Relative polarity of PP and PS events in the registration process and approximations to the PS reflection coefficient

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ABSTRACT

For most sedimentary interfaces, the PP and PS reflection coefficients, \( R_{PP} \) and \( R_{PS} \), have opposite sign. In such cases, the corresponding PP and PS events have the same display polarity on their respective seismic sections. In processing and interpreting PS seismic sections, one normally relies on correlation of corresponding PS and PP events in the so-called registration process. Getting the polarity wrong on an event on the PS section can lead to a mistie of at least half a cycle between correlated events on the two sections and, consequently, over- or underestimated \( V_P/V_S \) ratios for the affected intervals. An unexpected polarity switch could conceivably even inhibit the correct correlation altogether and lead to a totally spurious correlation.

We examine under just what conditions an event will be recorded with opposite display polarities on PP and PS and show that the sign of \( R_{PS} \) depends on the sign of \( \Delta U \), where \( U = \rho \beta^h \) and \( h \) is a function of P and S velocities as well as densities. In this analysis we derived an accurate small-angle \( R_{PS} \) approximation, one that we show is better than the Aki-Richards expression in many situations, not just for small angles.

INTRODUCTION

The SEG polarity standard (Thigpen et al., 1975) implies, when using for display a minimum-phase wavelet from a compressive source, that a PP reflection from an interface with a positive PP reflection coefficient (\( R_{PP} > 0 \)) will begin with a downward (negative) deflection on the recorded seismogram (Sheriff, 2002). Recommended SEG standards for horizontal-component geophones (Landrum et al., 1994) and subsequent proposed standards (Brown et al., 2002) imply that a PS reflection from the same source, when the interface has a negative PS reflection coefficient (\( R_{PS} < 0 \)), will also lead to a downward (negative) deflection on the inline geophone on positive-offset traces. Negative-offset traces, which are flipped in preprocessing, originally have the opposite polarity (see e.g. Tessmer & Behle, 1988; Brown et al., 2002). Therefore, because for most interfaces, \( R_{PP} \) and \( R_{PS} \) have opposite sign, an event usually has the same display polarity on PP and PS. In this paper, we study the ‘unusual situation’, where events on the two sections display opposite apparent polarities, i.e., where \( R_{PS}/R_{PP} > 0 \).

That sedimentary interfaces may fairly commonly exhibit opposite display polarities on PP versus PS, i.e., that \( R_{PP} \) and \( R_{PS} \) may have the same sign, has been shown abundantly by Vant (2003) for several lithologic-interface types. Figure 1 demonstrates this for two lithologic-interface examples of the many given by Vant (2003). In each example, a number of published values of the rock parameters, \( V_P, V_S \) and \( \rho \), have been gathered from the literature, for both of the lithology types constituting the interface. These were then combined to simulate many possible examples of this type of lithologic interface. Of the pairs of reflection coefficients plotted (Figure 1), those points falling in quadrants 1 and 3 represent cases of the unusual situation, i.e., opposite PP-PS polarities.
Preliminary (or first-order) estimates of $V_P/V_S$ ratios, which are needed in the PS processing flow, can be obtained after the registration process in which PP and PS events are correlated. Typically, we generate synthetic seismograms using available log data for $V_P$, $V_S$ and $\rho$ from nearby wells – if available – then match PP and PS events on the field records with correlated PP and PS events on these synthetics. This correlation will usually have started with a zeroth-order $V_P/V_S$ estimate, namely $V_P/V_S = 2$. First-order $V_P/V_S$ ratios can then be calculated from traveltime ratios over selected intervals.

However, if nearby logs for $V_S$ and/or $\rho$ are not available, this procedure must be modified by estimating $V_S$ and/or $\rho$, thereby introducing greater uncertainty. Without logs, $\rho$ may be estimated from $V_P$ using Gardner’s empirical relationship (Gardner et al., 1974) or modified versions thereof for specific lithologies (Castagna et al., 1993). If $V_S$ logs are unavailable, less reliable user-defined estimates of interval $V_P/V_S$ ratios must be used to create PS synthetic stacks (Lawton & Howell, 1992), which could then be fine-tuned for optimal correlation with the stacked field data (Miller, 1996). But, as shown in Figures 2 and 3, without all the logs one could easily arrive at the wrong relative PP-versus-PS polarity for a particular event.

**APPROXIMATIONS TO THE ZOEPPRITZ EQUATIONS**

In adopting approximations for reflection amplitudes, we do not wish to restrict ourselves to low contrasts, or small changes in rock parameters, so we assume small angles of incidence. Such approximations should also be valid for high-contrast interfaces. We also assume the polarity relationship at small angles to be representative of the polarity relationship of the stacked events in all but the rarest of cases. Figure 4 shows a rather extreme case in which there is a polarity change in $R_{PP}$. However, it occurs at sufficiently large offset that the stacked-trace polarity is the same as the small-offset polarity, even though this stack includes some rather long-offset opposite-polarity traces.

Many approximations to the Knott-Zoeppritz equations governing P-SV waves at a welded interface have been derived (mainly for $R_{PP}$ and $R_{PS}$), some of the earliest being...
FIG. 2. AVO responses and synthetic stacks [P-P (blue) and PS (red)] for a model of a wet sand over a coal, an example of opposite PP-PS display polarities. In the upper figure, all three logs were used to model the interface. In the lower figure, for the scenario of a missing density log, Gardner’s equation was used to supply a substitute for the density log.

those of Bortfeld (1961), Richards & Frasier (1976) and Aki & Richards (1980), whose notation we essentially follow. We use the symbol $r$ to represent rock parameters generally (e.g. $\alpha, \beta, \rho, \sigma$ or $\mu$) and we express any rock parameter and its change over an interface in terms of $r$ (the average of $r_1$ and $r_2$) and $\Delta r$ (the difference $r_2 - r_1$), where 1 and 2 denote media 1 and 2, respectively. Contrary to what has been implied by some authors, the definitions of $r$ and $\Delta r$ are exact and do not require any assumption of small parameter changes. However, caution is required in expressions like:

$$\Delta Z = \Delta (\rho \alpha) = \rho \Delta \alpha + \alpha \Delta \rho \quad \text{and} \quad \Delta \mu = \Delta (\rho \beta^2) = \rho \Delta \beta^2 + \beta^2 \Delta \rho$$ (1)

(where $\mu$ is rigidity). Both are exact if one defines notation like $\beta^2$ or $\rho \alpha$ to be the average of the squares $\beta_2^2$ and $\beta_1^2$ or the average of the products $\rho_2 \alpha_2$ and $\rho_1 \alpha_1$, not the square of $\beta$ (the average of $\beta_1$ and $\beta_2$) or the product of the averages $\rho$ and $\alpha$. The difference between these two is second-order in $\Delta \beta$ or in $\Delta \rho \Delta \alpha$, so no such caution is needed in first-order low-contrast theory.

For $R_{PP}$, many approximations have been published (e.g. Bortfeld, 1961; Richards & Frasier, 1976; Aki & Richards, 1980; Shuey, 1985; Zheng, 1991; Wang, 1999;
FIG. 3. AVO responses and synthetic stacks [P-P (blue) and PS (red)] for a model of a gas sand over a limestone, an example of opposite PP-PS display polarities. In the upper figure, all three logs were used to model the interface. In the lower figure, for the scenario of a missing shear-wave log, a $V_P/V_S$ ratio of 2.0 was used to supply a substitute for the shear-wave log.

FIG. 4. AVO responses and synthetic stacks [P-P (blue) and PS (red)] for an interface model of water-saturated sandstone over chalk at a depth of 1000 m. The polarity of PP changes at a moderate offset but the stack retains the small-offset polarity.

Ursenbach, 2002) However, for $R_{PP}$, we use the zero-offset expression as sufficient for characterizing polarity. Some of the published $R_{PS}$ approximations assume small
parameter changes (Aki & Richards, 1980; Zheng, 1991; Xu & Bancroft, 1997; Gulati & Stewart, 1997; Donati & Martin, 1998; Ursenbach, 2002), some, like ours (below) assume small angles (Bortfeld, 1961; Richards & Frasier, 1976; Zaengle & Frasier, 1993; Wang, 1999; Ramos & Castagna, 2001; Carcuz, 2001; Geldart & Sheriff, 2004). It turns out that the differences between the two are not as great as one might think because in the Taylor expansions the terms of higher order in $\sin \theta$ tend also to be the terms of higher order in $\Delta r/r$.

**OUR APPROXIMATION FOR $R_{PS}$**

In deriving our own approximation to $R_{PS}$ (Figure 5), we started with the exact formula given by Aki & Richards (1980, p. 150):

$$R_{PS} = -2 \left[ \frac{\cos i_1}{\alpha_1} \left( ab + cd \frac{\cos i_2}{\alpha_2} \frac{\cos j_2}{\beta_2} \right) p \alpha_1 \right] / (\beta D)$$

(2)

where $p$ is horizontal slowness. The definitions of $a$, $b$, $c$, $d$ and $D$ (Aki & Richards, 1980) were reprinted by Xu & Bancroft (1997), Ramos & Castagna (2001) and Vant (2003), though Ramos & Castagna err in their expression for $d$. It should be:

$$d = 2 \left( \rho_j \beta_j^2 - \rho_i \beta_i^2 \right) \quad \text{and not} \quad d = 2 \left( \rho_j \beta_j^2 p^2 - \rho_i \beta_i^2 \right).$$

(3)

To get our $R_{PS}$ approximation we first rewrite (2) as:

$$R_{PS} = - \left[ \sin 2i_1 \left( ab + cd \frac{\cos i_2}{\alpha_2} \frac{\cos j_2}{\beta_2} \right) \right] / (\alpha \beta D)$$

(4)

then apply the small-angle approximation by setting $\sin \theta \approx \theta$ and $\cos \theta \approx 1$ for sines and cosines of all incidence angles, $i_1$, $i_2$, $j_1$ and $j_2$, in the expressions for $a$, $b$, $c$, $d$ and $D$, giving:

![Graph](a)
FIG. 5. Comparison of the exact $R_{ps}$ curve (solid black) with three approximations: our equation (5) (dashed magenta); Geldart & Sheriff (2004) (dotted red) [equivalent to (5) with $\sin 2i_1 \to 2i_1$], and Aki & Richards (1980) (coarsely dashed blue). The interface models are (a) young shale over old shale, (b) shale over gas-sand and (c) sandstone over salt (Brown et al., 2002).

$$R_{ps} = \frac{-\sin 2i_1 (\alpha_2 \beta_2 \rho_2 \Delta \rho + 2 \rho_1 \Delta \mu)}{(\rho_1 \alpha_1 + \rho_2 \alpha_2) (\rho_1 \beta_1 + \rho_2 \beta_2)}. \quad (5)$$

If we also approximate the explicit sine factor, it simply reduces from $\sin 2i_1$ to $2i_1$ and we have an expression equivalent to one given by Geldart & Sheriff (2004, p. 70). However this is only a slight simplification and it costs significantly in accuracy at moderate-to-large angles (Figure 5); so we usually choose to retain the explicit sine factor.
Wang (1999) started with the exact formulae for $R_{PP}$ and $R_{PS}$ (Aki & Richards, 1980) and developed Taylor-series expansions in powers of $\rho$, and therefore in powers of $\sin \theta$, or $\theta$, i.e., small-angle approximations. He presents one $R_{PS}$ approximation [his equation (C-3)] that is correct up to terms in $\rho^5$. However, to obtain a second simplified approximation [his equation (C-5)], Wang introduces two assumptions, one of which is quite unjustified [coming from his equation (A-10)]. For $R_{PS}$ this amounts to assuming that $\Delta \rho = 0$, which eliminates one of the two first-order terms in his expressions, even though he retains other terms up to fifth order. In comparing the accuracy of Wang’s and other $R_{PS}$ approximations for reasonable interfaces, Vant (2003) found Wang’s first fifth-order approximation [his (C-3)] to be extremely accurate but the second [his (C-5)] to be quite inaccurate. Truncation of Wang’s two $R_{PS}$ approximations after first order gives, for the first, an expression whose accuracy is about the same as that of our equation (5) but which is much more complicated; and for the second, an expression that is much less accurate than (5).

**CONDITIONS FOR OPPOSITE PP AND PS POLARITIES**

We shall use the two small-angle approximations to $R_{PP}$ and $R_{PS}$ to determine under what conditions we get opposite display polarities on the same event on PP versus PS data, that is, when $R_{PP}/R_{PS} > 0$. Thus, we start with our own approximation for $R_{PS}$ [equation (5)] and the normal-incidence expression for $R_{PP}$:

$$R_{PP} = \frac{\rho_2 \alpha_2 - \rho_1 \alpha_1}{\rho_2 \alpha_2 + \rho_1 \alpha_1}. \quad (6)$$

With respect to equation (6):

$$\text{sgn } R_{PP} = \text{sgn} \left( \rho_2 \alpha_2 - \rho_1 \alpha_1 \right) = \text{sgn} \left( \rho \Delta \alpha + \alpha \Delta \rho \right) = \text{sgn} \left( \frac{\Delta \alpha}{\alpha} + \frac{\Delta \rho}{\rho} \right). \quad (7)$$

And with respect to equation (5):

$$\text{sgn } R_{PS} = -\text{sgn} \left[ \alpha_2 \beta_2 \rho_2 + \rho_1 \left( \beta_2^2 + \beta_1^2 \right) \right] \Delta \rho + 2 \rho_1 \left( \rho_2 \beta_2 + \rho_1 \beta_1 \right) \Delta \beta$$

$$= -\text{sgn} \left[ f \frac{\Delta \rho}{\rho} + g \frac{\Delta \beta}{\beta} \right] \quad (8)$$

where \( f = \rho \left( \alpha_2 \beta_2 \rho_2 + \beta_2^2 + \beta_1^2 \right) \) and \( g = 2 \rho_1 \beta \left( \rho_2 \beta_2 + \rho_1 \beta_1 \right). \quad (9)$$

So, for opposite display polarities, or $R_{PP}/R_{PS} > 0$, we need either:

$$\frac{\Delta \alpha}{\alpha} + \frac{\Delta \rho}{\rho} > 0 \text{ and } \frac{f \Delta \rho}{\rho} + g \frac{\Delta \beta}{\beta} < 0; \text{ i.e. } \frac{\Delta \alpha}{\alpha} < \frac{\Delta \rho}{\rho} < -\frac{g \Delta \beta}{f \beta} \text{ for } R_{PP} > 0 \text{ and } R_{PS} > 0 \quad (10)$$

or:
\[
\frac{\Delta \alpha}{\alpha} + \frac{\Delta \rho}{\rho} < 0 \text{ and } \int \frac{\Delta \rho}{\rho} + g \frac{\Delta \beta}{\beta} > 0; \text{i.e. } -\frac{g \Delta \beta}{f \beta} < \frac{\Delta \rho}{\rho} < -\frac{\Delta \alpha}{\alpha} \text{ for } R_{pp} < 0 \text{ and } R_{ps} < 0. \tag{11}
\]

Equations (9) to (11) give conditions for occurrence of the unusual situation, that is, reversed display polarity of a reflection event on PS versus PP. We can formulate these conditions from (10) and (11) in another way by first noticing that:

\[
\frac{\Delta \rho}{\rho} + \frac{g \Delta \beta}{f \beta} = \frac{\Delta \rho}{\rho} + \Delta \ln \beta^h = \frac{\Delta U}{U}, \quad \text{where } h = \frac{g}{f} \text{ and } U = \rho \beta^h. \tag{12}
\]

Then we will get the unusual polarity situation, \( R_{pp}/R_{ps} > 0 \), if either:

\[
\frac{\Delta U}{U} < 0 \quad \text{(i.e. } R_{ps} > 0) \quad \text{when } \frac{\Delta Z}{Z} > 0 \quad \text{(i.e. } R_{pp} > 0) \tag{13}
\]

or:

\[
\frac{\Delta U}{U} > 0 \quad \text{(i.e. } R_{ps} < 0) \quad \text{when } \frac{\Delta Z}{Z} < 0 \quad \text{(i.e. } R_{pp} < 0). \tag{14}
\]

It seems to be ‘conventional wisdom’ for many that the sign of \( R_{ps} \) is determined by the sign of \( \Delta Y \), where \( Y (= \rho \beta) \) is shear impedance. However, we have shown that it actually depends on the sign of \( \Delta U \), where \( U = \rho \beta^h \). The exponent \( h \), given by (12) and (9), involves not only \( \beta_1 \) and \( \beta_2 \), but also \( \rho_1, \rho_2 \) and \( \alpha_2 - \alpha_1 \) explicitly. The quantity \( U \) could be termed the converted-wave impedance, or PS impedance. We believe \( U \) to be a more fundamental and diagnostic parameter in converted-wave analysis than \( Y \) or \( Z \). It should actually not be surprising that the amplitude of a reflection involving both P and S waves should not depend only on shear-wave velocities.

**CONCLUSIONS**

We have reaffirmed the possibility of opposite polarities on corresponding PP and PS reflections and some of the related pitfalls in the registration process. To quantify when this might occur, we have derived mathematical expressions that give the conditions for opposite PP and PS polarities in terms of the interface rock parameters. In the course of this work, we required an approximation to \( R_{ps} \) for small angles of incidence. The \( R_{ps} \) approximation we thus derived turns out to be more accurate than the Aki-Richards approximation, at least for three interfaces tested, and not just for small angles: beyond 25° incidence in two cases and beyond 40° in the third case.

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REFERENCES


