Reflections on Q

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ABSTRACT

Seismic reflections are generally caused by contrasts in acoustic impedance. However, in media where there is significant absorption of seismic energy, reflections can also be caused by contrasts in the seismic absorption coefficient (or inverse-Q values). This note derives the reflection coefficient for a normally incident acoustic wave and then uses seismic modeling of SH-wave reflections for absorptive media using the finite-difference codes of Carcione (2007). As predicted by theory and modeling, there are weak reflections due to contrasts in seismic absorption that show phase differences from reflections caused by impedance contrast.

INTRODUCTION

Exploration seismologists normally consider seismic reflections to be caused by contrasts in acoustical impedance (product of seismic velocity and density). However, in media where there is significant absorption of seismic energy, we can have reflections that are caused by contrasts in the seismic absorption coefficient, α. Contrasts in absorption can also be considered as contrasts in the quality factor, which is inversely proportional to absorption.

Figure 1 shows two reflection seismograms for SH mode propagation obtained using seismic modeling codes by Carcione (2007). In Figure 1 (left), the reflections are caused only by a contrast in acoustical impedance in a two-layer model where the velocity increases from 2000 m/s to 3500 m/s. In Figure 1 (right), there is no impedance contrast but there is a contrast in Q from Q=40 to Q=6.283. We see reflections that are not as strong for the second case, but reflections nonetheless. Hence, it would appear that reflections in the second seismogram are due solely to contrasts in the absorption, α, which is related to the quality factor, Q, by the relationship:

$$\alpha = \frac{\pi}{Q\lambda}. \tag{1}$$

Here $\lambda = \frac{v}{f}$ is the wavelength, v is the seismic velocity and f is the temporal frequency of the wave.

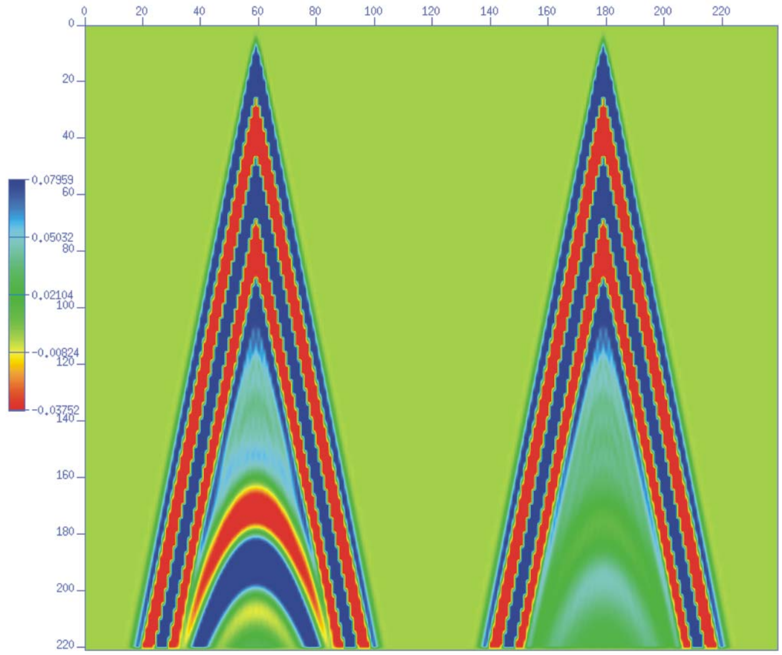

FIG. 1. The seismogram on the left arises from a model with an impedance boundary at a depth of 400 m (2:3.5 impedance contrast), source at 250 m depth in the middle of the model and receivers at 260 m depth with 10 m spacing across the model. The seismogram on the right is from the same model with no impedance contrast across the boundary, but with a Q contrast of 40:6.28.

METHODOLOGY AND RESULTS

In order to understand how these reflections arise, we shall extend the derivation for reflection coefficients elastic media as given by Robinson and Treitel (1980, p. 296) to allow for absorption. Following their notation, we consider a layer boundary at y=0 where y, the depth dimension is increasing downward. The incident, reflected and transmitted sinusoidal wave displacements are given by:

$$g_i = A_i \exp(2\pi ft - \kappa_1 y) \text{ (incident),} \tag{2}$$

$$g_r = A_r \exp(2\pi ft + \kappa_1 y) \text{ (reflected)} \tag{3}$$

and

$$g_t = A_t \exp(2\pi ft - \kappa_2 y) \text{ (transmitted).} \tag{4}$$

For an elastic medium, the wavenumbers κ_1, κ_2 are real and given by:

$$\kappa_1 = \frac{2\pi f}{v_1} \tag{5}$$

and

$$\kappa_2 = \frac{2\pi f}{v_2} \tag{6}$$

respectively, where v_1, v_2 represent the seismic velocities in layers 1 and 2. These wavenumbers are real for a medium that is purely elastic with no energy loss to absorption, which is the case where $\alpha = 0$.

However, we can readily consider absorption by introducing a wavenumber that is complex and given by:

$$k = \kappa + i\alpha \tag{7}$$

or

$$k = \frac{2\pi f}{v} + i\alpha . \tag{8}$$

Toksoz and Johnston (1980), among others, explained this concept of a complex wavenumber whose imaginary component is absorption.

In order to derive the reflection coefficients for this absorptive medium for a normally incident plane wave, we follow Robinson and Treitel's method and require that the displacement and the normal stress be continuous at a boundary. The continuity of displacement at y=0 requires that:

$$A_i + A_r = A_{t.} \tag{9}$$

(Here we divided out a common exponential factor, $\exp(2\pi i f t)$.)

The continuity of normal stress requires that $E\frac{\partial g}{\partial y}$ be continuous where E is the elastic constant relating stress to strain. Therefore,

$$E_1\frac{\partial g_i}{\partial y} + E_1\frac{\partial g_r}{\partial y} = E_2\frac{\partial g_t}{\partial y} . \tag{10}$$

If we replace κ by k in the displacement expressions of (2), (3) and (4) and substitute these into equation (10), after dividing out a common factor of $\exp(2\pi i f t)$ we obtain the following boundary condition at y=0:

$$E_1(-k_1A_i + k_1A_r) = -E_2k_2A_t . \tag{11}$$

Now if we combine equation (9) with equation (11), we obtain the expression for the reflection coefficient:

$$R = \frac{A_r}{A_i} = \frac{E_1 k_1 - E_2 k_2}{E_1 k_1 + E_2 k_2}. \qquad (12)$$

We can express Ek in terms of density, ρ , and velocity, v while recalling that $E = \rho v^2$. Therefore Ek can be expressed as:

$$Ek = \rho v^2 \left[\frac{2\pi f}{v} + i\alpha \right] \qquad (13)$$

If we express equation (13) in terms of Q, we get

$$Ek = \rho v^2 \left[\frac{2\pi f}{v} + i\frac{\pi f}{Qv} \right], \qquad (14)$$

which can be rewritten as:

$$Ek = 2\pi f \rho v \left[1 + \frac{i}{2Q} \right]. \qquad (15)$$

If we substitute equation (15) into equation (12), we obtain the following expression for the reflection coefficient in absorptive media, after dividing numerator and denominator by $2\pi f \rho v$:

$$R = \frac{\rho_1 v_1 \left[1 + \frac{i}{2Q_1} \right] - \rho_2 v_2 \left[1 + \frac{i}{2Q_2} \right]}{\rho_1 v_1 \left[1 + \frac{i}{2Q_1} \right] + \rho_2 v_2 \left[1 + \frac{i}{2Q_2} \right]}. \qquad (16)$$

We note that if there is no absorption in the medium such that $\frac{1}{Q_1} = \frac{1}{Q_2} = 0$, equation (16) reduces to the difference of acoustical impedances divided by the sum as given by Robinson and Treitel (1980). On the other hand, if there is no acoustical impedance between medium 1 and medium 2, we can still obtain reflections due to a contrast in absorption (inverse Q). In such a case, the reflection coefficient is given by:

$$R = \frac{i\left[\frac{1}{Q_1} - \frac{1}{Q_2} \right]}{2\left[2 + \frac{i}{2}\left[\frac{1}{Q_1} + \frac{1}{Q_2} \right] \right]}. \qquad (17)$$

Therefore, even with no impedance contrast, we can have reflections due to a contrast in the absorption properties of a two layers.

In Figure 1, we show the results of some finite-difference calculations for SH-waves for two cases: where the impedance contrast is 2:3.5, and where the Q contrast is 40:6.28. The reflection coefficient for the first case (where Q is constant) would simply be equation (16) with the Q and density contributions cancelling out leaving the difference in velocity divided the sum of the veolcities, $\frac{v_1 - v_2}{v_1 + v_2}$=-1.5/5.5 = -0.27. The reflection coefficient for the second case (with no impedance contrast) would be obtained by substituting into equation (17) to give:

$$R = \frac{i\left[\frac{1}{40} - \frac{1}{6.283}\right]}{2\left[2 + \frac{i}{2}\left[\frac{1}{40} + \frac{1}{6.283}\right]\right]}, \tag{18}$$

or

$$R = \frac{0.134i}{4 + 0.184i} = 0.034 - 0.00150i\ . \tag{19}$$

This a complex number whose amplitude is about 1/8 as big as the reflection coefficient for the impedance contrast, and there is a phase shift due to absorption. If we include both an impedance contrast and a Q contrast, we will see that the resulting seismic trace is not significantly different from that of the impedance contrast only. Figure 2 shows the normal incidence traces of the seismograms for impedance contrast only (leftmost trace), Q contrast only (middle trace) and for impedance contrast and Q contrast. We note that the reflection amplitude for the left trace is nearly identical to the right trace. Although the absorption contrast reflection is nonzero and phase shifted from the impedance reflection, it is barely noticeable.

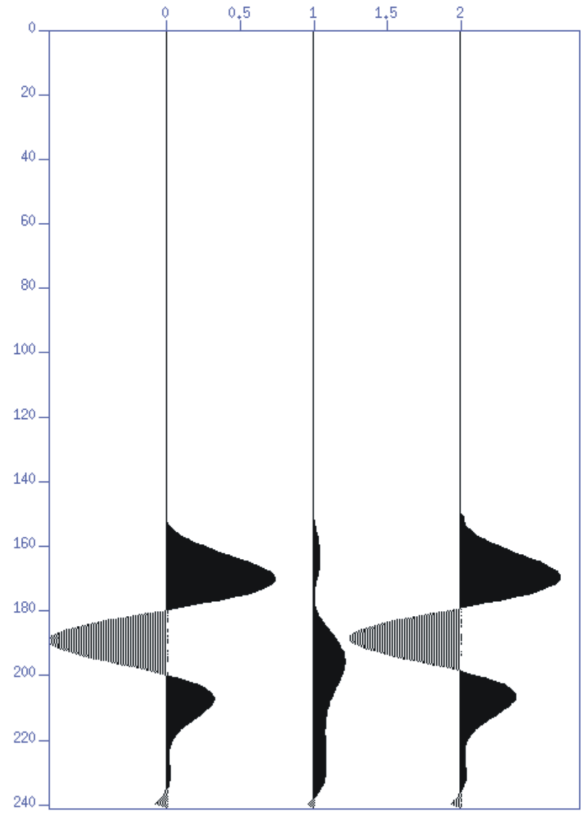

FIG. 2. Zero-offset seismic traces for the layered model of Figure 1 for the cases of the impedance contrast only (impedance contrast = 2:3.5, left), absorption contrast only (Q contrast = 40:6.28, middle), and impedance and absorption contrasts (same as left and middle traces, right).

CONCLUSIONS

From both theory and numerical modeling we note that the inclusion of absorption in our reflection computations, both in boundary value analysis and finite-difference calculations, produce reflection amplitudes that are slightly phase shifted but not noticeably different from those derived from elastic reflection coefficient calculations.

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REFERENCES

Carcione., J., 2007, Wavefields in real media: Elsevier Publishing Co., New York, NY.

Robinson, E.A., and Treitel, S., 1980, Geophysical signal analysis: Prentice-Hall Publishing, reprinted by the Society of Exploration Geophysicists, Tulsa, Oklahoma.

Toksoz, M. N., and Johnston, D.H., 1980, Seismic wave attenuation: Society of Exploration Geophysicists, Tulsa, Oklahoma.