Choice of parameters in an AVO inversion

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ABSTRACT

In AVO inversion the choice of parameters is not neutral in the sense that, although theoretically equivalent, if they are not correctly chosen, the numerical algorithms in the inversion can be inefficient. In this paper we show that in inverting converted shear wave (PS) data choosing S-impedance and density as parameters rather than P-impedance and density, produces a more stable AVO inversion.

INTRODUCTION

A combination of three parameters is needed to describe a perfectly elastic, isotropic earth: for example, density, ρ, and the Lame parameters, λ and μ, or the density, ρ, and the P-wave and S-wave velocities, V_P and V_S (Tarantola, 1986). For the first combination, several authors have commented that more physical insight is provided by the rigidity modulus, μ (Wright, 1984; Thomson, 1990; Castagna et al., 1993). Stewart (1995) discussed the potential usefulness of the Lame parameters, λ and μ, to better differentiate rock properties. For the second combination, a number of authors have observed a link between V_P, V_S, and pore fluid content. The relationship of V_P and V_S values to various rocks and their saturants can be complex, but can provide useful information about reservoir rock properties when used in conjunction with well-log or other independent measurements (Tatham and McCormick, 1991). There are other parameter choices describing rock properties, such as P-wave impedance, $I = \rho V_P$, S-wave impedance, $J = \rho V_S$, and density, ρ, which was the choice advocated in Jonnane et al. (1988) or Tarantola (1986). The choice of parameters is not neutral in the sense that although theoretically equivalent, if they are not properly chosen, the numerical algorithms in the inversion can be inefficient (Tarantola, 1986).

In this paper we show that in inverting converted shear wave (PS) data the choice of S-impedance and density parameters rather than P-impedance and density parameters, produces more stable PS inversion. An AVO inversion of converted shear wave data is called PS inversion here.

METHODOLOGY

The presented PS inversion is based on the AVO inversion method described in Mahmoudian and Margrave (2003). The Aki-Richards (1980) linear approximations for PS reflection coefficient, R_{PS}, can be formulated as a function of density and S-wave impedance as

$$R_{PS} = D(\theta,\rho)\frac{\Delta\rho}{\rho} + E(\theta,\rho)\frac{\Delta J}{J}, \tag{1}$$

where R_{PS} is the angle dependent PS reflection coefficient and ρ is density. The coefficients *D* and *E* are functions of the average P-wave incident angle, θ, the average S-wave reflected angle, φ, and the ratio of S-velocity to P-velocity, V_S / V_P, across the interface (Larsen and Margrave 1999) given by

$$D(\theta,\varphi) = -\frac{V_P \tan\varphi}{2V_S}\left(1 + 2\sin^2\varphi - 2\frac{V_S}{V_P}\cos\theta\cos\varphi\right), \tag{2}$$

$$E(\theta,\varphi) = \frac{V_P \tan\varphi}{2V_S}\left(2\sin^2\varphi - 2\frac{V_S}{V_P}\cos\theta\cos\varphi\right). \tag{3}$$

Assuming Gardner's rule between density and P-wave velocity ($\rho = kV_P^{1/4}$, where *k* is a constant; Gardner at al., 1974) the density contrast term can be rewritten as a function of P-wave impedance,

$$\frac{\Delta\rho}{\rho} = \frac{1}{5}\frac{\Delta I}{I}. \tag{4}$$

With this assumption, the Aki-Richards (1980) approximation for PS reflection coefficient becomes

$$R_{PS} = C(\theta,\rho)\frac{\Delta I}{I} + E(\theta,\rho)\frac{\Delta J}{J}. \tag{5}$$

Inverting the PS data using equation (1) will result in the *J* and *ρ* estimates and is called PS inversion for *J* and *ρ*. Also, inverting the PS data using equation (5) will result in the *I* and *J* estimates and is called PS inversion for *I* and *J*. The PS inversion is done using the SVD method as described in Mahmoudian (2006). For a simple synthetic example which obeys Gardner's rule, we show that a simple change in the inversion parameter makes a big change in the stability of the inversion. The PS inversion for a synthetic data example is presented.

TESTING

Figure 1 shows the velocity-depth model, in which the density model obeys Gardner's rule and the S-wave velocity obeys the Mudrock relation ($V_P = 1360 + 1.16V_S$ (velocities in m/s)). The PS synthetic has an initial 5-10-80-100 Hz zero-phase wavelet, and the offsets range from 0 to 500 m.

Using the SVD method in the PS inversion, the condition number as an indicator to the singularity of the inversion problem is examined. Condition number is the ratio of the largest to the smallest singular values (see Mahmoudian and Margrave (2006) for more details). A matrix is well-posed when its condition number is not far from 1 (Jin at al., 2002), and an ill-posed matrix is a matrix with very large condition number.

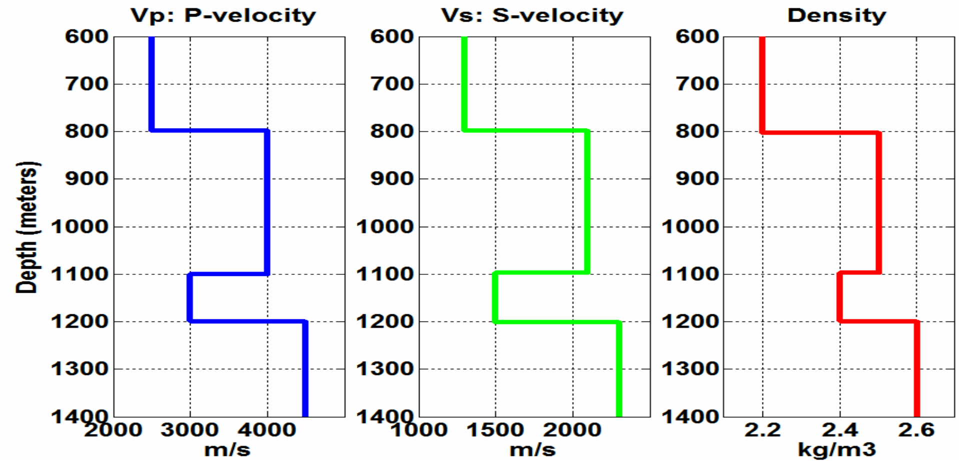

FIG. 1. Synthetic 1, velocity-depth model.

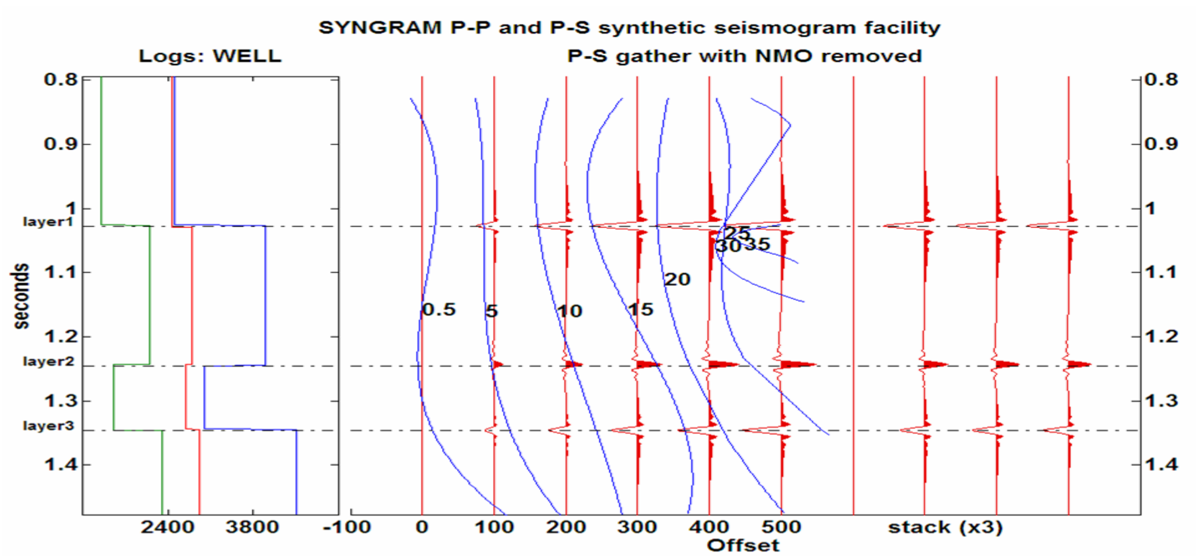

FIG. 2. Synthetic PS gather from the velocity model in Figure 1, in PS time. The three traces on the right are three repetitions of the stacked trace. The contours of incident angles (degrees) of PS rays are displayed.

Figures 3 and 4 show the two singular values versus the depth from the PS inversions of the synthetic example. In singular value plots, below, the condition number of the matrix is shown in red and the righthand vertical axis shows its value. While the singular value plots are shown in red, the lefthand vertical axis shows the magnitude of the singular values. The Condition number varies from 50 to 250 for the PS inversion for I and J, while it varies from10 to 50 for the PS inversion for J and ρ. The decrease in condition number in the PS inversion for J and ρ indicates that the converted shear PS data is capable of being inverted with favorable estimates of J and ρ.

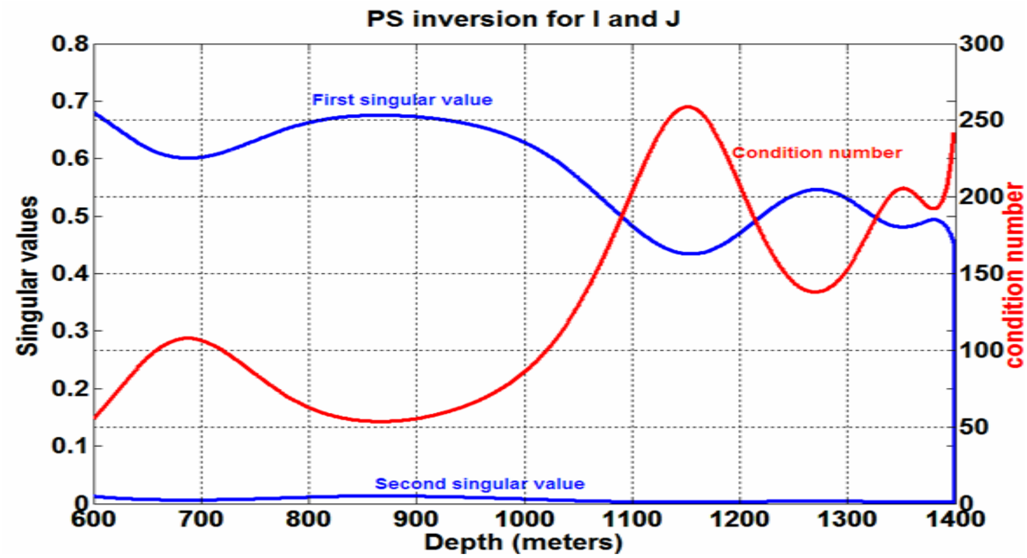

FIG. 3. Singular values (in blue) and the condition number (red curve) versus depth, of the PS inversion of synthetic 1, for *I* and *J*.

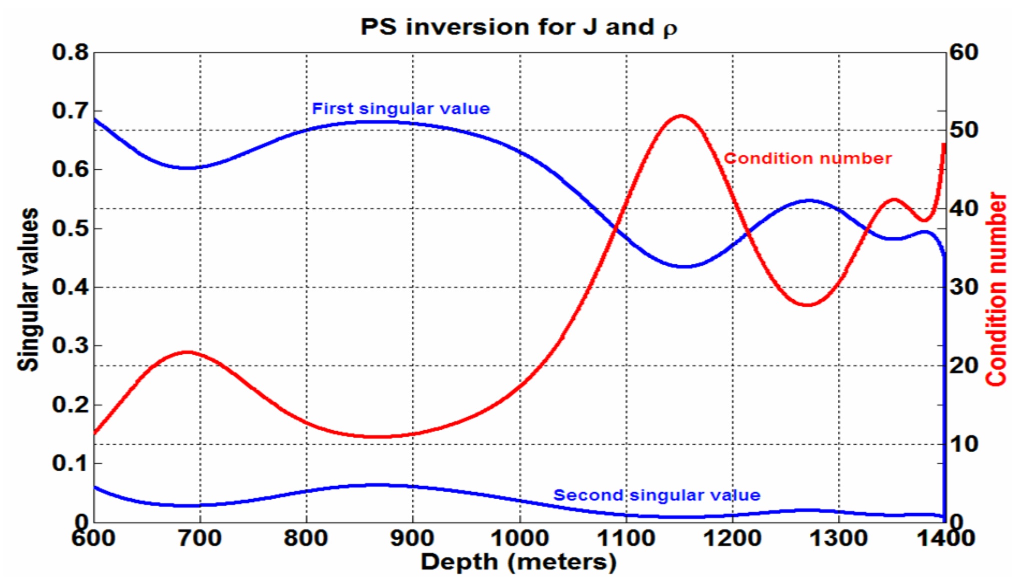

FIG. 4. Singular values (in blue) and the condition number (red curve) versus depth, of the PS inversion of synthetic 1, for *J* and *ρ*.

Therefore, even for this synthetic example which obeys Gardner's rule, the choice of parameters is not neutral; the PS inversion has more stable results for the choice of *J* and *ρ* parameters, rather than *J* and *I*. Figure 5 shows the *I*, *J* and *ρ* estimates from the PS inversion of synthetic example; the red plots are the result of the PS inversion for *J* and *ρ*, and the green plots are the result of the PS inversion for *J* and *I*. Comparing the estimates shows better PS inversion results for the choice of *J* and *ρ* parameters.

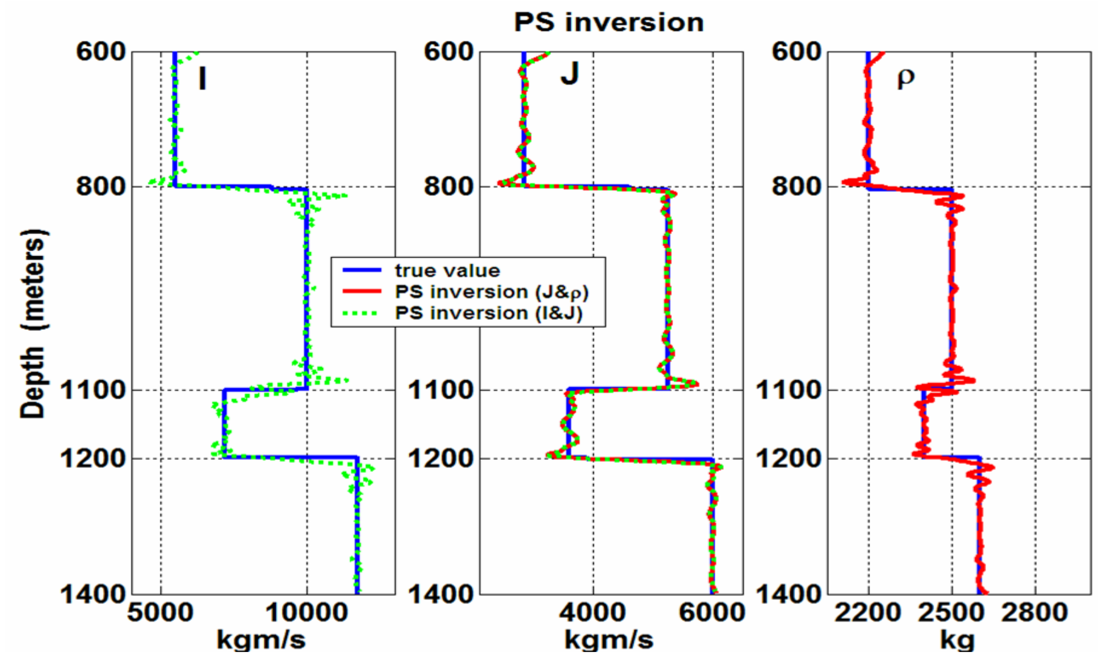

FIG. 5. P-impedance: I, S-impedance and density estimates from the PS inversions.

We tested several synthetics and real data example for the PS inversions, and for all of the examples the PS inversion using the J and ρ estimates was more stable than the PS inversion for the I and J estimates.

CONCLUSIONS

In PS inversion, the choice of parameters is not neutral. Although theoretically the same, the PS inversion using I and J has a much higher condition number compared to the PS inversion using J and ρ; the PS inversion for the J and ρ estimates is more stable than the PS inversion for the I and J estimates.

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