Bicorrelation and random noise attenuation

Arnim B. Haase

ABSTRACT

Assuming that noise free auto-correlations or auto-bicorrelations are available to guide optimization, signal can be recovered from a noise background to some extent. A synthetic example is employed to demonstrate the procedure of noise rejection by signal optimization. Except for noise bursts at higher frequencies, the auto-bicorrelation approach gives better results.

INTRODUCTION

High frequency loss due to attenuation decreases bandwidth and resolution. The process of frequency enhancement seeks to restore bandwidth and thereby improve resolution. Chopra (2003) uses information from VSP's to compensate frequency loss in surface seismic data. Countiss (2002) utilizes a proprietary process to recover high frequency signal from under noise. Mendel (1991) in his tutorial on higher-order statistics (known as cumulants) states that cumulants are blind to any kind of a Gaussian process whereas correlation is not. By contrast, correlation is phase blind but cumulants are not. This sounds like an excellent recipe to recover seismic signal from below a random noise floor. The first step beyond second-order statistics is bicorrelation (third order statistics). Cross-bicorrelation has been used by Lu and Ikelle (2001) to image beyond seismic wavelengths. The key to these methods appears to be signal processing in the bicorrelation/bispectral domain where Gaussian noise is minimized and resolution is increased. This report describes an attempt to recover a synthetic signal from a Gaussian noise background.

SIGNAL AND NOISE GENERATION

The synthetic signal used throughout this study is shown by the green curve in Figure 1. Attenuation is modelled by applying an exponential high frequency roll-off. Noise is generated employing an algorithm for square distributions found in Numerical Recipes (Press et al., 1999). A Gaussian distribution is simulated via the central limit theorem. Figure 2 displays the result of adding signal and noise when both are normalized to unit maximum absolute amplitude. Total trace length is 2.048 seconds. One half of this trace is set to zero in order to prevent wrap-around when computing correlations in the frequency domain. No ramping is applied at either end of the nonzero trace part which implies a boxcar time domain window. Note that, at this signal-to-noise amplitude ratio of unity, the synthetic signal is almost hidden in the noise.

AUTO-CORRELATION AND AUTO-BICORRELATION

The auto-correlations of both the noise free synthetic signal from Figure 1 and the noise contaminated synthetic signal from Figure 2 are given in Figure 3. The process of auto-correlation reduced the noise level considerably. Increasing the length of the noise contribution beyond 1.024 seconds allows the auto-correlation procedure to reduce the noise level further. Figure 4 shows the log-magnitude spectra of the auto-correlations of Figure 3. A side-lobe of the noise free auto-correlation spectrum is barely visible at approximately 45Hz. When looking at the noise contaminated spectrum in Figure 4 it appears almost hopeless to attempt signal recovery.

The normalized auto-bicorrelation of the noise free synthetic signal is displayed in Figure 5. An interesting pattern of side-lobes is visible. Note that the depth of troughs is limited to a maximum negative excursion for display purposes. The equivalent plot for the noise contaminated synthetic signal can be seen in Figure 6. It is not as easy as it was for the auto-correlation (Figure 3) to make out the signal. The corresponding normalized auto-bicorrelation log-magnitude spectra are shown in Figures 7 and 8. Again, the noise contaminated spectrum in Figure 8 appears to be an almost hopeless case. The formulae used to compute auto-bicorrelations and auto-bispectra are given by Lu and Ikelle (2001) as:

$$r(\tau_1, \tau_2) = E[x(t)x(t+\tau_1)x(t+\tau_2)] \quad \text{and}$$

$$B(\omega_1, \omega_2) = E[X(\omega_1)X(\omega_2)X^*(\omega_1+\omega_2)]$$

where E[ ] is the expectation operator.

SIGNAL OPTIMIZATION

The assumption made for the optimization procedure is that the auto-correlation and auto-bicorrelation of the noise free synthetic signal are known. Given this information, the auto-bicorrelation of the noise contaminated synthetic signal can be fitted to the “ideal” noise free auto-bicorrelation in, for example, the LSE (least square error) sense. The procedure is to modify the noisy input signal, one sample at a time, such that the LSE is minimized at each step. When all input samples are modified, the algorithm returns to the first sample and starts over. Figure 9 displays the result of 99 recursions applied to the auto-correlation LSE. The noise level is reduced but the signal optimum (red curve) shows considerable error. By contrast, when applying the procedure to the auto-bicorrelation LSE, the signal optimum (red curve) matches the original noiseless synthetic signal better and the remaining noise level is also lower as can be seen in Figure 10. Again, 99 recursions are used. Figures 11 and 12 show the log-magnitude spectra equivalent to Figures 9 and 10. Clearly, the noise level is reduced in both cases. The bispectrum optimization result (Figure 12) matches the original signal more closely in the notch between the main-lobe and the first side-lobe. In fact, that match is improved considerably when the optimization is allowed to proceed beyond 99 recursions. The tuning effect visible in Figure 12 changes little with the number of recursions. At the time of writing the cause of this tuning effect is not known.

CONCLUSIONS

Assuming that noise free auto-correlations or auto-bicorrelations are available to guide the procedure, signal optimization can reduce a noise background to some extent. The additional information available in the bicorrelation domain leads to improved noise rejection by signal optimization when compared to the second order process of auto-correlation. A tuning effect is observed at higher frequencies in the optimized bispectrum. Whether the assumptions made for signal optimization are too restrictive or bicorrelation signal optimization can be expanded to include a reflection coefficient series would be an interesting topic for future research.

REFERENCES

Chopra, S., 2003, Enhancing seismic frequency bandwidth using VSPs-a case study: Oil and Gas Journal, Oct.20 2003, 38-43.

Countiss, M.L., 2002, Frequency-enhanced imaging of stratigraphically complex, thin-bed reservoirs: A case study from South Marsh Island Block 128 Field: The Leading Edge, **21**, No.9, 826-836.

Lu, W., and Ikelle, L.T., 2001, Imaging beyond seismic wavelength using higher-order statistics: Preliminary Results: Journal of Seismic Exploration, **10**, 95-119.

Mendel, J.M., 1991, Tutorial on Higher-Order Statistics (Spectra) in Signal Processing and System Theory: Theoretical Results and some Applications: Proceedings of the IEEE, **79**, No.3, 278-305.

Press, W.H., Teukolsky, S.A., Vetterling, W.T., and Flannery, B.P., 1999, Numerical Recipes in Fortran77: Cambridge University Press.

ACKNOWLEDGEMENTS

Support by the CREWES team and its industrial sponsorship is gratefully acknowledged.

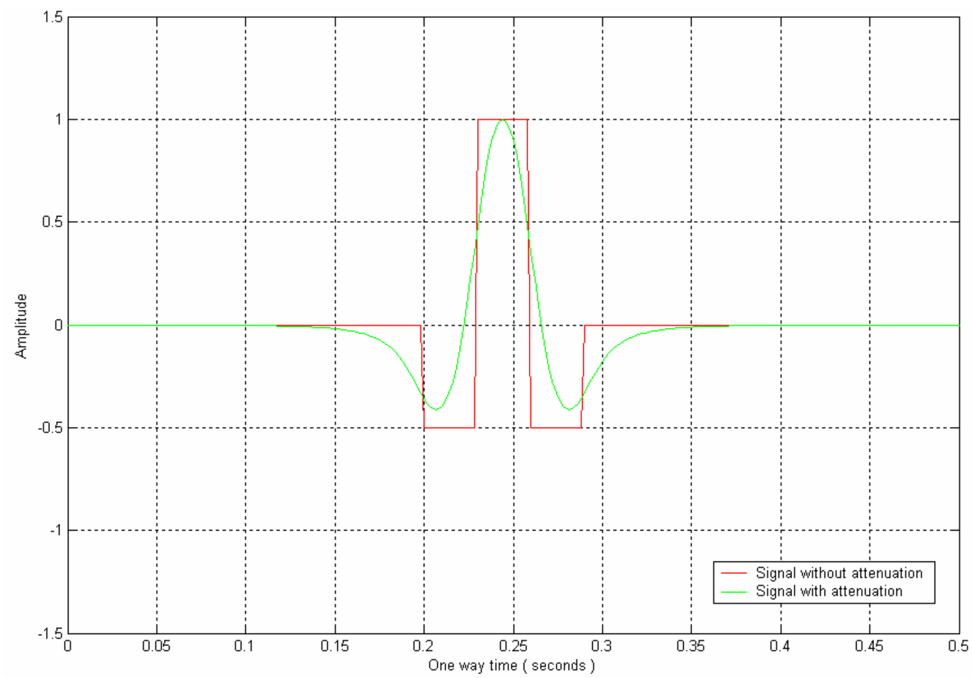

FIG. 1. Normalized synthetic signals.

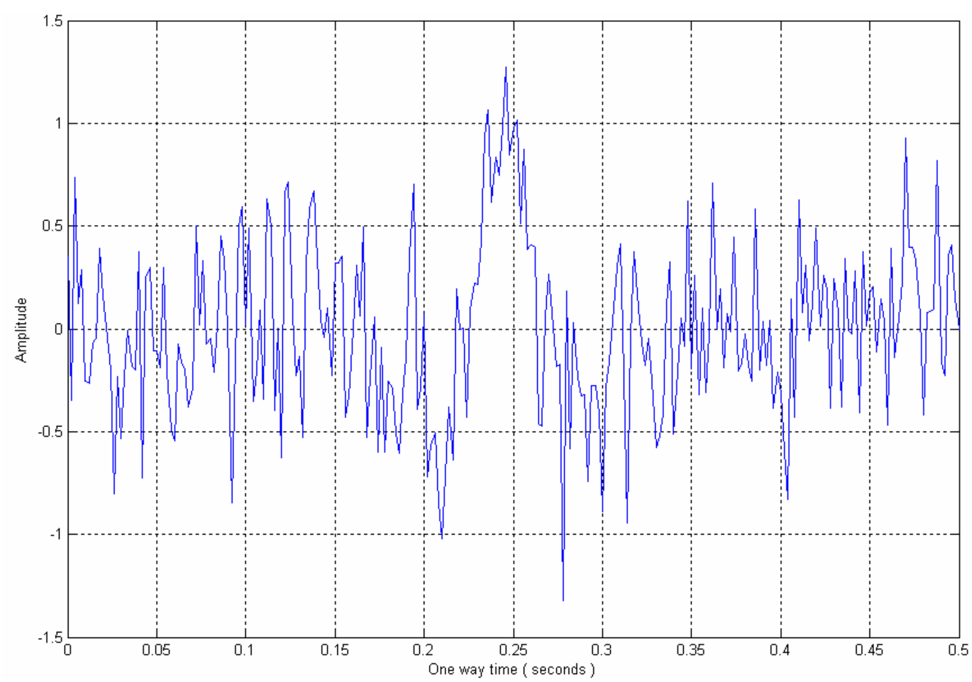

FIG. 2. Noise contaminated synthetic signal.

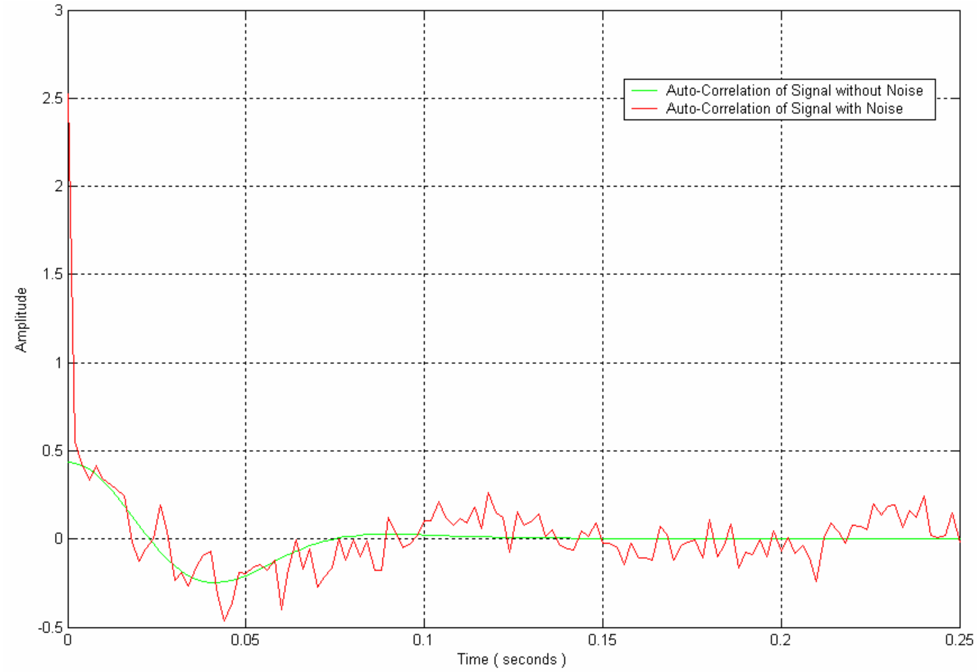

FIG. 3. Auto-correlation of synthetic signals.

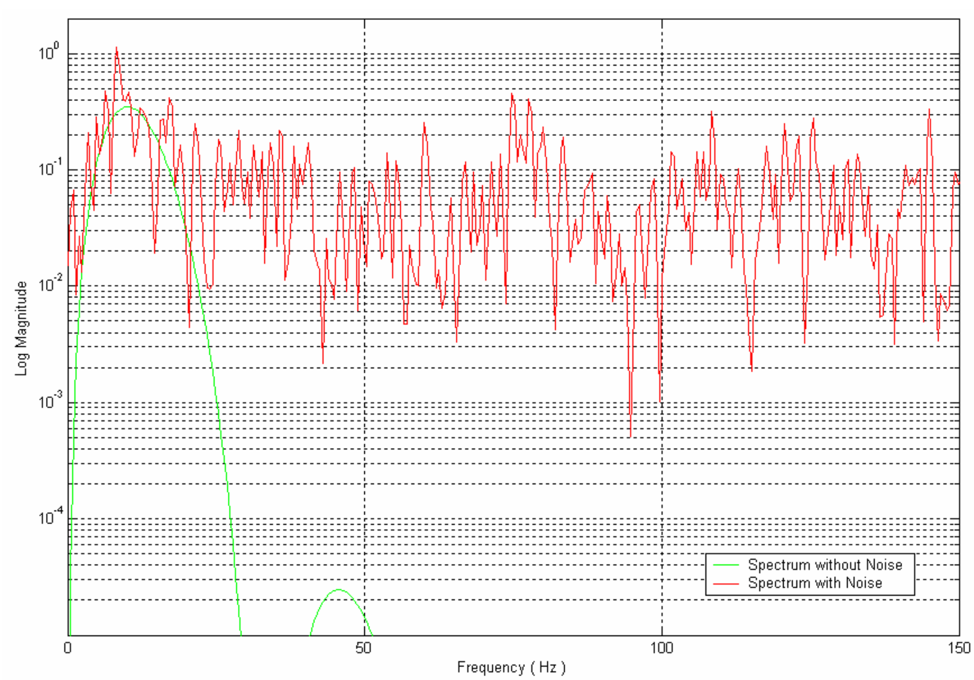

FIG. 4. Auto-correlation magnitude spectrum.

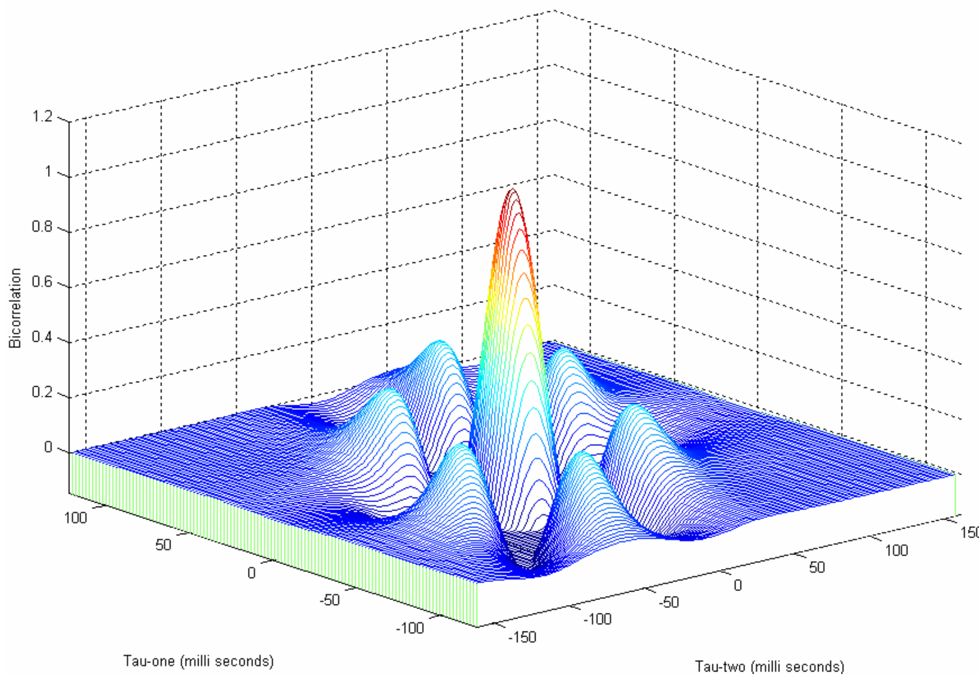

FIG. 5. Normalized auto-bicorrelation of signal without noise.

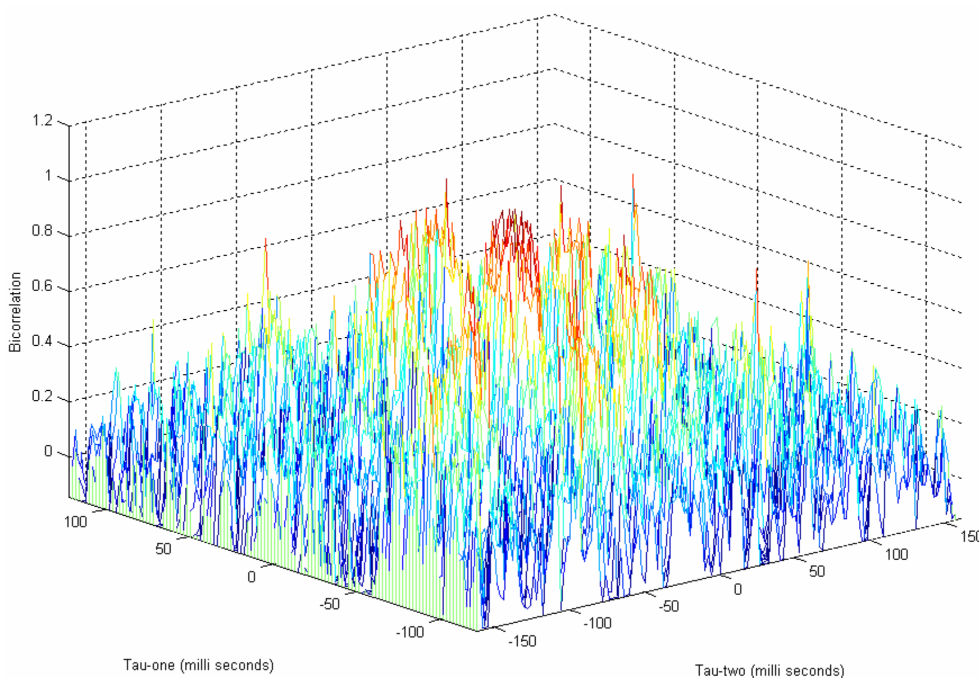

FIG. 6. Normalized auto-bicorrelation of signal with noise.

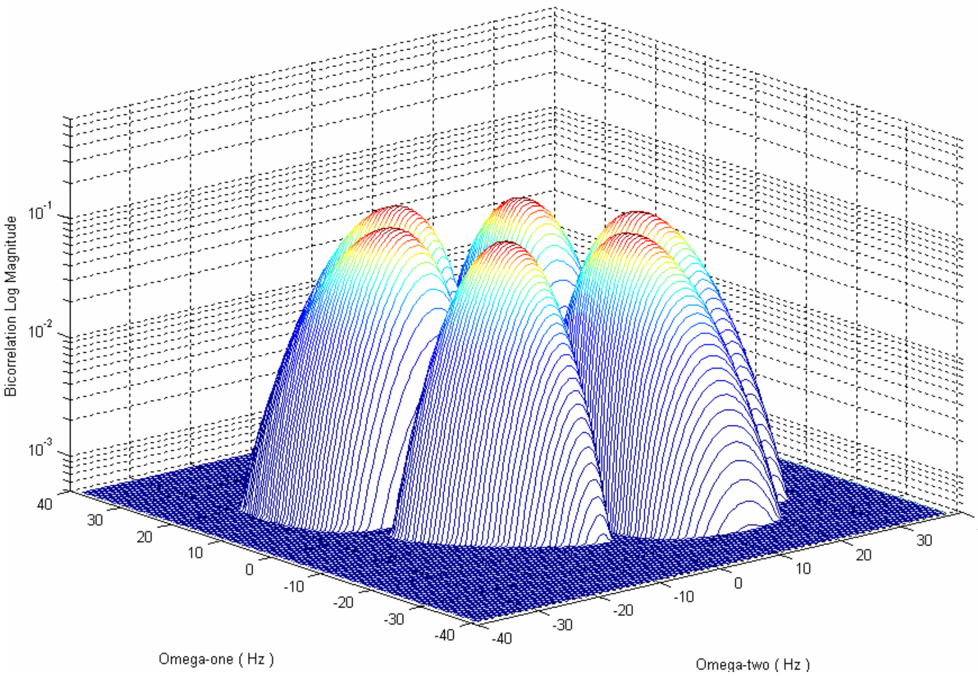

FIG. 7. Normalized auto-bicorrelation spectrum (noise free input).

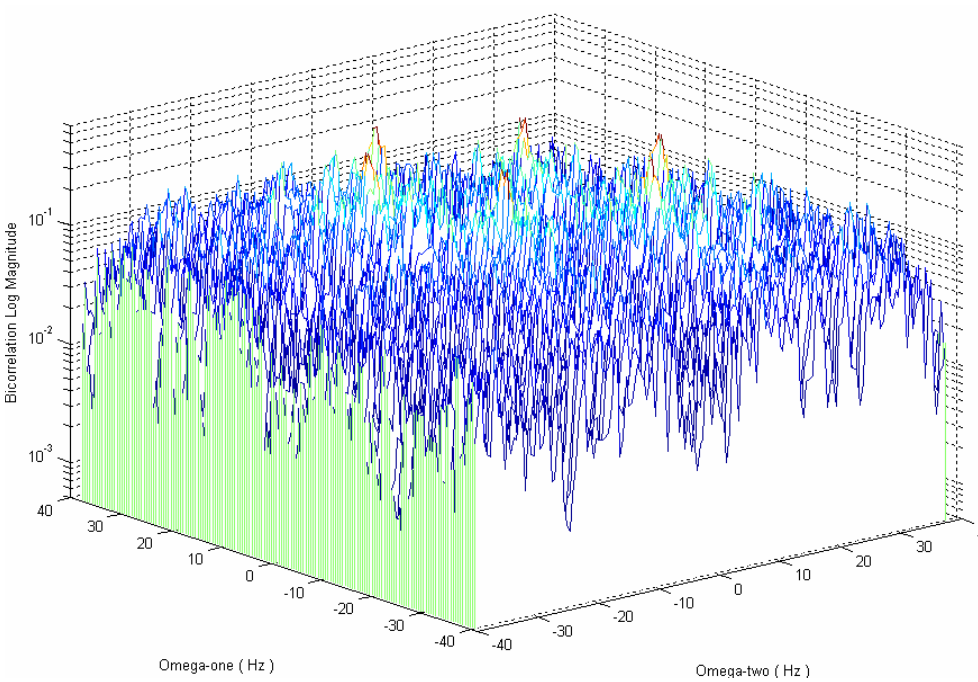

FIG. 8. Normalized auto-bicorrelation spectrum (noisy input).

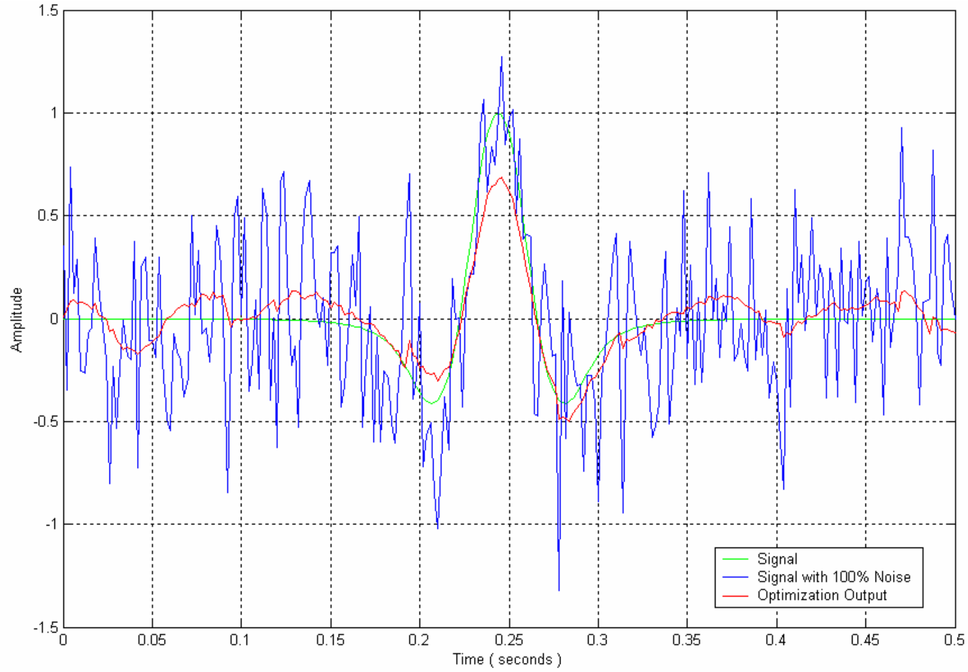

FIG. 9. Auto-correlation optimization.

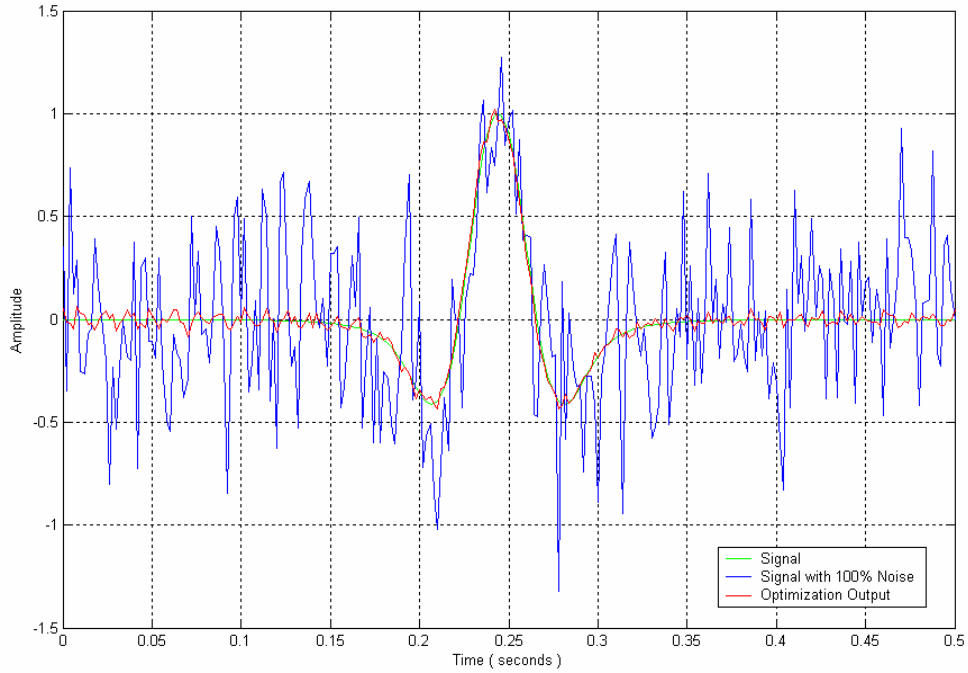

FIG. 10. Auto-bicorrelation optimization.

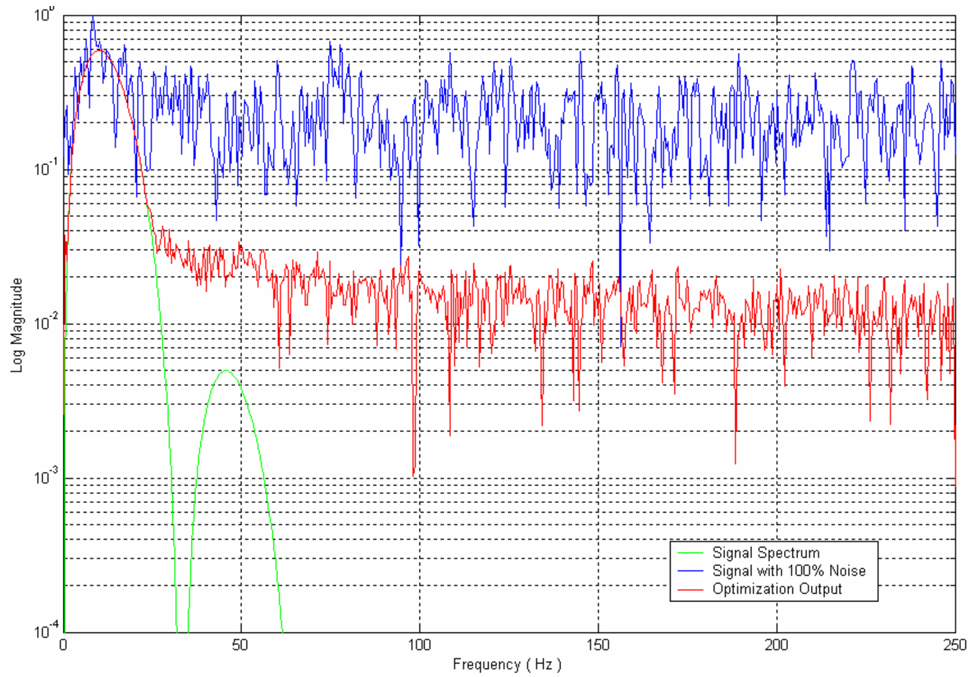

FIG. 11. Auto-correlation optimization spectrum.

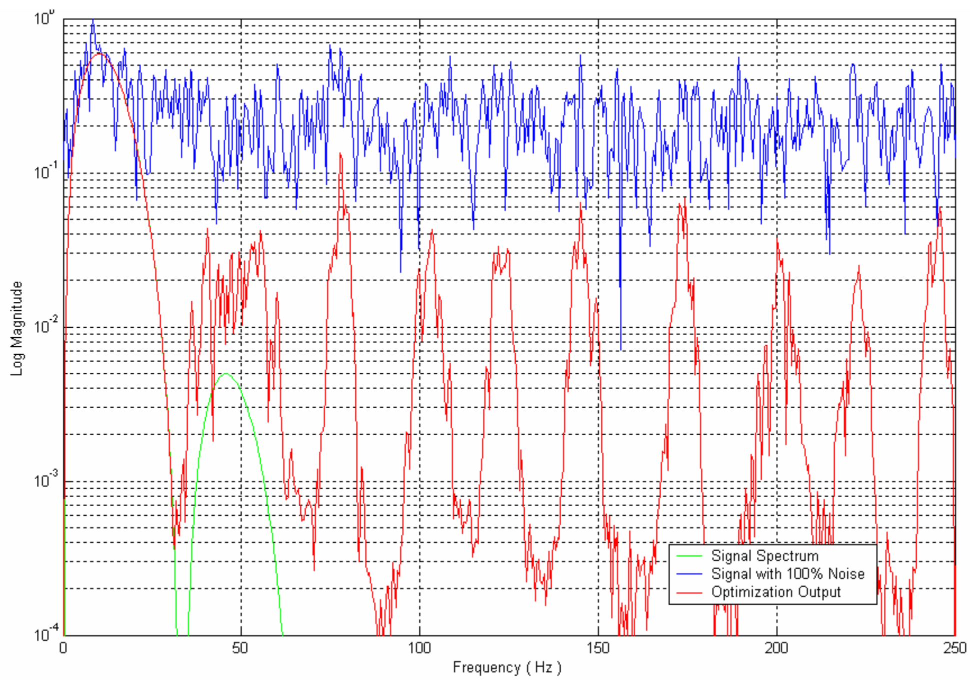

FIG. 12. Auto-bicorrelation optimization spectrum.