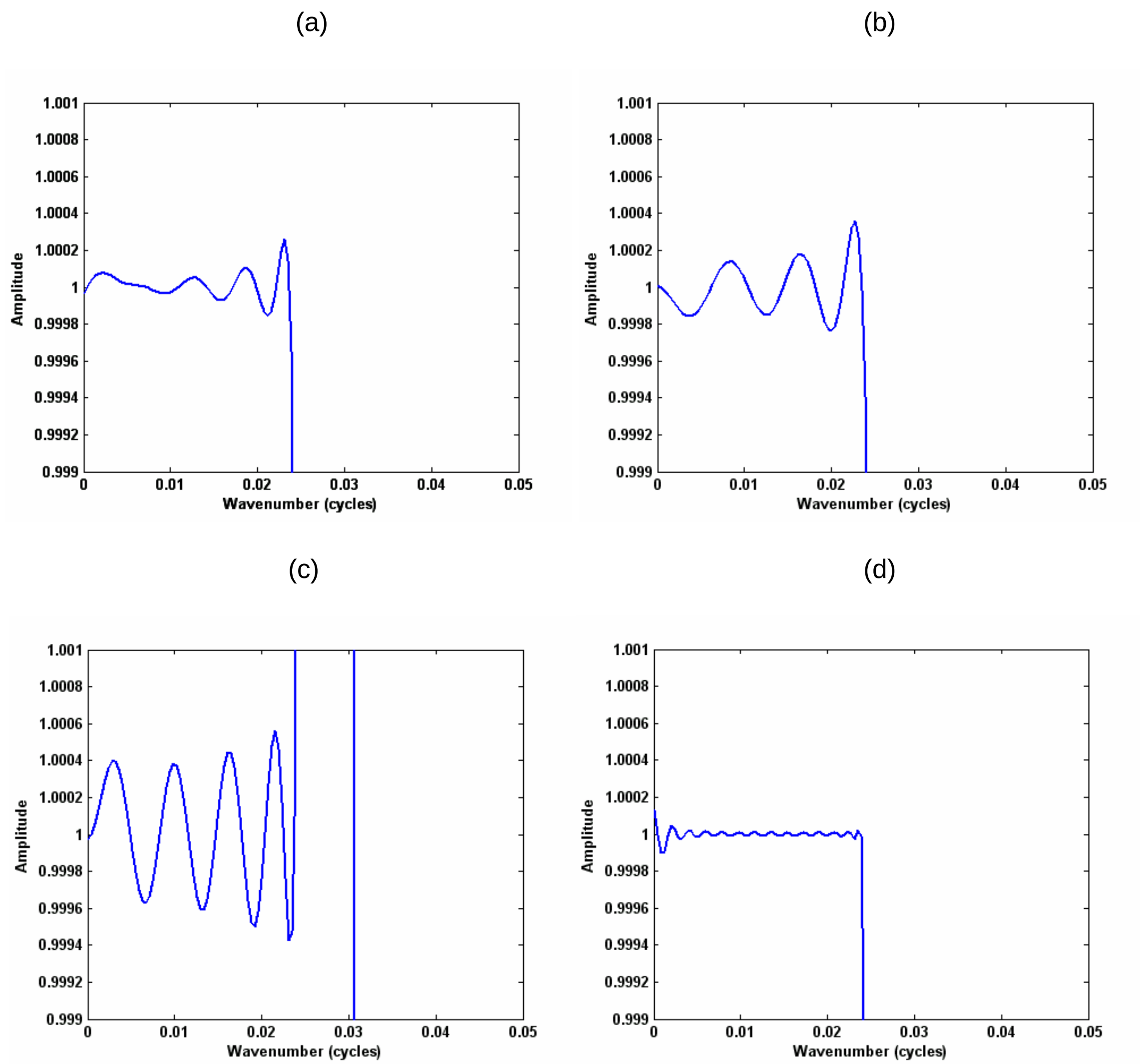


FIG 2. Stability of WLSQ extrapolator is a function of the size of depth step and operator length. (a) $\Delta x = 10m,\ \Delta z = 2m$, operator length=25 points, velocity=2000m/s, frequency=50 Hz, and $\alpha_{\max}$ =75 degrees. (b) Same parameters as (a) but with operator length of 19-point. (c) Same parameters as (a) but with Δz equal to 10m. (d) Same parameters as (c) but with operator length of 101 points.

FOCI is not perfectly stable but also has a controlled instability. It is less stable than Hale's but also has less phase error and can migrate high angles of propagation. Also, it does not have to have a value for each frequency range as in Hale's method where each range is assigned a different *M* value. Further, the table of operators can be calculated in a much faster time. On other hand, FOCI is more stable than the WLSQ and its stability is not as sensitive to the size of the depth step or operator length as in WLSQ. Figure 3a shows a comparison among the three extrapolators. FOCI's extrapolator exhibits a better stability than WLSQ's even for small depth steps. However, the FOCI extrapolator is less stable than Hale's but with a broader amplitude spectrum, which means that it is more effective in handling the high angles of propagation. When increasing the depth step size from 2m to 10m, the stability of FOCI's extrapolator does not change as much as the

WLSQ's (Figure 3b). Moreover, the FOCI's extrapolator has less phase error than Hale's and WLSQ's extrapolators (Figure 3c).

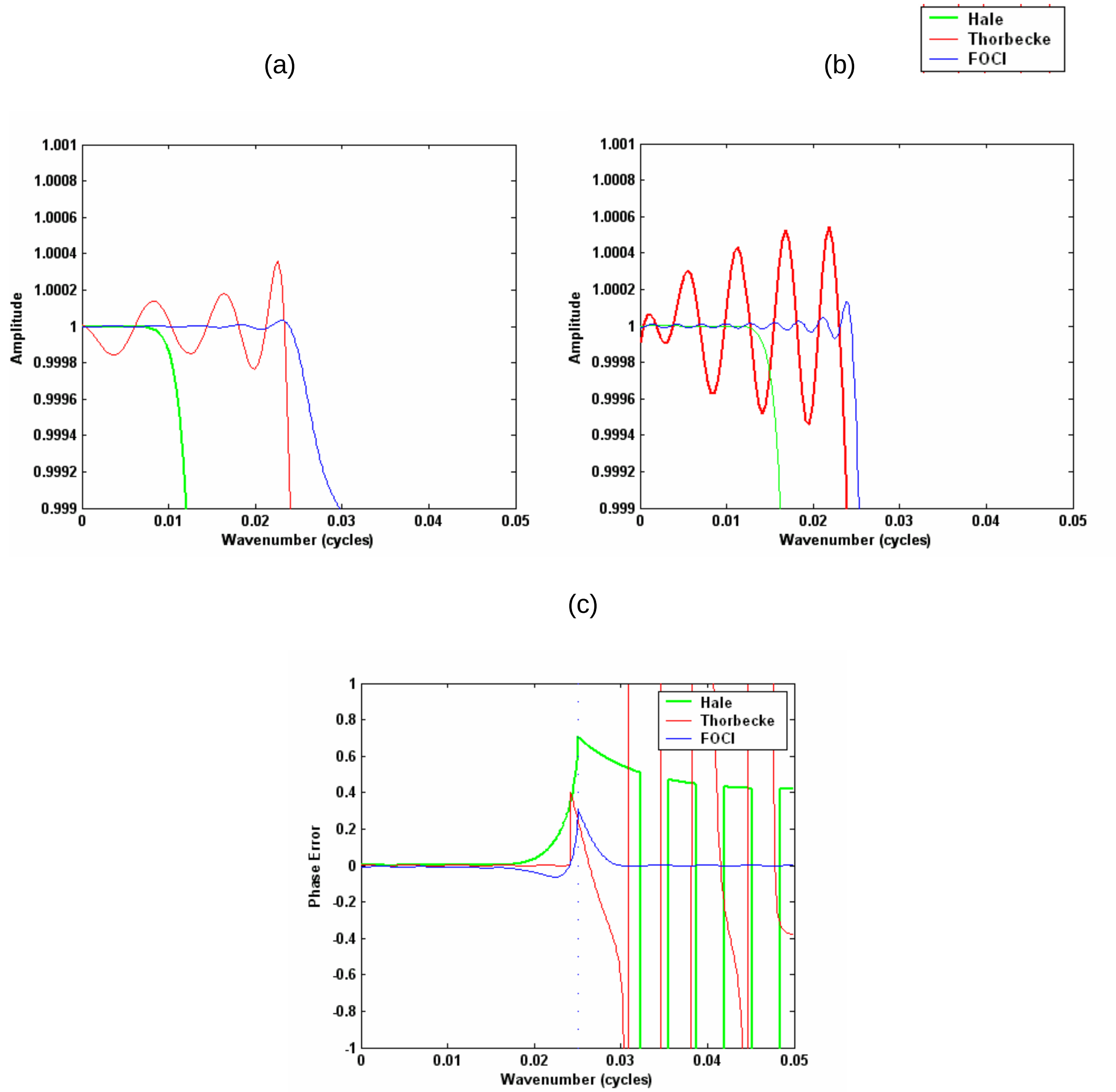


FIG 3. A comparison between the amplitudes of Hale's , FOCI's, and WLSQ's extrapolators. (a) $\Delta x = 10m,\ \Delta z = 2m$, operator length=19 points, velocity=2000m/s, frequency=50 Hz, and $\alpha_{\max}$ =75 degrees for Thorbecke's extrapolator and ρ =0.01 for FOCI. (b) $\Delta x = 10m,\ \Delta z = 10m$, operator length=31 points, velocity=2000m/s, frequency=50 Hz, and $\alpha_{\max}$ =75 degrees for WLSQ's extrapolator and ρ =0.01 for FOCI. (c) The phase error of the three extrapolators using the parameters of (b) where the dashed line is to show the evanescent boundary.

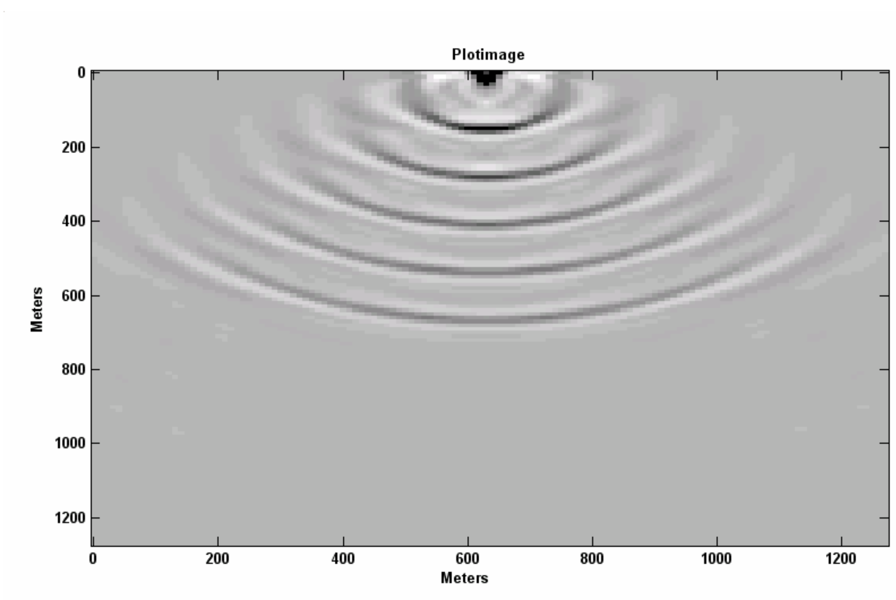
Despite the facts that the FOCI extrapolator has less phase error than Hale's and is more stable than Thorbecke's it does not have a strong attenuation for the evanescent waves. In designing a FOCI extrapolator, the parameter, ρ, that controls the degree of evanescent attenuation in the final operator is set to a small number (<1). However, it is not necessary to apply the full evanescent attenuation with every step because the evanescent wavenumbers are rapidly driven into numerical insignificance by just one or two filter applications. Since operator stability increases as ρ decreases, it makes sense to build two operator tables, one that applies strong attenuation (say with ρ=1) and the other that applies very little attenuation (say with ρ=0.01). Most marching steps can then be taken with the ρ=0.01 table and the ρ=1 table can be invoked, for example, every 10th step.

MIGRATION RESULTS OF SYNTHETIC DATA

Impulse responses of Hale, WLSQ, and FOCI extrapolators

The impulse responses of the three extrapolators are used to analyze their accuracies. The zero-offset experiment is done with operator length of 31 points in a homogenous medium, a receiver spread of 1280 meter, a maximum extrapolation depth of 1280 meters, a velocity of 2000 m/s, and a spatial and vertical spacing of $\Delta x = 10m,\ \Delta z = 10m$. The trace in the center of the zero-offset section contains five Ricker wavelets at 0.0600, 0.1240, 0.1880, 0.2520, and 0.3160 seconds. The sample rate is 4 ms and the dominant frequency of the Ricker wavelet is 30 Hz. Figure 4 shows the impulse responses of Hale's, WLSQ's, and FOCI's extrapolators. While Hale's extrapolator could not migrate high angles of propagation, WLSQ's and FOCI's extrapolators show that they can better handle high angles of propagation.

(a)



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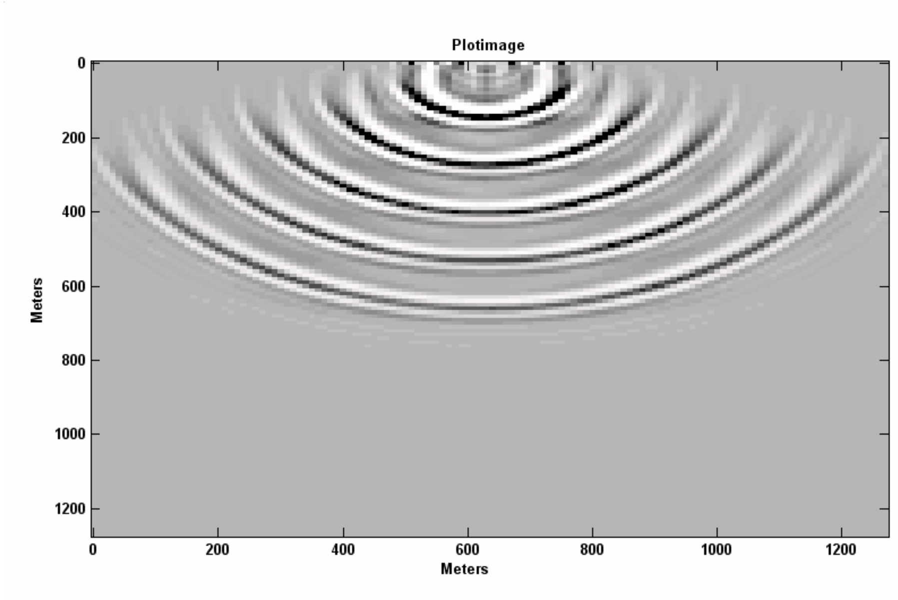


(c)

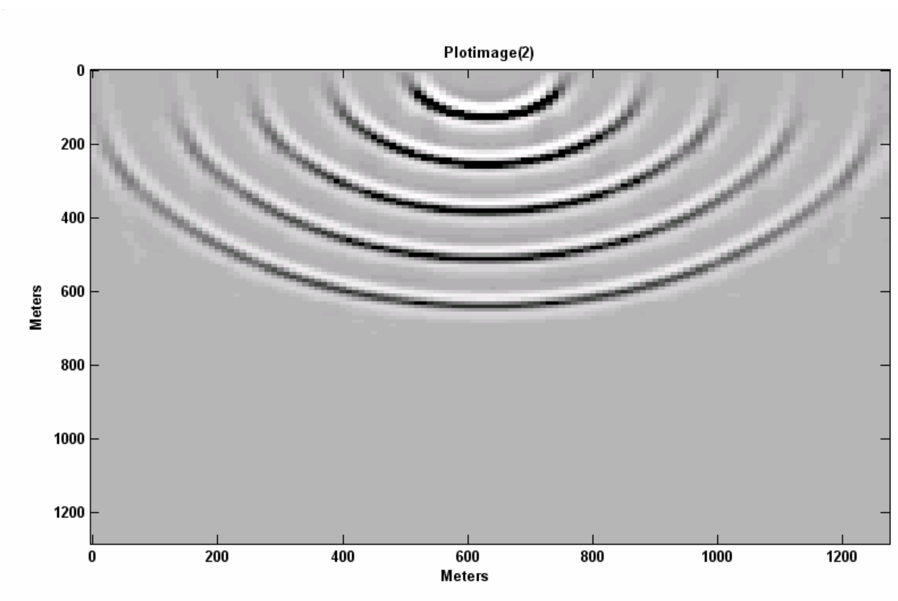


FIG 4. Impulse responses of (a) Hale's, (b) WLSQ's, (c) and FOCI's extrapolators for velocity=2000 m/s, $\Delta x = 10m, \Delta z = 10m$, 31-point operator, $\alpha_{\max}$ =75 degrees for WLSQ's extrapolator and two tables for FOCI with ρ =0.01 and ρ =1.0 applied every 10th step, length of inverse operator for FOCI=71 points, The FOCI result was obtained using the spatial resampling technique.

Migration results of Marmousi data using Hale, WLSQ, and FOCI extrapolators

The Marmousi dataset is usually used to test the accuracy of migration algorithms due to its strong lateral velocity variations and steeply dipping events. These data will be used as a second test to further analyze Hale, WLSQ, and FOCI extrapolators in the presence of strong lateral velocity variations and steeply dipping events.

The prestack depth migration of this data will be done by the reflector mapping method. The principle of reflector mapping was first introduced by Claerbout in 1971. The basic principle of reflector mapping is that reflectors exist in the subsurface when the first arrival of the downgoing wave is time coincident with the upgoing wave. This can give the correct phase but not the amplitude at the reflector. To get the correct amplitude at the reflector, Claerbout defines it as the ratio of the upgoing and downgoing wavefields at the subsurface imaging location. Geiger (2001) derived Claerbout's reflector mapping as a reformulation of the Kirchhoff integral

$$\hat{R}_\theta\left(x_G, x_s\right) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega \frac{P_S^-\left(x_G, x_s, \omega\right)}{P_i^+\left(x_G, x_s, \omega\right) + stability} d\omega, \tag{26}$$

where the *stability* factor is added to avoid dividing by zero after normalizing the data. Equation 26 can estimate a reflectivity map and is called the deconvolution imaging condition where $P_S^-\left(x_G, x_s, \omega\right)$ is the extrapolated wavefield to a subsurface location, x_G and $P_i^+\left(x_G, x_s, \omega\right)$ is the modeled source from the surface to x_G .

Figure 5 shows the velocity model for the Marmousi data. The FOCI method will be analyzed against the Hale and WLSQ methods separately. Figures 6a and 6b show the migration results of the Marmousi data using Hale's and FOCI's extrapolators. The operator length for both is 19 points. The computation times are 3.5 hours for Hale's and 2 hours for FOCI's results on a desktop PC. The FOCI result has less computational cost because of the spatial resampling, which reduces the data volume. The images shown have $\Delta x = \Delta z = 25m$ spacing. The WLSQ image is not shown because we could not stabilize the operators for these specific parameters. The images have some aliasing due to a coarse sampling but the objective of this exercise is to investigate the stability of these methods with short operators and coarse sampling. Whilst both Hale and FOCI methods are stable for parameters, the FOCI image is superior to Hale's and cheaper computationally. This example shows that even for short operators and coarse sampling, FOCI can yield a reasonable result, which shows the robustness of this method.

Figures 7a and 7b show the migration result using WLSQ's and FOCI's extrapolators, respectively. The operator length used for both is 69 points and the images have $\Delta x = \Delta z = 12.5m$ spacing. The run times are 23.7 and 15.8 hours for the WLSQ and FOCI results, respectively. The two results are comparable but the WLSQ is a little bit better. We think that the discrepancy between them can be attributed to the way that FOCI performs the evanescent filtering. Further, better ways can be developed to improve the evanescent filtering which will lead into better images.

However, building a table of operators using the WLSQ approach is not easy because it requires the inspection of each operator for each frequency to ensure its stability. This is one of the disadvantages of the WLSQ extrapolator. On the other hand, the FOCI method can build more stable operators for different set of parameters. However, in the FOCI method, two tables are calculated where the first table is used for extrapolation and the second one is used for evanescent filtering. Calculating tables using FOCI is relatively very fast and can be done in minutes so it is not a big issue.

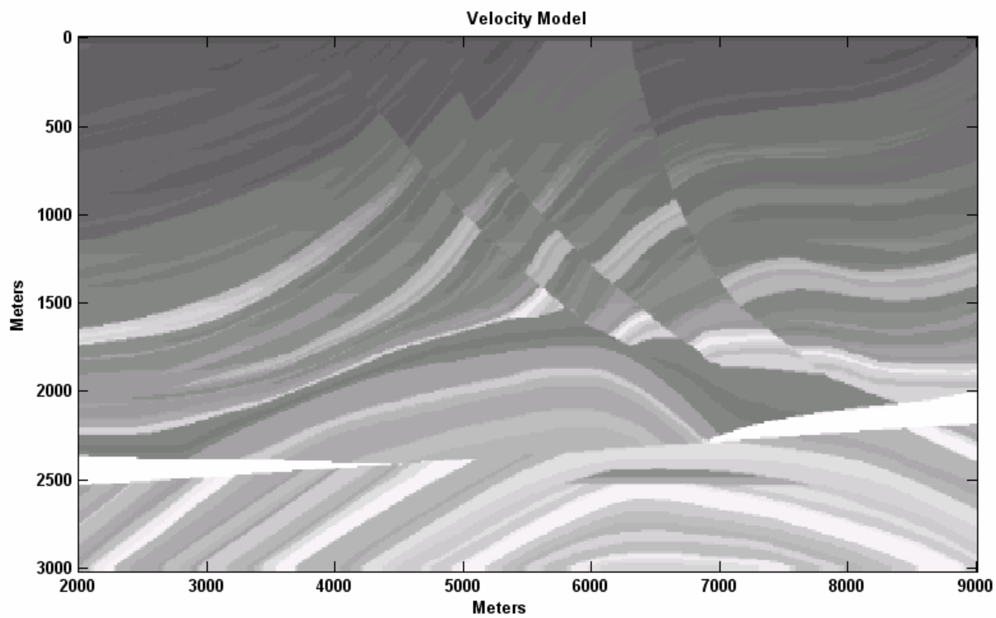
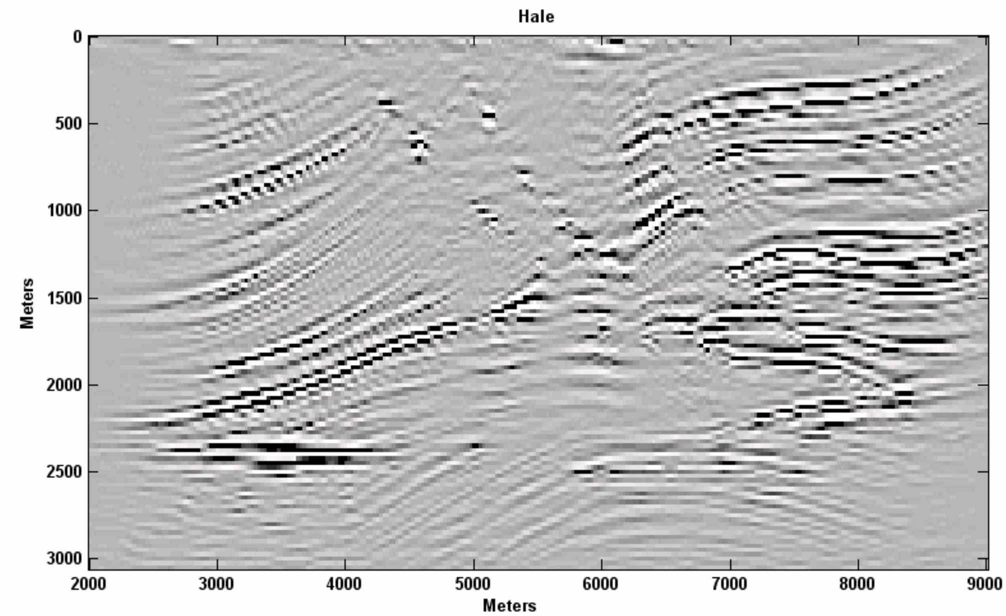


FIG 5. Velocity model that will be used to migrate the Marmousi data.

(a)



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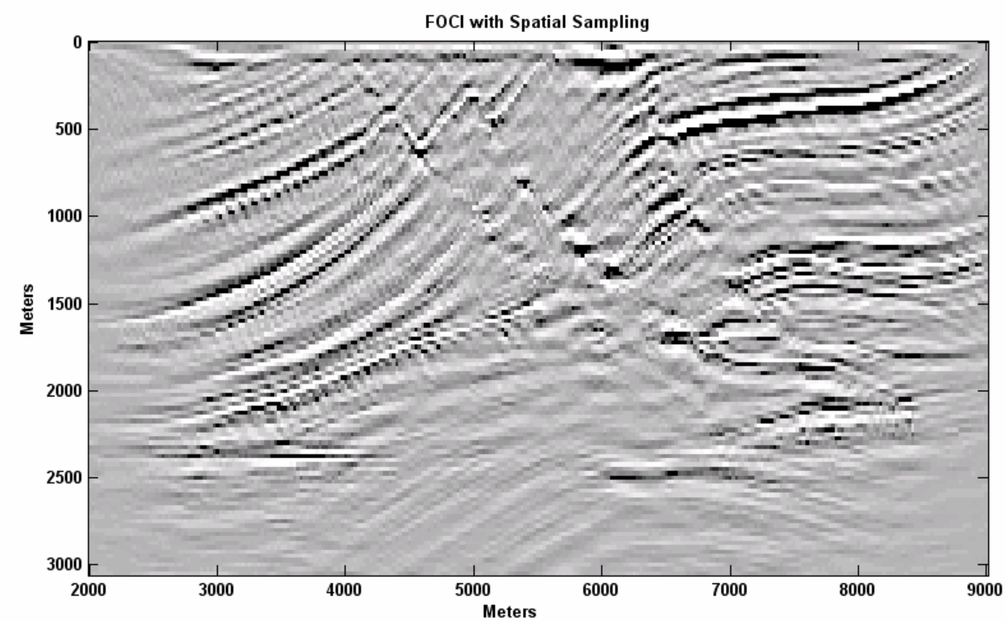
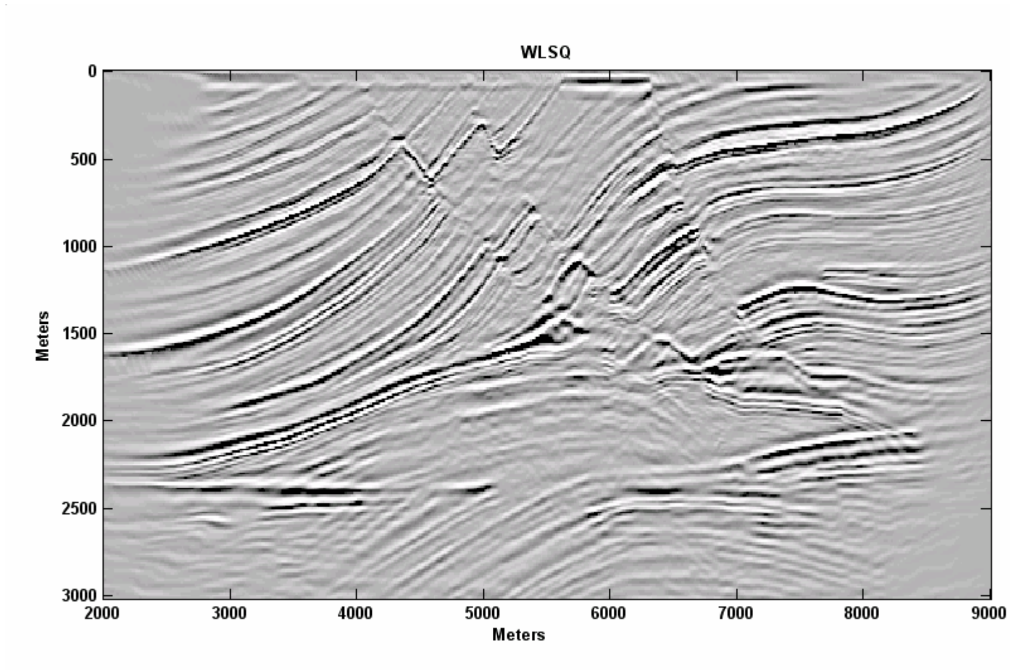


FIG 6. (a) Prestack depth migration results using (a) Hale's extrapolator with 19 coefficients and (b) FOCI's extrapolator (with spatial resampling) with two tables: ρ =0.01 used for extrapolation and ρ =1.0 applied every 10th step for evanescent filtering. The lengths of the forward, inverse, and windowed (final) operators are 19, 41, 19 points.

(a)



(b)

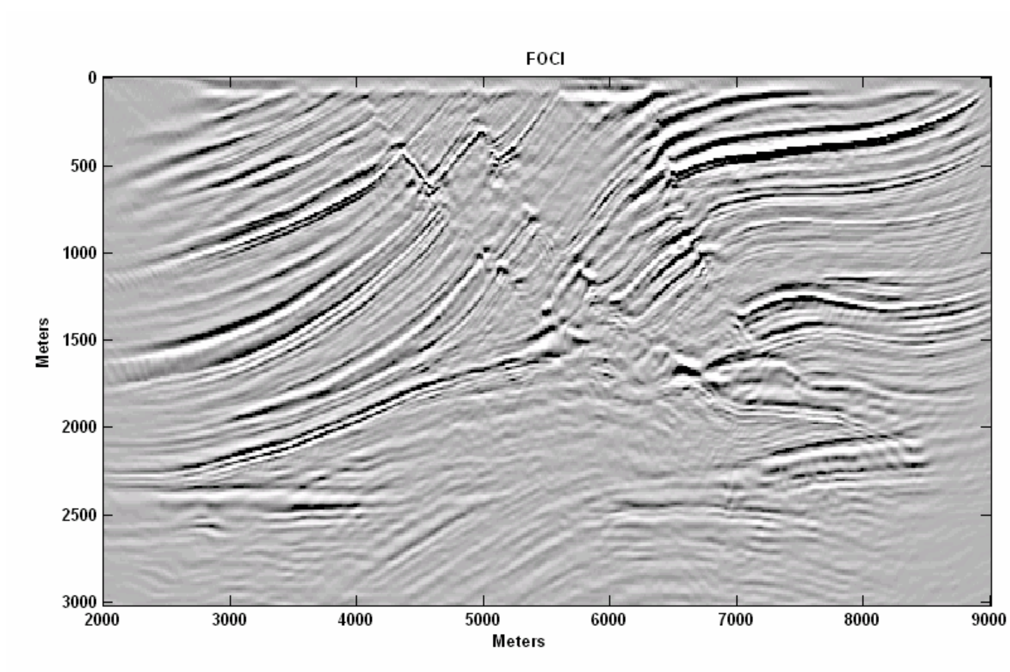


FIG 6. Prestack depth migration results of (a) the WLSQ method with operator length=69 points, $\alpha_{\max}$ =75, and run time=23.7 hours and (b) the FOCI method (with spatial resampling) using two tables with ρ =0.01 and ρ =1.0 applied every 10th step, length of the inverse and forward operators 45 and 25 points, respectively and run time=15.8 hours

CONCLUSIONS

In this paper, the FOCI method was evaluated and compared with other wavefield extrapolation methods. The other extrapolation methods are Hale and WLSQ. The Hale method has several problems: (1) each frequency range has to have a distinct M value and this is done subjectively; (2) it can not handle high angles of propagation; (3) it has a relatively large phase error; and (4) it is computationally expensive to calculate a table of operators.

The WLSQ method is not perfectly stable but has a controllable stability. Its stability is directly related to the size of the depth step and operator length. Large depth steps and short operators could decrease its stability. Calculating a table of extrapolators using the WLSQ method is computationally very fast but requires the inspection of the amplitude of each operator to ensure its stability for a specific depth range.

The FOCI method has also a controllable instability and less phase error than the Hale method. Unlike the WLSQ method, its stability is less sensitive to changing the size of the depth step and the operator length. Further, the FOCI method with spatial resampling is computationally less expensive than the other two methods. This can make a big difference for 3-D prestack depth migration. However, it requires calculating two tables one for extrapolation and one for evanescent filtering. Unlike the Hale's method, calculating the extrapolation tables can be done in a much shorter time. The FOCI's images were superior to Hale's and comparable to WLSQ's.

The Hale method was introduced in 1991, and the revised WLSQ method was introduced in 2004; but the original WLSQ method was first introduced in 1994 by Thorbecke and Rietveld. On the other hand, the FOCI method is only two months old and its preliminary results show that it is a very promising technique for seismic imaging that combines both stability and efficiency. Furthermore, the FOCI can even be improved further by developing new techniques for evanescent filtering.

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