

MEMS for the masses part 1: comparison to geophones in theory

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ABSTRACT

The operation of geophones, delta-sigma analog-to-digital converters and digital feedback MEMS accelerometers is explored. It is found that as long as nonlinearities in mechanical springs and electric and magnetic fields are negligible, the sensors should operate very much as predicted by their frequency responses. Since the frequency response to acceleration for a MEMS accelerometer is flat in acceleration and zero in phase, the geophone frequency response in acceleration can be used to produce geophone equivalent data from MEMS accelerometer data, or vice versa. When doing so, the noise added to the data by the recording system will be shaped along with the signal amplitudes.

INTRODUCTION

Micro Electro-Mechanical Systems (MEMS) accelerometers are widely known to acquire acceleration data. That is, over the seismic signal band a MEMS chip has flat amplitude and zero phase lag response to ground acceleration. The operation of these devices, however, is not well understood within the geophysical community. The functioning of geophones and MEMS will be explored in practical terms to help determine over which frequencies and amplitudes the sensors are expected to perform differently and, possibly, which sensor has the advantage under those conditions.

While all of the discussion here will center on raw data, it is important to note something at the outset. Elements of seismic processing can be generally described as signal processing. The goal of these processes is to recognize and maximize signal, and as a consequence minimize noise. Of particular interest to this discussion is deconvolution, which largely seeks to determine which frequencies are dominated by signal, and flatten that frequency range to whiten the spectrum and collapse the recorded wavelet. Once this has been carried out, the output is no longer any particular ground motion domain. No matter in what domain the amplitudes were input to deconvolution (whether they were flat in displacement, velocity or acceleration, or not flat in any), if they were identified as signal and optimally whitened the output is identical. As such, processed data (particularly post-deconvolution) cannot appropriately be described as being 'in' any ground motion domain. Some processes may respond differently depending on the domain of the input data, but processed and stacked MEMS data is not in the acceleration domain, nor is processed and stacked geophone data in the velocity domain (this is highlighted below). Both poststack datasets would simply be processed data.

ANALOG GEOPHONES

The operation of analog geophones is generally well understood in the geophysics community. Geophones are usually moving-coil, so the magnet is fixed to the geophone case, and the coil represents the inertial reference mass (generally called a proof mass).

The velocity of the coil relative to the magnet generates an electric signal, by Faraday’s law of magnetic induction. It is a second-order spring-mass system, damped using electric shunts that provide stronger inductive feedback as the velocity of the coil relative to the magnet increases. Its response relative to ground velocity in the frequency domain is

$$\frac{\partial X}{\partial t} = \frac{\omega^2}{-\omega^2 + 2i\lambda\omega_0\omega + \omega_0^2} \frac{\partial U}{\partial t} \quad (1)$$

where X is the displacement of the coil relative to the magnet inside the case, and U is the displacement of the ground away from its initial position. The transfer characteristics for each ω (the frequency of ground motion) are based on ω_0 (the resonance of the spring-mass system), and λ (the damping coefficient). This result calculates the velocity of the coil relative to the magnet give the velocity of the ground. As such, it need only be scaled by a constant based on the number of loops in the coil and the strength of the magnetic field to predict the output voltage from the geophone. A graphical representation of the velocity response is shown in Figure 1.

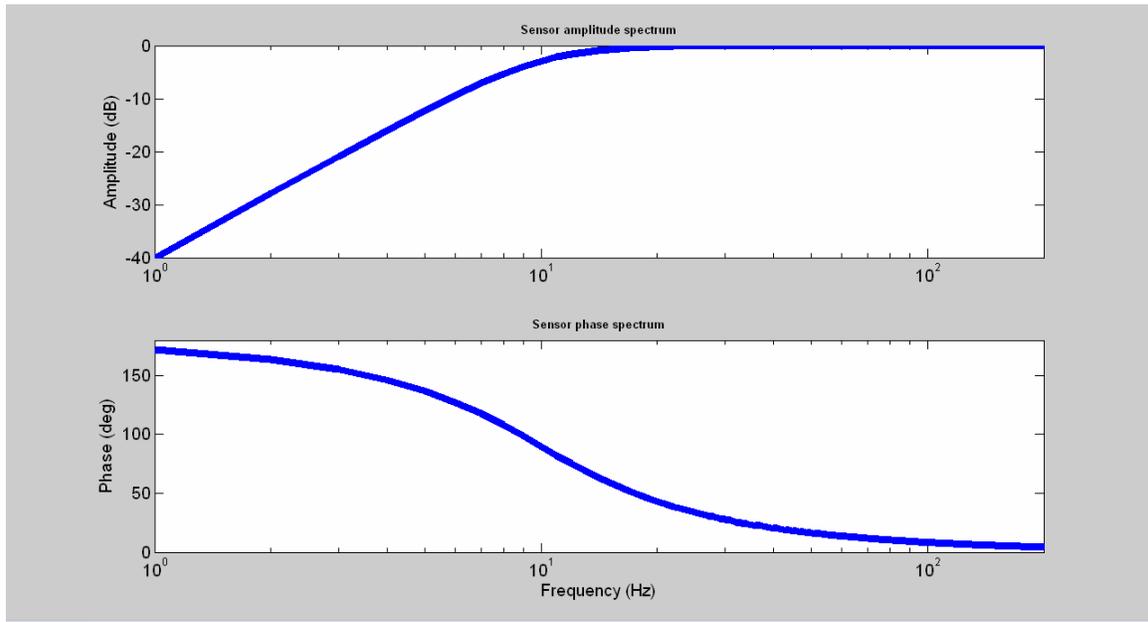


FIG. 1. Frequency response of a 10 Hz resonance, 0.7 damping ratio geophone relative to ground velocity.

If the resonance and damping of the geophone are known accurately, the amplitude and phase lag applied to each frequency can be determined. Deviation from this relationship is due to details not considered in its derivation. Examples are friction losses, nonlinearity in the spring, varying magnetic field strength at larger proof mass displacements, and motion in any direction other than the intended axis of sensitivity (spurious resonances are generally rotational) (Faber and Maxwell, 1997). Nonlinearities in the spring and in the magnetic field are most likely to be expressed under very strong ground motion. These issues are minimized and compensated for as much as possible during design and manufacturing, so over a fairly wide range of excitation amplitudes a

commercial geophone element can be expected to perform nearly exactly how its stated resonant frequency and damping would predict.

From equation (1) it is simple to find the response relative to ground acceleration, as a factor of $i\omega$ can be replaced by a time derivative on the right hand side:

$$\frac{\partial X}{\partial t} = \frac{-i\omega}{-\omega^2 + 2i\lambda\omega_0\omega + \omega_0^2} \frac{\partial^2 U}{\partial t^2}, \quad (2)$$

which can be rewritten as

$$\frac{\partial X}{\partial t} = \frac{\omega}{i(\omega_0^2 - \omega^2) - 2\lambda\omega_0\omega} \frac{\partial^2 U}{\partial t^2}. \quad (3)$$

This acceleration response is shown graphically in Figure 2. This represents the amplitude and phase changes applied to an input acceleration by a geophone.

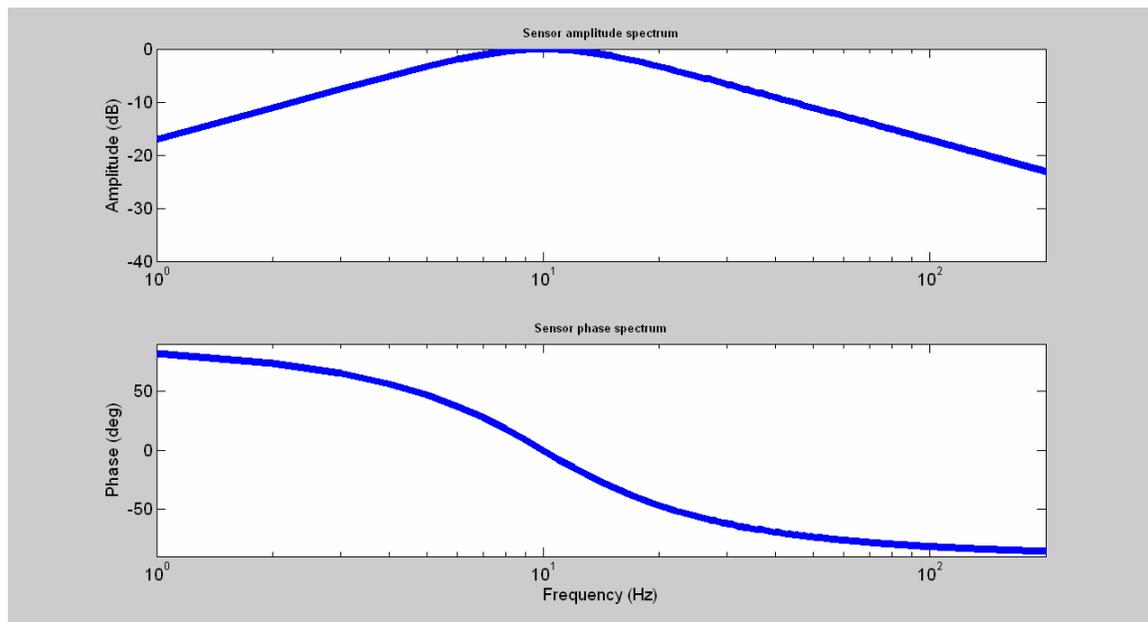


FIG. 2. Frequency response of a 10 Hz, 0.7 damping ratio geophone relative to ground acceleration.

The response of geophones has traditionally been stated relative to ground velocity. Amplitudes of the velocity of ground vibrations are recorded with flat sensitivity above resonance by a geophone. This has led to the perception that geophones are ‘ground velocity’ sensors. However, since low frequency amplitudes of the true ground velocity are not correctly represented, and significant and varying phase lags are introduced through much of the usual surface seismic signal band (up to 100 Hz or more), the output from a geophone bears little time-domain resemblance to the true ground velocity. Raw geophone output is not truly ground velocity, or any other domain of ground motion. It is a specific mix of frequency amplitudes and phase lags that might be labeled ‘geophone domain’. However, as long as the geophone performs exactly as its response predicts, the

recorded amplitude and phase can be easily corrected to represent that of any domain desired. Expressing the geophone's response in 'velocity domain' (as in equation (1)) essentially means comparing the response of the geophone to a sensor with flat amplitude and zero phase lag response to ground velocity. Expressing the geophone's response in another domain is similarly identical to comparing the response of the geophone to a sensor with flat amplitude and zero phase lag response in that domain.

DELTA-SIGMA ($\Delta\Sigma$) ANALOG-TO-DIGITAL CONVERTERS

Once the analog voltage from the geophone has been generated, it must be converted to digital information for transmission and storage. Delta-Sigma analog-to-digital converters (ADCs) are used in modern 24-bit field boxes because of their low noise and high accuracy. They are also known as 'oversampling' converters because they sample the data very quickly (at least 256 kHz) with low resolution, and use a running average algorithm to converge to the correct value over many samples. In the simplest case, the $\Delta\Sigma$ system consists of a difference, a summation and a 1-bit ADC (Figure 3; Cooper, 2002). The 1-bit ADC essentially provides feedback of a constant magnitude, with a 1 representing a positive sign and a 0 representing a negative sign. At every clock cycle the previous feedback voltage is subtracted from the incoming signal voltage (this is the 'delta'). Then this difference is added to a running total (this is the 'sigma'). If the running total is negative, the 1-bit output is a 0 (representing negative). If the running total is positive, the 1-bit output is a 1 (representing positive). The feedback voltage from this clock cycle is used to update a running average of all feedback voltages within some longer sample (e.g. 1 or 2 ms). Over time, this running average converges to very near the input voltage value (for an example see Table 1). If the $\Delta\Sigma$ converter is running at 256 kHz, and the desired sample rate is 1 kHz (1 ms), then 256 samples contribute to the output at each seismic sample, and the oversampling ratio (OSR) is 256.

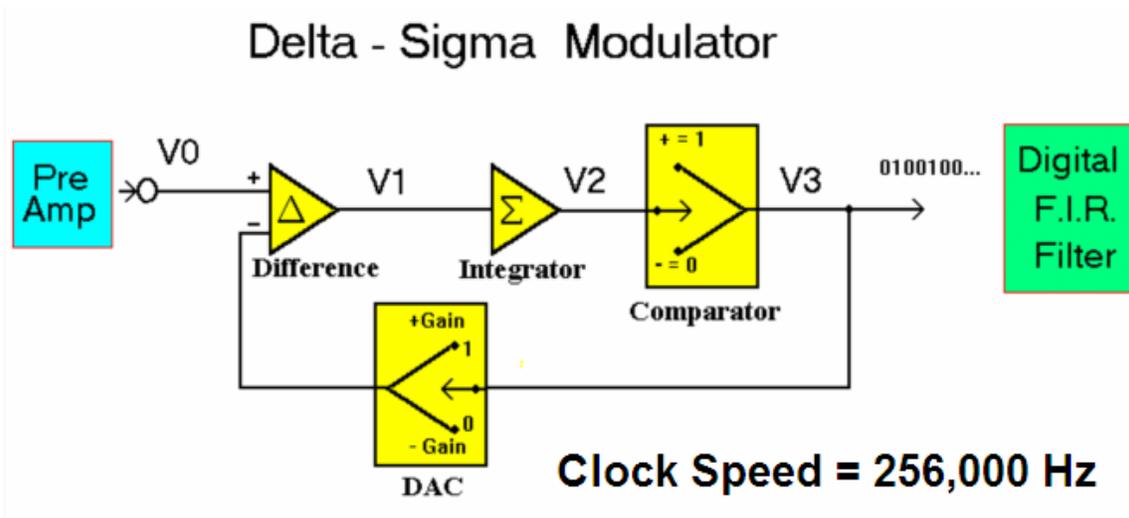


FIG. 3. Diagram of a Delta-Sigma analog-to-digital converter (Cooper, 2002).

Table 1. Example of a Delta-Sigma loop in operation.

Cycle Number	Input Voltage	Difference	Running Sum	1-bit A/D	Running Average
0	0	0	0	0	0
1	-0.45	-0.45	-0.45	-1	-1
2	-0.45	0.55	0.1	1	0
3	-0.45	-1.45	-1.35	-1	-0.333
4	-0.45	0.55	-0.8	-1	-0.5
5	-0.45	0.55	-0.25	-1	-0.6
6	-0.45	0.55	0.3	1	-0.333
7	-0.45	-1.45	-1.15	-1	-0.429
8	-0.45	0.55	-0.6	-1	-0.5
9	-0.45	0.55	-0.05	-1	-0.556
10	-0.45	0.55	0.5	1	-0.4

Each individual clock cycle represents poor resolution and a single output with large error relative to the actual input, but the average of many such cycles over time converges to very near the true average input value. For this reason, this process can be thought of as loading most of the digitization error into the high frequencies, resulting in lower quantization error in the desired frequency bandwidth. By adding more integrators it is possible to increase the ‘order’ of the system and shape even more noise into the high frequencies, further reducing noise in the desired bandwidth. However, the digital bitstream out of the $\Delta\Sigma$ system described to this point is still sampled at the higher rate and contains the high frequencies with their associated noise. The high frequencies, and most of the digitization noise, are removed using a finite impulse response (FIR) anti-alias filter to downsample to the desired sample rate. This is usually a minimum phase or linear phase filter as the bitstream is continually output from the $\Delta\Sigma$ system and is usually not digitally stored before filtering. However, applying the same filter to the time reversed data is a simple way to restore the phase prior to transmission and storage. The cutoff frequency of the filter should be as low as possible to capture all desired frequencies, but not allow more quantization error than necessary.



FIG. 4. Noise shaping of Delta Sigma ADCs (Cooper, 2002).

MEMS ACCELEROMETERS

Any sensor that detects proof mass position, and has a resonant frequency far above the measured frequencies is an accelerometer. If the resonance is very high relative to the frequencies to be recorded, the stiff springs keep the proof mass centered unless the case accelerates. Only under acceleration of the case do the springs stretch, and the magnitude of the change in proof mass position relative to its unforced position is proportional to the magnitude of the applied acceleration. If the sensor was an inductive sensor like a geophone, however, it would not qualify as an accelerometer, in part because there would be zero signal recorded at a DC acceleration. MEMS accelerometers use capacitors to sense the displacement of the proof mass, so their response can be written

$$X = \frac{1}{-\omega^2 + 2i\lambda\omega_0\omega + \omega_0^2} \frac{\partial^2 U}{\partial t^2}. \quad (4)$$

A graphical representation of this response is shown in Figure 5. Clearly at low frequencies there is a flat response down to DC acceleration. This response and Figure 2 (for a geophone) can be directly compared because they are both responses to ground acceleration.

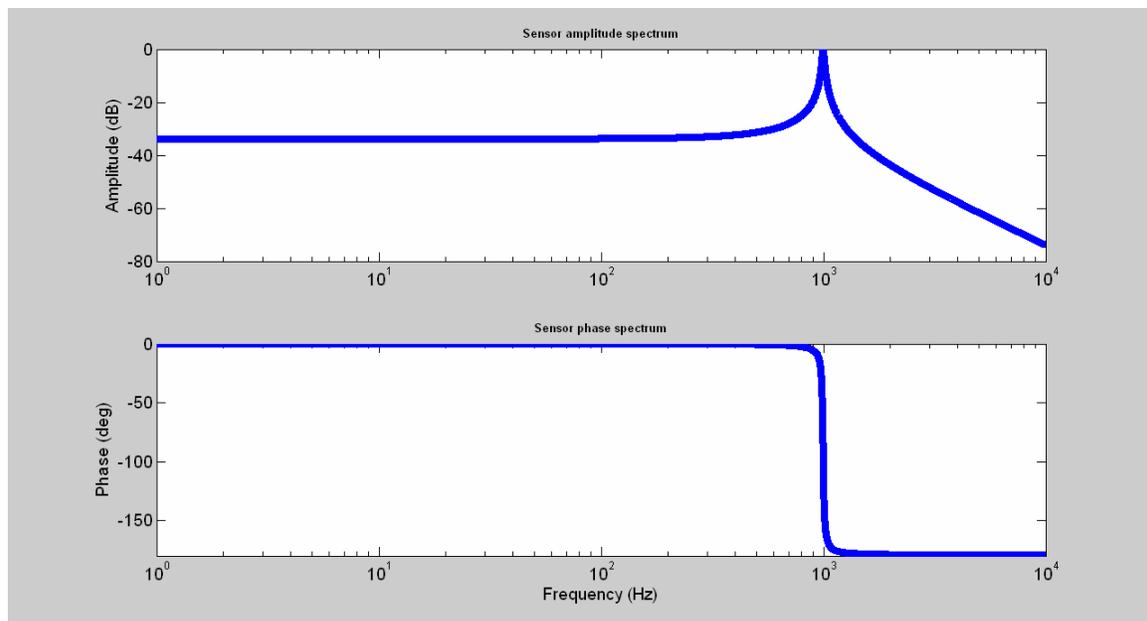


FIG. 5. Frequency response of a 1000 Hz resonance, 0.01 damping ratio accelerometer to ground acceleration.

For MEMS-based seismic sensors, layers of silicon are cut to form a proof mass and two outer sandwich layers. A simple cartoon is shown as Figure 6. The mass is very small (on the order of many micrograms to a few milligrams) and the springs are fairly stiff, resulting in a very high mechanical resonance. The mechanical springs are simply the arms of silicon remaining to anchor the proof mass to the middle silicon layer. They are designed to allow a small amount of motion, and act as linear springs over a small range of displacement.

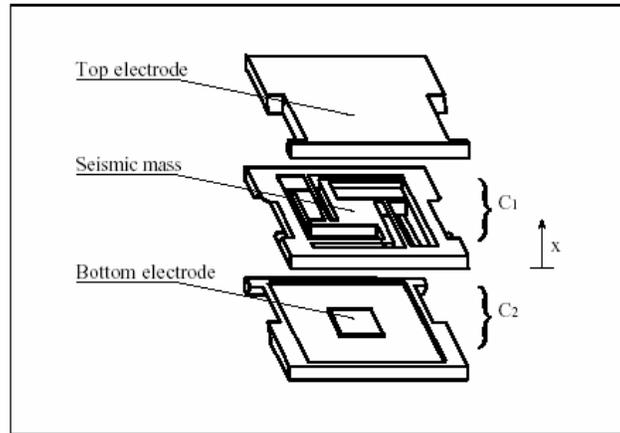


FIG. 6. Cutout cartoon of a capacitive MEMS accelerometer chip.

Noise is a significant problem in MEMS devices because the proof mass is extremely small. The Brownian motion of air molecules, or any viscous fluid, causes unacceptable interference with the proof mass. In the case of the geophone, the influence of air molecules in the chamber can be ignored. MEMS chips, however, must be vacuum-packed to reduce the mechanical damping and associated noise. This results in the mechanical system being very underdamped, so if it was left to operate under these conditions its response might look very much like Figure 5. One of the important reasons for using feedback in MEMS devices is to control the oscillations at the mechanical system's resonance.

A small area on the inside of each outer layer, and on each side of the proof mass, is plated with metal to form capacitors. Two capacitors are formed: one between the proof mass and the upper silicon layer, and one between the proof mass and the lower silicon layer. The capacitors are able to detect very small changes in the spacing between the metal plates. There are other means of detecting the movement of the proof mass, such as piezoelectric or piezoresistive materials, optical methods (like gratings) and electron tunneling. Capacitors lend themselves most easily high precision detection as well as to low-noise feedback.

Without feedback (open-loop), it is fairly easy for the proof mass to close the gap between the capacitor plates (which is only several μm wide), leading to a full-scale reading. There is only one way to prevent this, and that is to apply feedback to keep the position of the proof mass near the center. This can be done mechanically by making the silicon springs thicker and stiffer, or it can be done electrically by applying electrostatic feedback based on the position of the proof mass. Both solutions reduce the displacement of the proof mass under both strong and weak case motion, but the mechanical solution results in small displacements due to weak signals being below the position sensing resolution. The electrostatic feedback solution minimizes the proof mass displacement only after it has been detected above the noise floor, so small displacements are still detected. In other words, applying feedback does not reduce the sensitivity as much as mechanical stiffening of the springs.

Feedback is implemented in seismic MEMS accelerometers using electrostatic forces by applying a voltage across the capacitors. The electrostatic force between capacitor plates in a circuit is always attractive because it is not possible to build up charge of the same polarity in both plates. This is why it is necessary, and not just convenient, to build two capacitors within the MEMS sandwich; one on either side of the proof mass. The electrostatic force is given by

$$F_e = -\frac{1}{2} \frac{\epsilon A}{d^2} V^2, \quad (5)$$

where ϵ is the electric permittivity between the plates (approximately ϵ_0 for air), A is the area of one plate, d is the gap between the plates and V is the applied voltage. Clearly the force is nonlinear, and becomes stronger as a set of capacitor plates comes closer together (within a MEMS, the distance between the other set of capacitor plates will increase, and its feedback strength will decrease). As long as the proof mass stays near the center and the gaps are nearly symmetrical, the nonlinear nature of the electrostatic force is negligible.

Feedback can be applied in an analog implementation, where the force applied by the upper and lower capacitors is constantly balanced to bring the proof mass closer to the center. However, if the proof mass moves too far away from center (as the sensor case undergoes a strong acceleration), the balancing becomes extremely difficult. There can even come a point where the proof mass is attracted towards one side rather than back towards the middle, and the sensor fails. In other words, the sensor becomes ‘unstable’ beyond a certain proof mass displacement. Another problem with analog feedback is that since it is continuous, the feedback and sensing must take place on separate capacitors (making at least four separate capacitors necessary).

To overcome these problems (and others), the feedback in seismic sensors is implemented digitally, based on the $\Delta\Sigma$ system. Time is split into discrete sense-feedback intervals. First the position of the proof mass is sensed, and then this information is analog-to-digital converted (ADC) to give a digital output value. With a 1-bit ADC, the value is either +1 or -1 depending on whether the mass is above or below its reference position. Rather than continually balancing the electrostatic force of the capacitor plates, the digital output signal is used as feedback. For instance, a +1 could mean apply a feedback pulse to the lower plate and a -1 could mean apply the feedback pulse to the upper plate. The +1 or -1 is both the signal recorded and the feedback applied. Since feedback is provided digitally, electrical circuit noise is substantially reduced. The ‘width’, or working time of the pulse, is the length of the feedback phase.

If the sensor is experiencing a strong continuous acceleration, the mass will mostly be sensed on one side of the neutral position, and the feedback will mostly be applied to counteract it. As more and more of the feedback is applied to one side, the running average of the recorded data grows. Over the larger time interval, the average feedback is linearly proportional to the average input voltage, which, as long as the proof mass displacement is small, is linearly related to the position of the proof mass. Since the feedback is based on $\Delta\Sigma$ digitization, each component has a direct parallel to the ADC. Comparing with the purely electronic version used to digitize geophone data, the external

acceleration (represented by the average position of the proof mass) corresponds to the input voltage, the change in proof mass position due to the last feedback replaces the difference, the present position of the proof mass replaces the running sum, and the feedback voltage and running average of the feedback voltage are the same as the electronic version. Again, the oversampled bitstream must be sent through an FIR filter to remove the high frequencies contaminated with noise. In both cases it is not possible to record an input that is consistently larger than the feedback, because all feedback would be in one direction and the system would be ‘saturated’.

Over a 1 or 2 ms seismic sample, the average feedback force is proportional to the average position of the proof mass, which is representative of the acceleration applied to the case. So the feedback can be thought of as a supplementary spring exerting a restoring force. The force feedback adds to the restoring force of the mechanical spring, essentially an artificial ‘stiffening’ of the spring. So, if the spring can be said to have a linear coefficient k , and if the feedback is similarly assumed to be linear with proof mass displacement, the combination of the spring with the feedback system can be said to have an effective spring constant k_{eff} . Electrostatic feedback force can then be represented as:

$$F_{feedback} = k_{feedback} X . \quad (18)$$

This results in the total restoring force becoming

$$F_{restore} = F_{spring} + F_{feedback} = (k_{spring} + k_{feedback})X = k_{eff} X . \quad (19)$$

Similarly, the effective resonant frequency can be expressed as

$$\omega_{0(eff)} = \sqrt{\frac{k_{eff}}{m_{proof}}} . \quad (20)$$

This ‘resonant’ frequency should be relevant for predicting how far the flat response of the digital MEMS system extends. However, the digital nature of the feedback responds very differently to frequencies high enough that the feedback force is not effectively continuous. It should not be expected that the MEMS system will perform as a harmonic oscillator at frequencies nearing the feedback sampling ratio, and it is not likely that the system will exhibit resonance at the effective frequency described above. Once the approximation that the feedback acts nearly continuously is broken, a more detailed analysis of the system is required. For the seismic bandwidth, even up to 500 Hz, the continuous feedback approximation should be valid.

Also, during the sensing phase, the reference voltage applied across the capacitors exclusively for sensing purposes results in an unbalanced force if the proof mass is not exactly centered. Since the same voltage is used for both capacitors, equation (5) shows the capacitor with the smaller gap pulls strongest. This acts against the mechanical spring and is called electrostatic spring softening, but it is a smaller effect than the restoring feedback. In effect, the mechanical spring and the capacitors are all constantly acting on the proof mass, with the electrostatic forces working slightly against the mechanical spring during the sensing phase and strongly with it during the feedback phase. Note that the capacitor that is attracting the proof mass during the sensing phase

will go dead during the subsequent feedback phase, so it is impossible for the proof mass to become stuck to one side for more than one half of a sense-feedback period.

This does not mean digital feedback is immune to all problems. If the proof mass does displace substantially from center, nonlinearities in both the mechanical spring and the feedback will result in the amplitudes in the data failing to accurately reproduce the input acceleration. In the range of linear feedback, when the proof mass does not substantially displace from center, the sensor acts as a simple harmonic oscillator. If applied accelerations are too large, then the proof mass will be forced out of the range of linear feedback and the simple harmonic oscillator model will no longer hold.

In addition, the feedback itself effectively becomes a source of high frequency noise, like the quantization error in the electronic $\Delta\Sigma$ ADC. Overall, this oscillation can be thought of as a carrier frequency (Kraft, 1997), and the external applied acceleration results in a proof mass displacement bias. This can be seen by inspecting Figure 7 (Wu and Carley, 2001). The top trace shows the open loop time response to a 5g step applied to the sensor case, and the bottom trace shows the response with 1-bit feedback operating. In the open loop case, the mechanical spring reverberates at its resonance since there is little mechanical damping. It is exaggerated in this case as the damping ratio (λ) has been set to an extremely low value of 0.0005 (representing packaging in a very high vacuum). Figure (6) shows that the oscillation occurs about a central or ‘apparent’ proof mass position. The proof mass does not occupy this position for very much of the total time of a seismic sample, but it is nonetheless the average position that will be detected by the $\Delta\Sigma$ system over many sense-feedback cycles.

Note that applying the feedback prevents this oscillation, and significantly reduces the magnitude of the proof mass displacement (the scale on the y-axis changes). This can be interpreted as damping the mechanical system, though it is important to stress that there is no means to measure the instantaneous velocity of the proof mass, or to apply a force proportional to it. The sensor should not be considered to be damped in the traditional sense. The increase in the effective resonance pushes the peak in Figure 5 to considerably higher frequencies, and this is another way of explaining the lack of ringing. In any case, once the mechanical and electrostatic properties have defined a bandwidth where the amplitude response is flat and the phase response is zero, then the downsampling FIR filter is the final shaping of the output spectrum. This final filter limits the output and cuts high frequencies beyond a point chosen to include all desired frequencies but eliminate as much quantization noise as possible.

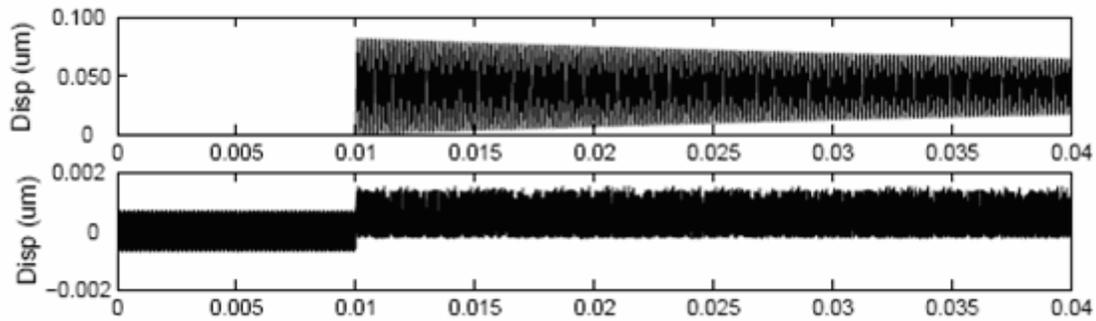


FIG. 7. Top: open loop response to an acceleration step at 0.01 seconds (mechanical resonance is about 6 kHz). Bottom: response to acceleration step with feedback applied (Wu and Carley, 2001).

CONVERTING BETWEEN GEOPHONES AND MEMS

As long as nonlinearities in the mechanical springs, and electric or magnetic fields can be ignored, then the data from each sensor should follow the appropriate frequency response. This assumption will likely fail for both sensors under strong ground motion, and it is impossible to suggest which sensor would be better without internal specifications or testing.

For a MEMS accelerometer, the frequency response is effectively flat in amplitude and zero phase. So comparing with Figure 2, it is clear that for a given ground acceleration, the geophone decreases in sensitivity to frequencies away from its resonance. Thinking about this another way, applying equation (3) to the acceleration amplitudes recorded by an accelerometer should exactly produce the amplitudes and phase lags as if a geophone had recorded the data. Essentially it transforms the MEMS recorded data into geophone equivalent data. Similarly, applying the inverse of equation (3) to geophone data should undo all of the transfer effects and correct the geophone data so it represents the ground acceleration with flat amplitude and zero phase response. The result is that we can consider equation (3) to be a geophone-to-MEMS transfer function.

The problem with this is that noise has been added into the data as it was recorded, at those amplitudes. If ground acceleration with an amplitude spectrum like that in Figure 8 is recorded through a geophone, the amplitudes of the recorded data will look like Figure 9, where the hashed area represents white noise added by the recording system. When the amplitudes are corrected to represent the acceleration again, the noise amplitudes are adjusted as well, as shown in Figure 10.

The noise in a geophone recording system can be estimated using publicly available datasheets. Above 10 Hz, equivalent input noise in commercial digitizing boxes is generally around $0.7 \mu\text{V}$ for a 250 Hz bandwidth (2 ms recording). Examples from datasheets are shown in Table 2. The noise inside a geophone is dominated by Brownian circuit noise, and comes out about an order of magnitude smaller. It will be ignored here. The equivalent input noise to a MEMS accelerometer is around 800 ng for a 250 Hz bandwidth. Converting the noise amplitudes in volts to g, using the sensitivity of the geophone in $\text{V}/(\text{m}/\text{s})$, and finding the appropriate acceleration for each frequency, the two noise floors can be directly compared (Figure 11). There are two crossovers: a 10 Hz

geophone should be less noisy than a digital MEMS accelerometer between ~3 and 50 Hz, and noisier outside this range. This analysis has assumed that the noise spectrum is white, but in reality at low frequencies electrical noise is often dominated by 1/f noise. It can be expected that noise below 5 Hz will be larger than that shown, but both sensors would likely be affected by a similar amount.

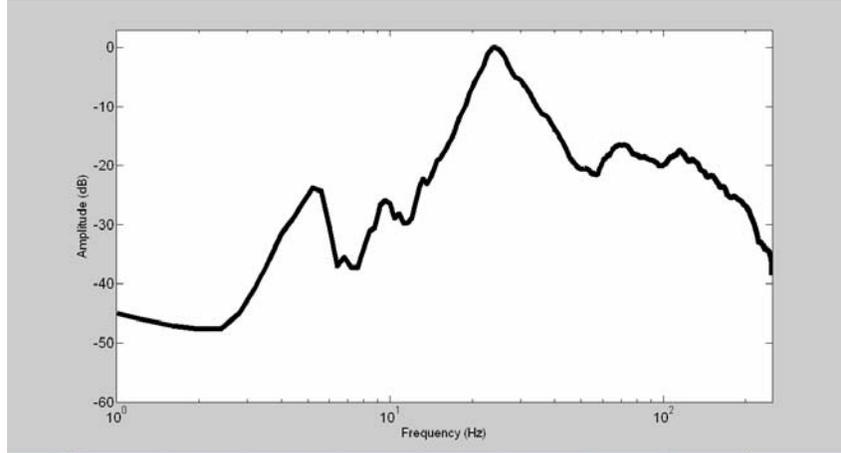


FIG. 8. Input acceleration amplitude spectrum.

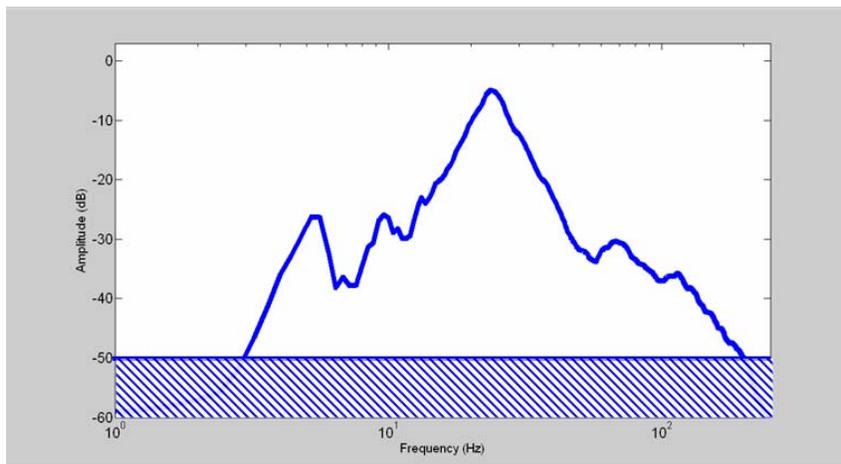


FIG. 9. Input acceleration amplitudes as recorded by 10 Hz, 0.7 damping ratio geophone.

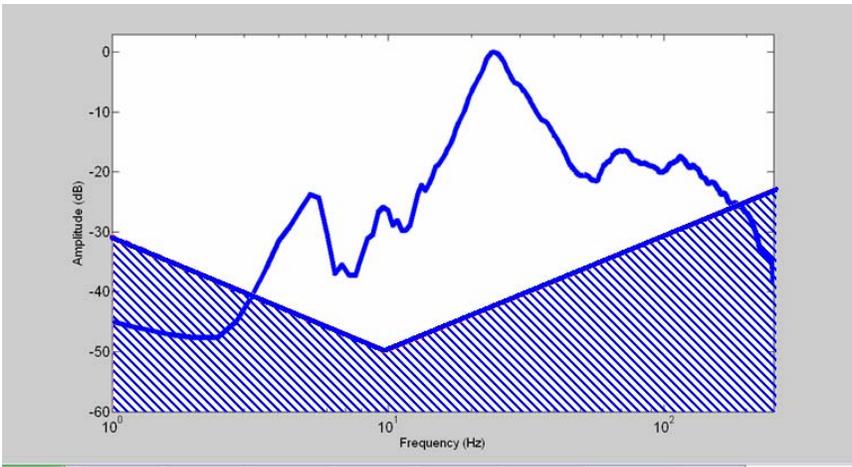


FIG. 10. Acceleration amplitudes restored.

Table 2. Equivalent Input Noise of digitizing units and MEMS accelerometers.

Recording system	Equivalent Input Noise
ION System Four	0.76 μV
Sercel 428XL	0.45 μV
ARAM Aries	0.61 μV
ION Vectorseis	607 ng
Sercel DSU	806 ng

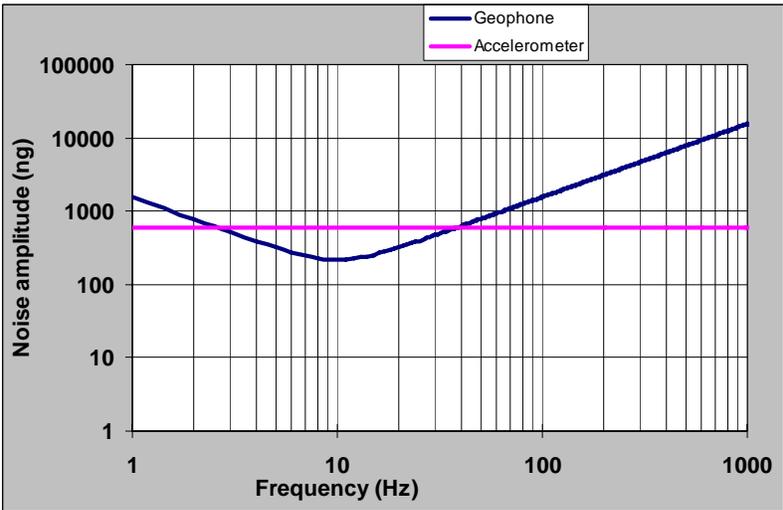


FIG. 11. Noise floors of a typical geophone and a typical MEMS accelerometer, shown as ng.

CONCLUSIONS

Modern analog geophones are a mature technology, manufactured to very close tolerances and expected to perform according to their modeled frequency response. Geophone data is digitized by very low noise $\Delta\Sigma$ ADCs. MEMS accelerometers use the same kind of $\Delta\Sigma$ loop to provide position stabilizing feedback at the same time as digitizing the output. The proof mass system with capacitive feedback is a physical analog to what happens electronically within a $\Delta\Sigma$ ADC, which is why it is a natural fit to implement the capacitive feedback in a MEMS accelerometer using a $\Delta\Sigma$ system. Because the MEMS accelerometer output is directly proportional to the ground acceleration, the geophone frequency response relating geophone output to ground acceleration can be used as a transfer function between geophone data and MEMS data. When this correction is applied, however, noise added to the geophone data during recording is shaped, and is modeled to be larger than MEMS accelerometer noise above ~50 Hz.

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