Colour correction in Gabor deconvolution

Carlos A. Montaña and Gary F. Margrave

ABSTRACT

White reflectivity is not a fundamental assumption either in the stationary or nonstationary convolutional model. However the deconvolution algorithms need to be modified slightly to honour color in the reflectivity series. This color correction is just possible when enough well-log information to build a mathematical model for the regional reflectivity is available. A method for correcting reflectivity color effects in frequency domain Wiener deconvolution is reviewed and extended to Gabor deconvolution.

INTRODUCTION

A generally accepted model for the seismic trace is to consider it as a convolution of the earth seismic response with a source wavelet. In turn, this wavelet can be regarded as the convolution of several effects: source signature, recording filter, earth filter, surface reflections and geophone response. Deconvolution is the process of removing the wavelet from the seismic trace to estimate the earth seismic response, which is composed of primaries and multiple reflections. The basic elements of the conventional deconvolution theory come from the theory of statistical communication (Wiener, 1949) in which a least least-square filter is the solution to the deconvolution problem. Robinson et al. (1967), established the basis for its application to Geophysics and introduced the Wiener or spiking deconvolution. The performance of the method depends on the fulfillment of a set of assumptions on which the convolutional model is based: stationarity, minimum phase wavelet, white reflectivity and white additive noise.

In presence of anelastic attenuation, the stationary assumption is not valid, and the results given by deconvolution methods are deviated from the ideal result proportionally with increasing attenuation. Inclusion of attenuation as a component of the trace model is achieved with the help of the constant Q theory for anelastic attenuation. A nonstationary convolutional model (e. g. Margrave and Lamoureux, 2002) is formulated using the constant-Q theory and the mathematical operation called nonstationary deconvolution (Margrave, 1998). An important application of the nonstationary extensions of the convolution operation and the convolutional model for the seismic trace is the Gabor deconvolution method (Margrave et al 2003, Margrave et al. 2004) that corresponds to a nonstationary extension of the Wiener deconvolution method.

Minimum phase, the second assumption of the convolutional model, continues occupying an essential place in Gabor deconvolution. Besides the minimum-phase character associated with the source wavelet generated by an explosive source, the constant-Q theory gives strong arguments to consider that the attenuation earth filter is endowed with a minimum-phase character as well. One of the consequences of this result is that even in the case when the seismic wave is generated by a vibrating source, in which case the autocorrelation of the source wavelet (the Klauder wavelet) is zero phase, the embedded wavelet remaining in the trace is still approximately minimum phase (Dong et al. 2004 SEG).

The assumption that the reflectivity is a white and stationary time series is not fundamental in the deconvolution methods, but it has to be addressed in order to avoid inaccuracies both in the amplitude and in the phase spectrum of the deconvolved trace. It has been largely reported that earth reflectivity does not have a white spectrum but instead shows considerable spectral color evidenced by a pronounced rollof in power at the low frequencies. Analysis of well logs in various regions of the world (e.g. Walden and Hosken, 1985) observed that in the majority of cases reflectivity tends to depart from the white noise behavior by a loss of power at low frequencies, thereby termed *blue reflectivity*. The assumption of white noise leads to a conventional deconvolution operator that can recover only the white component of reflectivity, thus yielding a distorted representation of the desired output, as Sagaff and Robinson, (2000) point out. The diagnostic and correction of these distortions in the application of Gabor deconvolution is the main subject of this report.

THEORY

Nonwhite reflectivity

The white character of a signal means that the signal is statistically random and uncorrelated. In the time domain this characteristic can be appreciated in the autocorrelation. The autocorrelation of an infinite-length, white signal is a perfect spike at the origin. In the frequency domain the white character implies constant power at all frequencies, which means a flat amplitude spectrum. Seismic processing however deals just with finite-length signals and the recording process imposes a constraint in the maximum frequency recorded, known as Nyquist frequency. The flatness of the amplitude spectrum of a bandlimited white signal is just a trend, as can be observed in figure 1, where a synthetic white reflectivity, its autocorrelation and its amplitude spectrum are shown.

Although the assumption of white reflectivity could be a good approximation for some areas, it has been reported (e.g, Walden and Hosken, 1985) that in many areas worldwide the reflections sequences are white just above a determined frequency. These authors found that below that frequency the power spectrum decreases according to a power law f^{β}, where β lies between 0.5 and 1.5. The same authors use a statistical tool known as ARMA (Box and Jenkins, 1970) to find a two parameter time-domain model for the reflectivity. The deviations from the white behaviour are associated especially with the stratigraphic configuration of the area. The sedimentation of sequences of rocks with repeating properties is the main cause of blueness observed in the reflectivity spectrum. Sagaaf and Robinson, (2000) synthesize the different statistical methods used to model nonwhite reflectivity in a unified method. Any of these methods allows creating a mathematical model for the reflectivity series as a function of time, or for its power spectrum as a function of frequency. This model is assumed to be valid for the area where the well logs used to build the model come from. An example of a nonwhite reflectivity series corresponding to the Wascala well in Alberta is shown in figure 2. Our personal experience is that we have observed non-white reflectivity behaviour in wells from throughout western Canada.

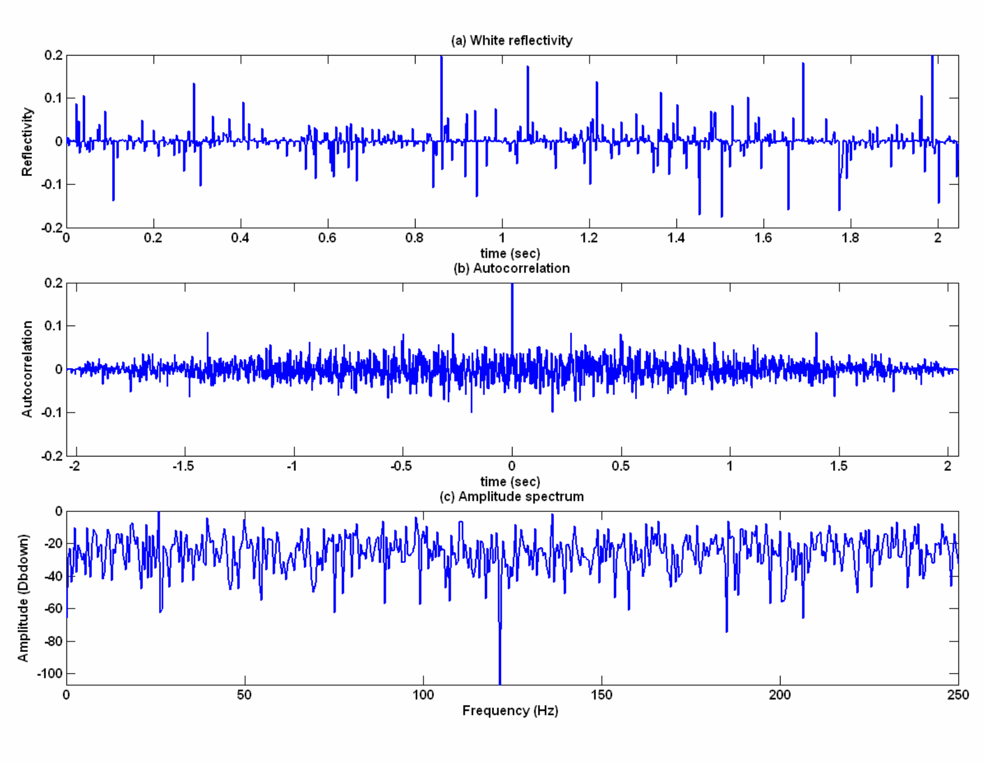

FIG. 1. (a): Synthetic white reflectivity. (b): Its autocorrelation is a spike at the origin; the noisy aspect comes from the reflectivity's finite length. (c): Amplitude spectrum, its character is essentially flat; the amplitude spectrum of an infinite length white signal would be a flat line.

The convolutional model, Wiener deconvolution and nonwhite reflectivity.

The convolutional model for a seismic trace (e.g. Robinson, 1985) establishes a structure for a seismic trace under the assumptions of nonstationarity, white reflectivity, minimum phase wavelet and additive white noise. The signature wavelet generated for the source is transmitted into the earth where suffers several effects that can be approximated as a sequence of convolutions. The most important of those events are: absorption, modification of the signature wavelet by primary and multiple reflections due to the earth layering, detection and recording. These effects can be expressed mathematically in the frequency domain by the equation

$$S(f) = \sigma(f)A(f)I(f)G(f) + N(f) \tag{1}$$

where the Fourier spectrum of the signal S is equal to the product of the spectra of the source signature σ, the near surface absorption effects A, the reflection impulse response I, and the instrument response G, plus the spectrum of the white noise N.

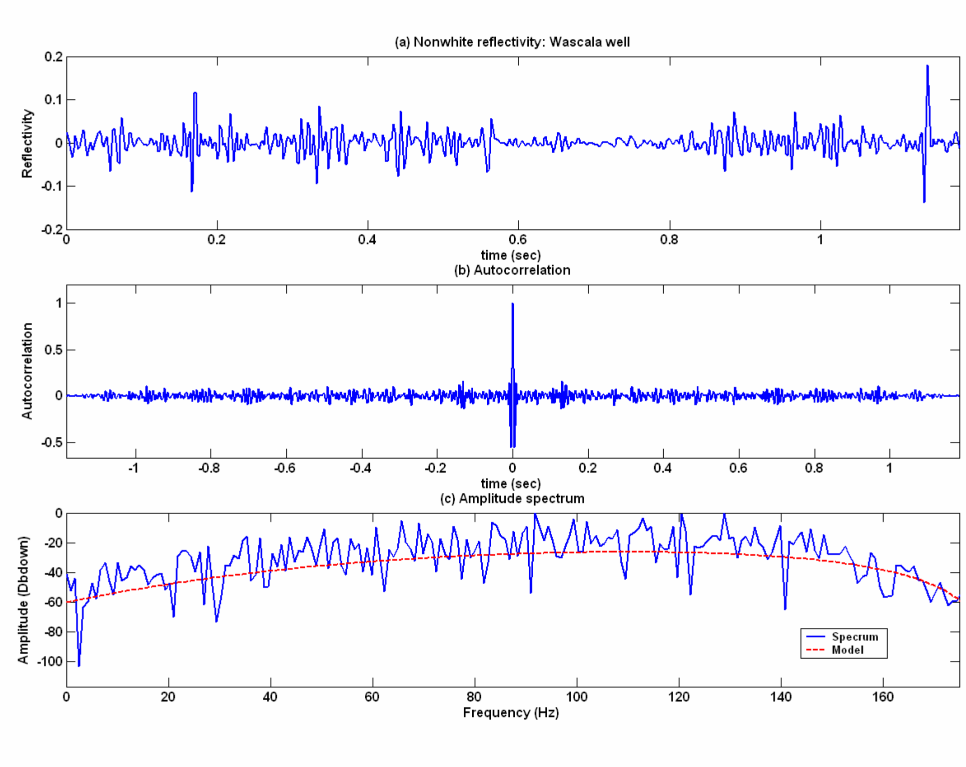

FIG 2. (a) Nonwhite reflectivity calculated from the sonic log of the Wascala well. The Gardner rule was used for the density. (b) The autocorrelation shows nonwhite character, evidenced by the presence of significant negative values at small lags. (c): The amplitude spectrum of the reflectivity shows a rolloff from 80 to 0 Hz.

Under the assumption of white reflectivity, the reflection response I can be approximated as the product

$$I(f) = R(f)M(f), \tag{2}$$

where R is the spectrum of the reflectivity coefficients (i.e. the reflectivity function, the desired geologic information) and M is the spectrum of the series of stationary multiples, such as ghost or marine multiples. By combining equations (1) and (2) the convolutional model can be expressed as

$$S(f) = \sigma(f)A(f)R(f)M(f)G(f) + N(f). \tag{3}$$

The elements affecting the reflectivity series can be grouped together and called the effective seismic wavelet, which leads to

$$S(f) = W(f)R(f) + N(f), \tag{4}$$

where the spectrum of the seismic wavelet $W(f)$ is

$$W(f) = \sigma(f) A(f) M(f) G(f) . \tag{5}$$

The components *A, M, G* are generally accepted as necessarily minimum phase, whereas σ can be minimum phase (explosive source) or zero phase (correlated vibroseis trace). The convolutional model as expressed by equation (4) is illustrated in figure 3.

Wiener deconvolution is based on the previously depicted convolutional model. The method in the frequency domain can be summarized in three main steps. (Step 1) The computation of the Fourier spectrum $S(f)$ of the seismic trace $s(t)$. (Step 2) The amplitude spectrum of the wavelet is estimated by smoothing the amplitude spectrum of the seismic trace; for example convolving it with a boxcar operator. Under the assumption of white reflectivity, this smoothed amplitude spectrum can be taken to estimate the amplitude spectrum of the unknown wavelet. For the estimation of the phase spectrum of the wavelet, the minimum-phase assumption is invoked. For such a minimum-phase wavelet, the phase spectrum is the Hilbert transform of the logarithm of the amplitude spectrum. (Step 3) The spectrum of the deconvolved trace is found by spectral division between the spectrum of the trace and the estimated spectrum of the wavelet. The process is sketched in figure 4. The Wiener deconvolution operator $D(f)$ is

$$D(f) = \frac{1}{\left|\overline{S(f)}\right| + K} e^{i\phi_D(f)} \tag{6}$$

where $\left|\overline{S(f)}\right|$ represents the smoothed amplitude spectrum of the trace, *K* is a numerically small, positive stability constant introduced to avoid numerical stability problems with values of $\left|\overline{S(f)}\right|$ equal or very near to zero, and $\phi_D(f)$, the phase spectrum, is given by the Hilbert transform, *H*, of the logarithm of the amplitude spectrum,

$$\phi_D(f) = H\left(\ln\left[\frac{1}{\left|\overline{S(f)}\right| + K}\right]\right) \tag{7}$$

A color correction can be applied after the smoothing of the amplitude spectrum of the seismic trace, dividing it by the mathematical model of the amplitude spectrum of the reflectivity $P(f)$. This function $P(f)$ is assumed to be a smooth representation of spectral color such as depicted by the smooth curve in Figure 2c. To include color correction in Wiener deconvolution, the operator equations (6) and (7) should be modified to

$$D(f) = \frac{P(f)}{|S(f)| + K} e^{i\phi_D(f)}, \tag{8}$$

$$\phi_D(f) = H\left(\ln\left[\frac{P(f)}{|S(f)| + K} \right] \right). \tag{9}$$

Application of this color corrected deconvolution operator will result in a reflectivity estimation that has the expected spectral shape. The presence of the color model function $P(f)$ in both amplitude and phase is important and indicates that the inappropriate use of the white reflectivity assumption results in both amplitude and phase errors.

An example of Wiener deconvolution with and without color correction is shown in figures 5 and 6, where the phase differences with respect to the original nonwhite reflectivity is plotted besides the deconvolved and reference traces. Two time-variant attributes are used as indicators of the time shift and phase rotation between two traces. The lag of the maximum coefficient of the crosscorrelation is used as a measure of the time shift. For the phase rotation, an angle can be estimated by minimizing the L2 norm of the difference between one signal and the other rotated by a constant angle. The two attributes are computed inside short windows. By shifting the window along the signal a time-variant characterization of the phase differences is generated.

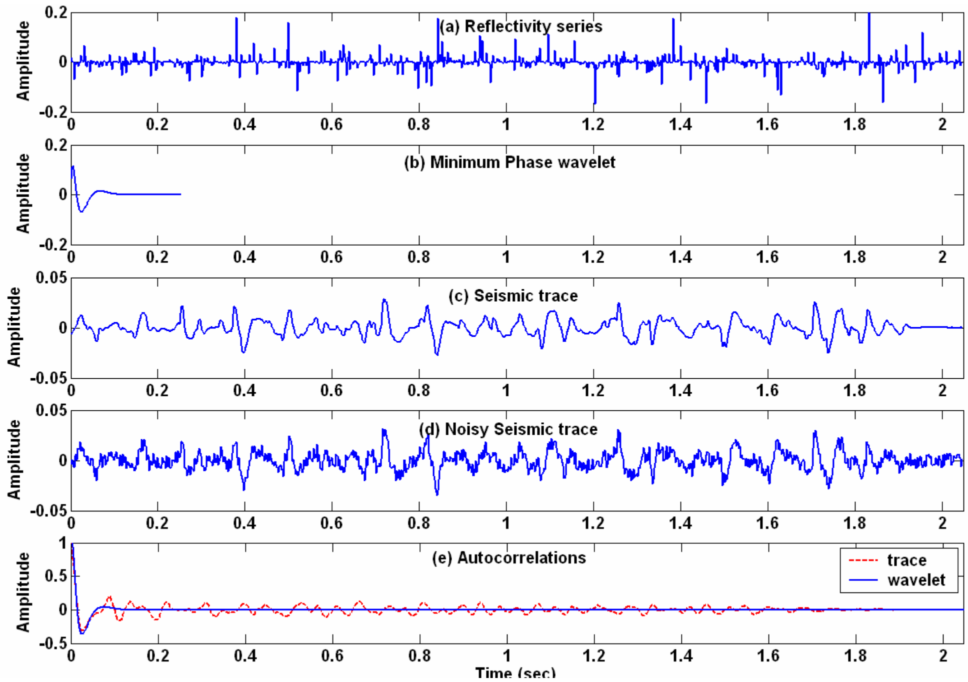

FIG. 3. Convolutional model for the seismic trace: (a): A white reflectivity series. (b): A mimimum phase wavelet that represents the source signature, the near surface minimum phase earth filter response, the stationary series of multiples and the recording instruments signature. (c) The seismic trace resulting from the convolution of the previous two signals. (d): The noisy seismic trace resulting of adding up white noise to (c). (e) The superposition of the autocorrelations of the trace and the wavelet shows an interesting and very useful result: the autocorrelation of the wavelet can be approximated by zeroing the autocorrelation of the seismic trace after a determined time.

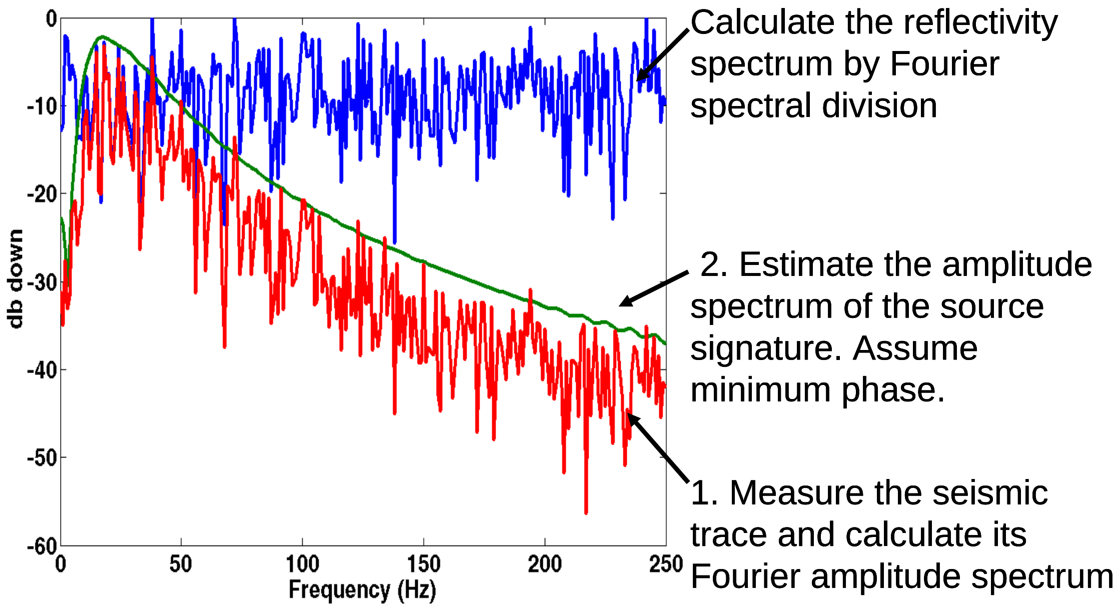

FIG. 4. The frequency domain formulation of the Wiener deconvolution method in a nutshell. The assumption of white reflectivity is implicit in the second step when stating that the spectrum of the wavelet is obtained by smoothing the spectrum of the trace. If this assumption is not true, an error is introduced both in the amplitude and in the phase spectra of the wavelet.

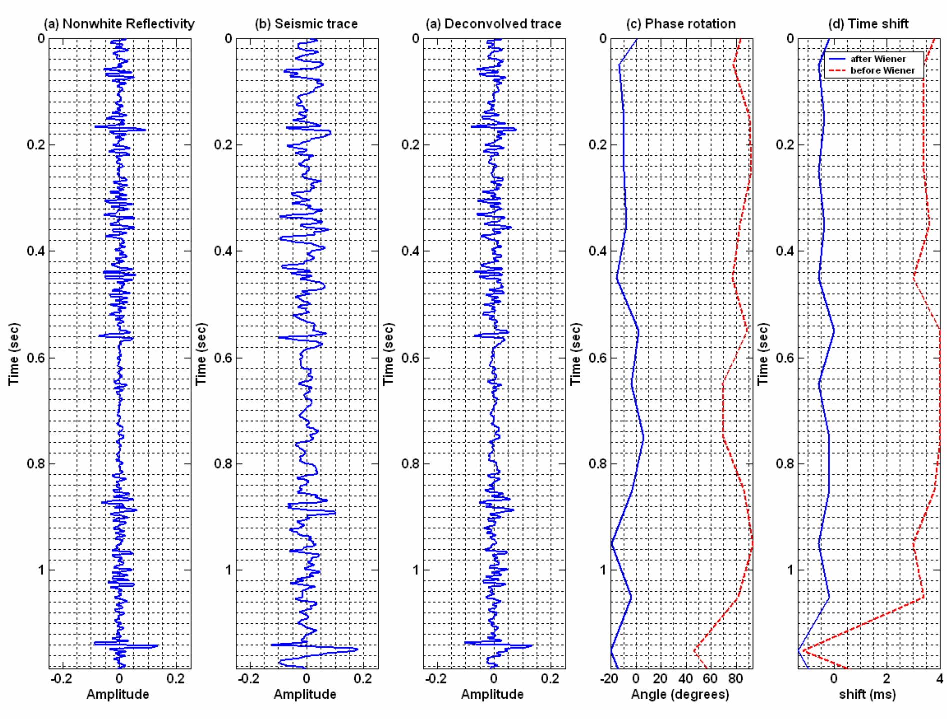

FIG. 5. Wiener deconvolution, without color correction. (a) Nonwhite reflectivity from the Wascala well. (b): Synthetic trace created by convolving the reflectivity series (a) with a minimum phase wavelet. (c): Wiener deconvolved trace. (d): Phase rotation before and after deconvolution. (e) Time shift. Phase rotations and time shifts were computed in windows of 120 ms.

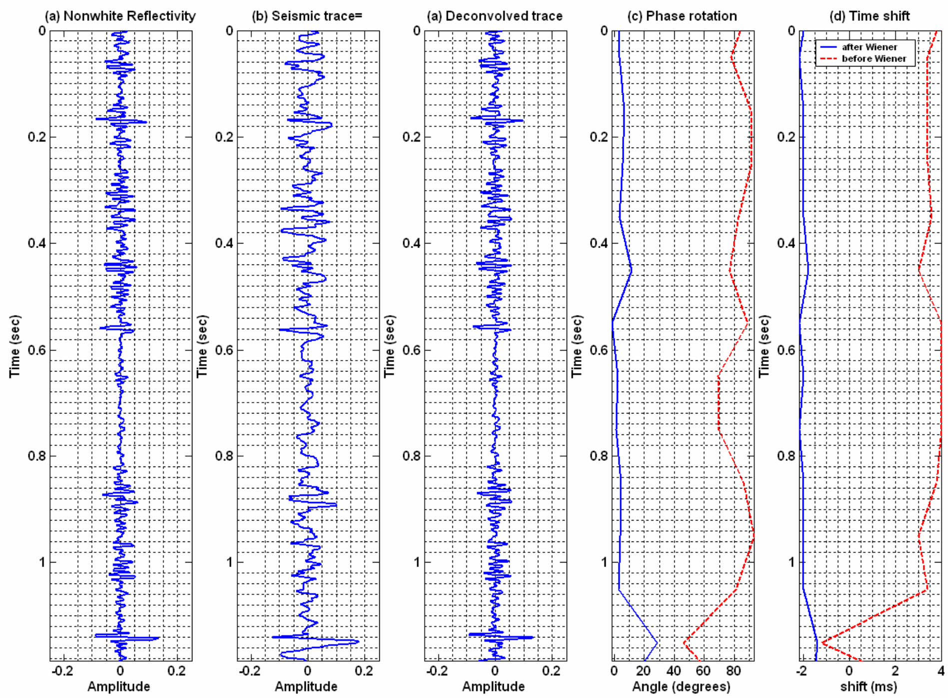

FIG. 6. Same example as in figure 5 but with color correction. (a) Nonwhite reflectivity. (b): Synthetic trace. (c): Wiener color corrected deconvolved trace. (d): Phase rotation before and after deconvolution, the color correction removed almost entirely portion of the phase rotations. (e) Time shift.

Nonstationary convolutional model, Gabor deconvolution and color correction

The constant-Q model (e.g., Aki and Richards, 2001) is the underlying theoretical support for the nonstationary convolutional model and the Gabor deconvolution method. Its basic assumptions are linearity, frequency-independent Q and velocity dispersion. A theoretical model for an attenuated trace can be derived from the constant-Q model which is useful for analyzing the effects of anelastic attenuation and for searching different methods of correcting them. These effects on the signal can be summarized as amplitude decay, due to energy absorption, waveshape modification, due to stronger absorption of higher frequencies, and phase delay, due to dispersion.

In contrast with the stationary convolutional model which can be formulated in a simple way either in the time or the frequency domain, the nonstationary convolutional model is easily depicted in mixed time-frequency domains. A mathematical model for an attenuated seismic trace $S(f)$ is, (e.g. Margrave and Lamoureux, 2002),

$$S(f) = \sigma(f)\int_{-\infty}^{\infty}\alpha_Q(f,\tau)r(\tau)e^{i2\pi f(t-\tau)}d\tau\,, \qquad (10)$$

where $\sigma(f)$ is the Fourier spectrum of the source signature, $r(\tau)$ is the reflectivity, and

$$\alpha_Q(f,\tau) = \exp\left(-\pi f\tau / Q + iH(f\tau / Q)\right), \tag{11}$$

where $\tau = x / \upsilon_\infty$ is the arrival time, f is the frequency, and Q is the attenuation parameter. Equation (10) states that the Fourier transform of the seismic trace is equal to the Fourier transform of the wavelet, multiplied by an integral that has the shape of a Fourier transform, but that given the presence of the time-frequency function $\alpha_Q(f,\tau)$, is rather a nonstationary extension of the Fourier concept, known in mathematics as a pseudodifferential operator. The function $\alpha_Q(f,\tau)$ contains the attenuation information and is endowed with minimum phase character as can be observed in the relation between the real and the imaginary part of the exponent, through the Hilbert transform. The nonstationary model for the seismic trace, according to equation (10) is illustrated in figure 7 for digital domains, where the time representation of the seismic trace is found to be equal to a nonstationary convolution of the time-frequency attenuation impulse response with the reflectivity series in time, followed by a stationary deconvolution with the source signature. As written, equation (10) assumes a spatially constant Q and models only primaries though both of these simplifications can be removed with a slight complication in the formula.

The theoretical relation between the Gabor transform and the pseudodifferential operators allows a asymptotic factorization of the nonstationary trace model, which can be considered a first order approximation to Equation (10), (Margrave and Lamoureux, 2002, Margrave et al., 2004),

$$Gs(\tau,f) \approx W(f)\alpha_Q(\tau,f)Gr(\tau,f), \tag{12}$$

which states that the Gabor transform of the seismic trace, $Gs(\tau,f)$, is approximately equal to the product of the Fourier transform of the source wavelet, $W(f)$, the time-frequency attenuation function, $\alpha_Q(\tau,f)$, and the Gabor transform of the reflectivity $Gr(\tau,f)$. Equation 12 is illustrated in figure 8.

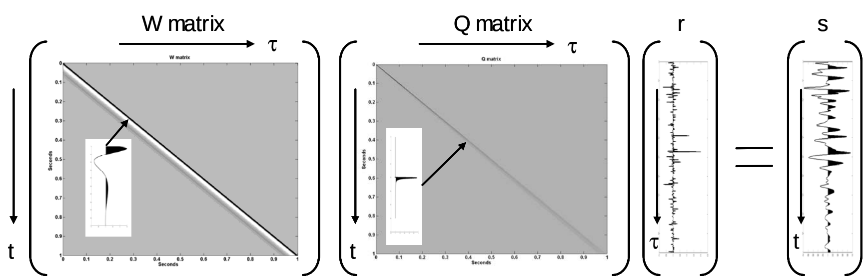

FIG. 7. Graphical representation of Equation (10) for discrete domains. The attenuated signal *s* can be modelled as the product of the source wavelet matrix, *W*, the *Q* matrix, which contains the time-frequency attenuation impulse response, and the reflectivity series.

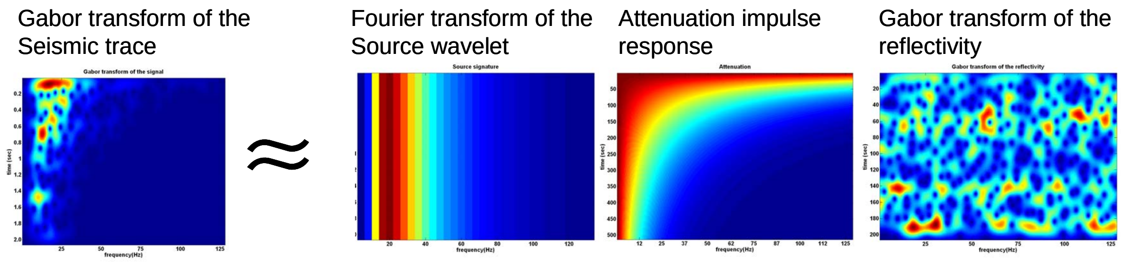

FIG. 8. Using the Gabor transform a more simple statement of the nonstationary convolutional model can be found. The Gabor transform of the seismic trace is approximately equal to the product of the Fourier transform of the source wavelet, the time-frequency attenuation function, and the Gabor transform of the reflectivity.

The Gabor deconvolution is a non-stationary extension of the Wiener deconvolution method in the frequency domain and implies a minimum-phase source wavelet and white reflectivity series, which is not fundamental but should be addressed to avoid miscalculations. The method assumes that $|Gr(\tau, f)|$ is a rapidly varying function in both variables τ and *f*, $|W(f)|$ is smoothly varying in *f,* and $\alpha_Q(\tau, f)$ is an exponentially decaying function in both variables τ and *f*, and constant over hyperbolic families of $\tau f = constant$. An approximation $|\theta(\tau, f)|$ of the product $|W(f)|\,|\alpha_Q(\tau, f)|$ is obtained by applying a smoothing operator to $|Gs(\tau, f)|$. Different kinds of smoothing are possible; Iliescu and Margrave (2002) use 2-D boxcar smoothing and hyperbolic smoothing, along curves of $\tau f = constant$. Grossman et al. (2002) use a different method to estimate $|W(f)|$ and $|\alpha_Q(\tau, f)|$ based on a least square fitting of Equation (12). As $\theta(\tau, f)$ represents the attenuated source wavelet, its minimum-phase function is estimated from its amplitude spectrum $|\theta(\tau, f)|$ using the Hilbert transform as

$$\varphi(\tau, f) = \int_B \frac{\ln|\theta(\tau, f')|}{f - f'} df' \qquad (13)$$

where B denotes the available spectral band.

Finally the Gabor spectrum of the reflectivity is estimated in the Gabor domain as:

$$Gr(\tau, f)_{est} = \frac{Gs(\tau, f)}{\theta(\tau, f)} . \qquad (14)$$

An example of the performance of Gabor deconvolution in the case of white reflectivity is shown in figure 9. Gabor deconvonvolution makes an excellent correction of the phase shifts and rotations introduced by attenuation. This example does not take into account the phase differences found when comparing synthetic traces generated from well logs and recorded traces. This additional phase errors are caused by fact that the frequency of the sonic well log tool is as high as 20000 Hz. As the maximum frequency of the recorded trace is limited to the Nyquist frequency, this difference will introduce additional phase delays and rotations which are caused by the imperfect estimation of the

Hilbert transform from the frequency bandlimited recorded trace. This subject is analyzed more in detail in Montana and Margrave, (2005).

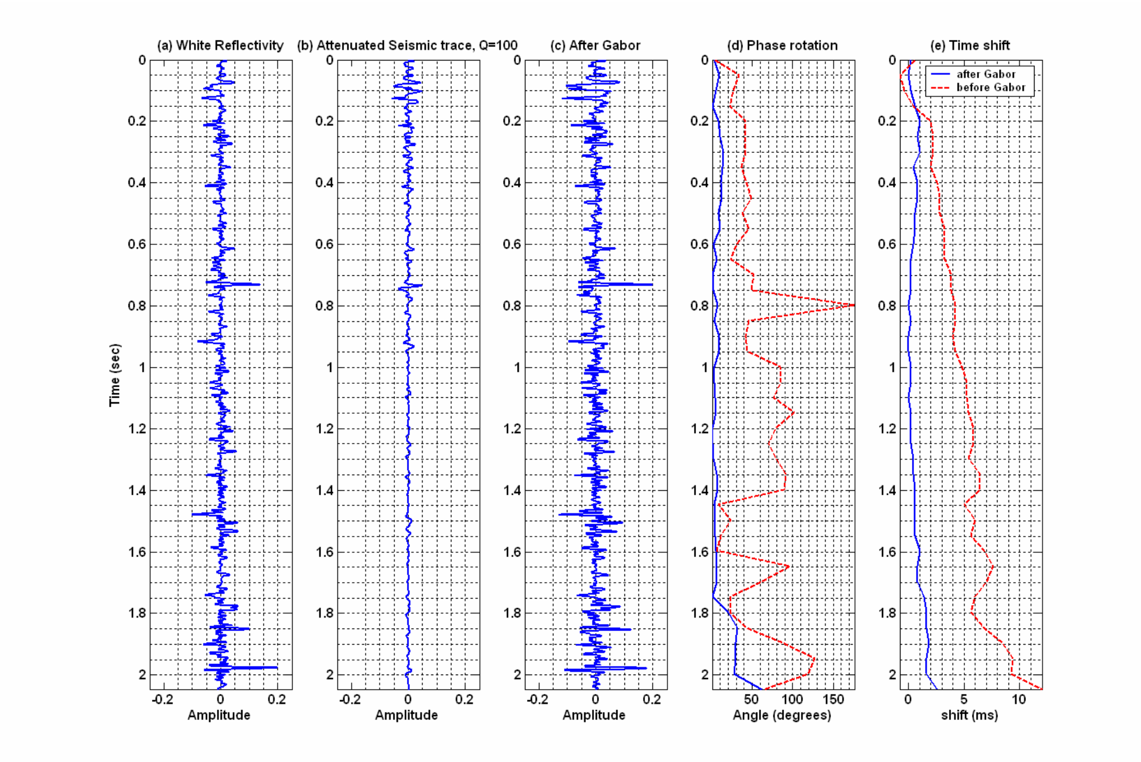

FIG. 9. Gabor deconvolution in the case of white reflectivity. (a): White reflectivity. (b): Attenuated seismic trace generated by applying a forward Q=100 filter and then a convolution with a minimum phase wavelet. (c): Gabor deconvolved trace. (d): Time-variant apparent phase rotation before and after Gabor decon, a big portion of the rotations is removed. (e): Time-variant phase shift.

The color correction can be addressed in an analogous manner as in the stationary Wiener case. If a time-frequency model $P(\tau,f)$ for the nonwhite reflectivity is available, the deconvolution operator with color correction is

$$\theta_c(\tau,f)=\frac{P(\tau,f)}{\left|\overline{Gs(\tau,f)}\right|+K}e^{i\phi_c(\tau,f)}, \tag{15}$$

where $\left|\overline{Gs(\tau,f)}\right|$ is the smoothed Gabor spectrum of the seismic trace, K is a stability constant and the phase spectrum $\phi_c(\tau,f)$ is given by the following Hilbert transform, H,

$$\phi_c(\tau,f)=H\left(\ln\left[\frac{P(\tau,f)}{\left|\overline{S(\tau,f)}\right|+K}\right]\right). \tag{16}$$

The application of color correction is illustrated in figures 10 and 11. Figure 10 shows what happens if the nonwhite reflectivity character is not taking into account at applying

Gabor deconvolution. An overcorrection of the phase shifts and rotations leaves the trace with phase and rotations equivalent to a big portion of the existing before Gabor deconvolution was applied.

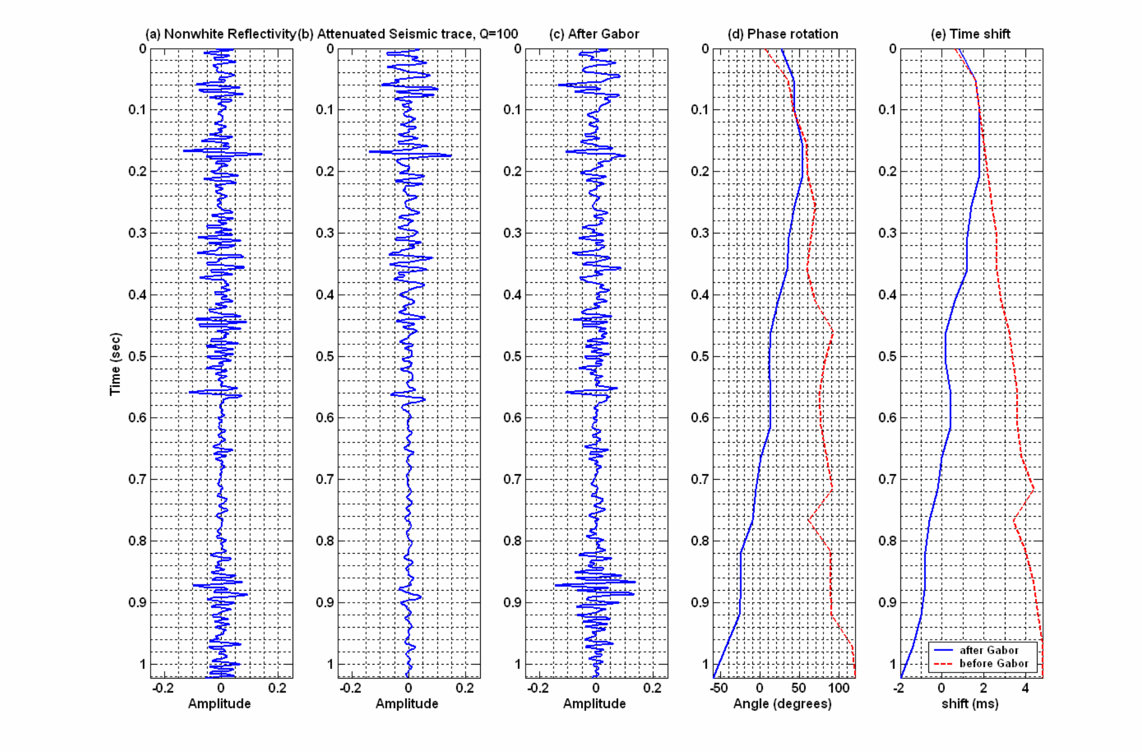

FIG. 10. Gabor deconvolution for nonwhite reflectivity. (a): Nonwhite reflectivity. (b): Attenuated seismic trace generated by applying a forward Q=100 filter and convolving the result with a minimum phase wavelet. (c): Gabor deconvolved trace without color correction. (d): Time-variant apparent phase rotation before and after Gabor decon, the phase rotations seem overcorrected. (e): Time-variant phase shift, the times-shifts appear overcorrected.

CONCLUSIONS

Gabor deconvolution is the nonstationary extension of Wiener deconvolution, based on the nonstationary extension of the convolution operation and the convolutional model. Color correction in reflectivity in Wiener deconvolution can be extended to Gabor deconvolution in an analog way. However this correction should be applied in conjunction with corrections to the other sources of phase differences in deconvolution.

When the color correction factor is applied to the Gabor deconvolution operator, the phase differences are reduced to a small portion of the phase differences existing before deconvolution and after deconvolution without phase correction, this can be observed in figure 11. It has to be pointed out at this point that the effect of color in the reflectivity series on the phase is just one component of the whole phase problem in deconvolution. Other factors affecting the phase are noise and the difference between the Nyquist frequency and the maximum frequency of the sonic well logging tool.

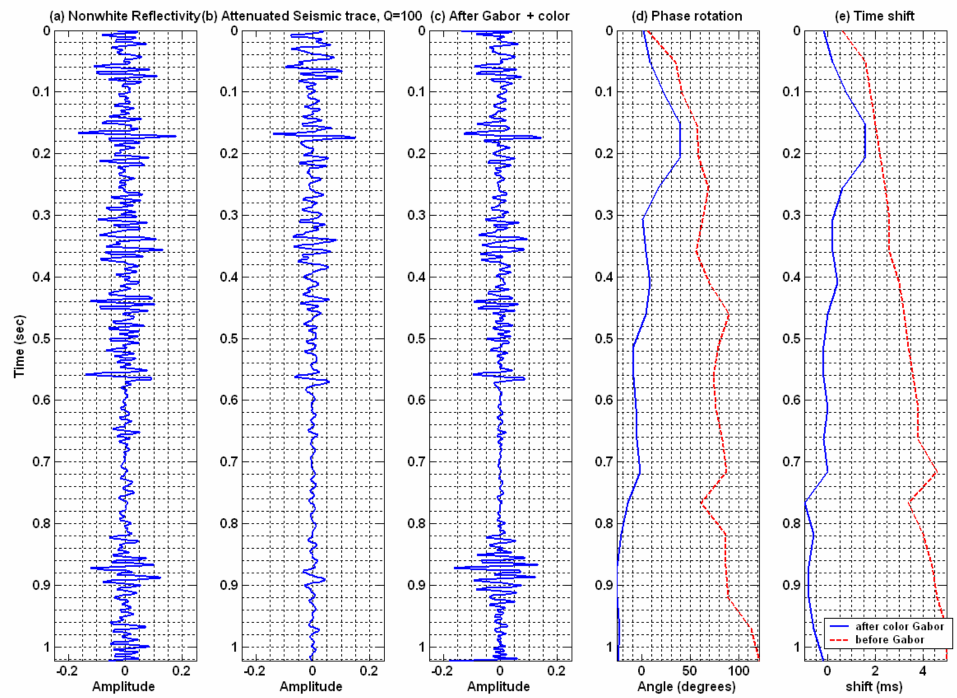

FIG. 11. Gabor deconvolution for nonwhite reflectivity. (a): Nonwhite reflectivity. (b): Attenuated seismic trace generated by applying a forward Q=100 filter and then a convolution with a minimum phase wavelet. (c): Gabor deconvolved trace with color correction. (d): Time-variant apparent phase rotation before and after Gabor deconvolution. (e): Time-variant time shift.

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