Choosing between the two Apollonius solutions when locating a microseismic event

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ABSTRACT

The location and clock-time of a microseismic event can be computed analytically using the first arrival clock-times at four known receiver locations, assuming the velocity is known. The process is based on the Apollonius method that produces two solutions, either being possible for the same receiver geometry. The difficulty in choosing the correct solution is presented along with a method for identifying the best solution.

INTRODUCTION

When a microseismic event can occurs, and the first arrivals of this event may be identified on seismic recordings from seismic receivers in relatively close proximity to the event. Since the time of the event is not known, the times on the seismic records are defined as clock-times. It is desirable to know the time of the source, also defined as a clock-time, and the location of the source. The traveltimes from the source location to the receivers are known as delta-times. Initially, these times are not known, but would be the same as the clock-times if the event occurred at a clock-time of zero.

An array of receivers is used to estimate the location of a microseismic event. There are many complicated computational methods that are used to locate the source. Some involve migration, inversion, iteration, or ray tracing, while others may assume a constant velocity and linear raypaths.

An accurate location of a seismic event and its clock-time can be computed from the first arrival clock-times at four receiver locations. The analytic computation is based on the Apollonius solution of a source sphere that is tangent to four receiver spheres. The receiver spheres are defined with the center at the receiver location and the radius equal to the receiver clock-time. The centre of the tangent sphere defines the (x, y, z) location of the source, and its radius defines the clock times (Bancroft et al. 2007). The method assumes the velocity is known and constant, and that the receiver locations and clock times are exact. The computed analytic solution will provide an accurate solution to the precision of the computer.

The above solution requires the correct velocity of the medium. This method does assume the velocity of the medium is constant, however, the velocity can be assumed to be of an RMS type that compensates for a variable velocity medium. The RMS velocity is assumed to be locally constant in the area of the source and receivers.

The Apollonius method produces two solutions, and either solution can be the correct solution, depending on the geometry of the receivers and the location of the source. Once the geometry is established with an approximate source location, one of the solutions typically provides a consistently correct answer. However, there may be cases that require additional effort to decide which of the two solutions is correct.

METHOD

The general Apollonius method produces eight possible solutions to finding a circle tangent to three other circles. The geometry of the problem for locating microseismic events limits the number of 2D solution to two. Similarly, the 3D solution becomes one of finding a sphere tangent to four other spheres, and also has two possible solutions. Either solution is possible for different geometrical configurations; however, only one solution may be correct for a given system geometry. I will illustrate the complexity of choosing the correct solution using 2D examples as they can be visualized.

Complexity of choosing the better of two possible solutions

I will assume a 2D geometry (x, z) where only three receivers are required. The receivers are close to the surface, (x, 0) and the microseismic source is located below the surface at (0,-4) as shown in Figure 1a. The receivers are located at (0.00, -0.10), (1.40, 0.00), and (3.00, -0.20).

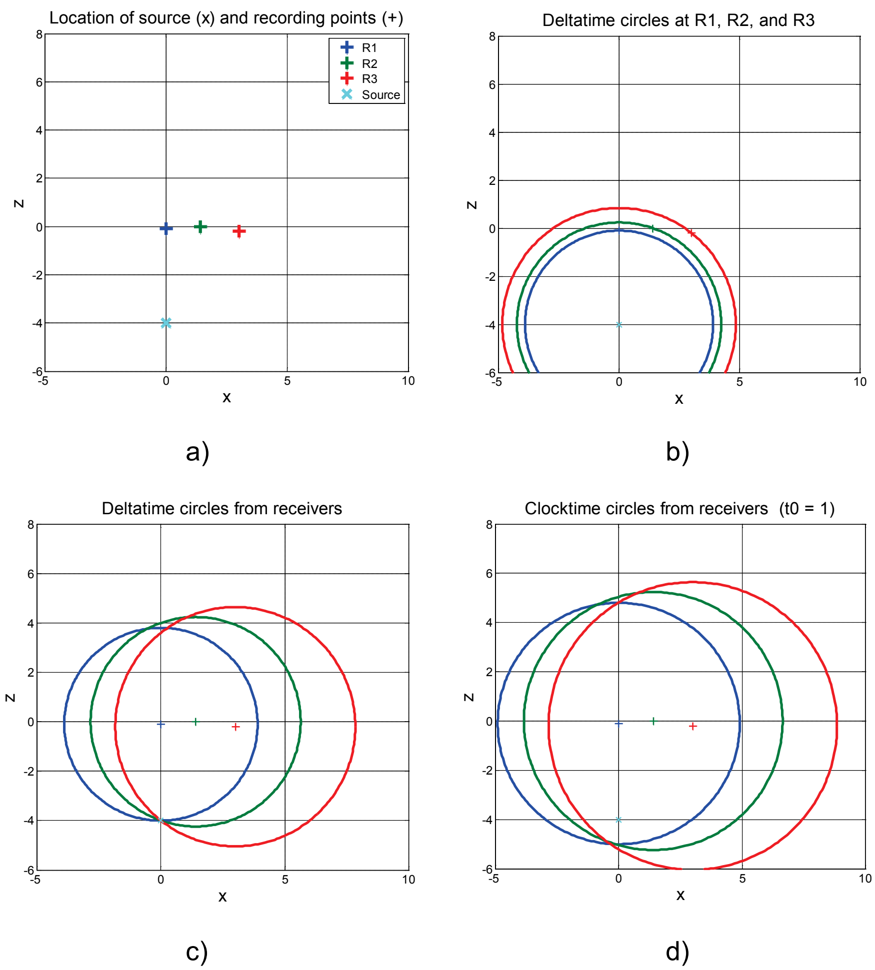


FIG. 1 Geometry of source and receiver locations in a) with b) showing the delta-times emanating from the source, c) the delta times emanating from the receivers and intersecting at the source, and d) the clock-times plotted as circles emanating from the receivers. (TT2DsurfaceRec.m)

Wavefronts emanating from the source and arriving at the receivers, determine the delta-times of the event, as illustrated in Figure 1b. The delta-times are not known at this point, and it is our objective to find them. The delta-times are plotted with their centers at the receiver locations in Figure 1c, and are observed to intersect at the source location. The velocity used is v = 1, enabling traveltimes to be equated to distance. Figure 1d shows circles of the clock-times plotted with centers at the receiver location. This information is known, and the method of Apollonius is used to find the source location and the clock time of the event. Knowing the clock-time of the event will give the delta-times to the receivers.

The two Apollonius solutions are included in Figure 2 as two circles; the first cyan circle has a center at the true source location while the second solution shows part of a magenta circle with its center well beyond the lower right. The Apollonius method finds these two circles that are tangent to the receiver clock-time circles. In this case the first solution is correct.

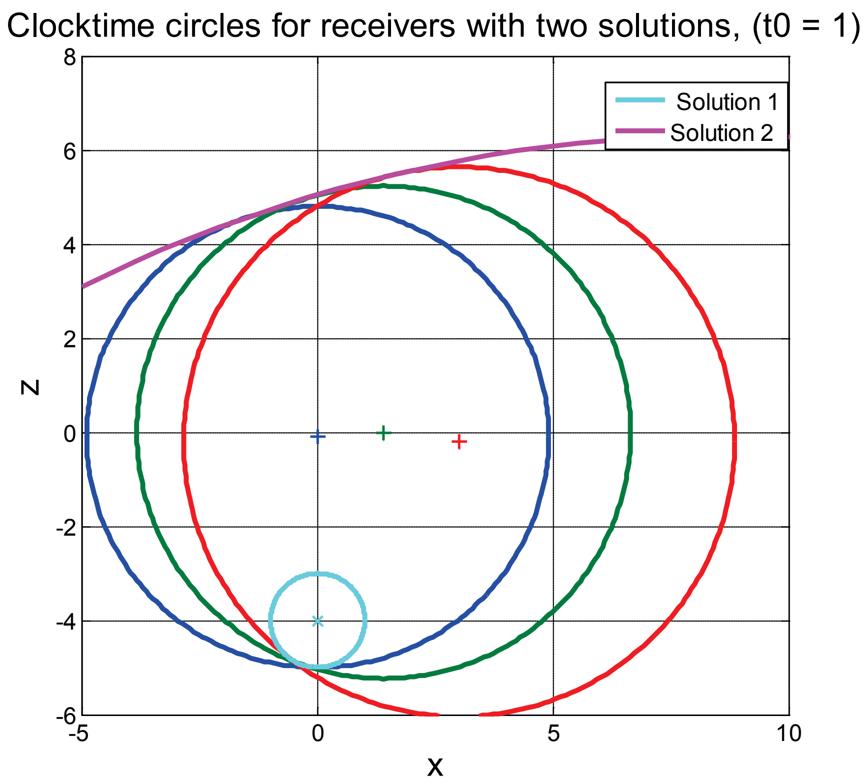


FIG. 2 Clock-times of the receivers and two Apollonius solutions shown in cyan and magenta.

In this case, both solutions have the correct polarity of the z component, but as shown in the next figure, a slight shifting of the source location to (0, -3) will produce circles with opposite polarities and a change in the correct solution.

Moving the source to a still shallower location (0,-1), and changing the clock-time of the source to t = 2, produces the result in Figure 3b where the second solution is correct but the radius of first solution is less than the second solution but the first solution is still incorrect.

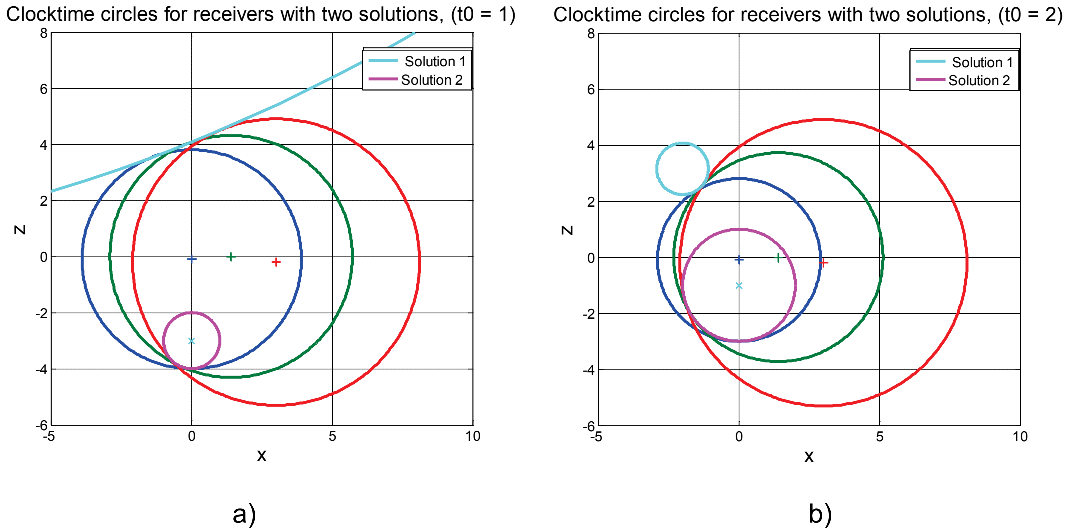


FIG. 3 A new source location (0, -3)in a) produces significantly different results with the second solution being correct (magenta) and the first solution (cyan) producing a circle with a positive z value above the surface. In b) the source is moved to (0, -1) and the clock-time of the source is *t* = 2.

The actual clock-times may be very large relative to the delta-times, producing very large circles, which in turn may produce inaccuracies when fitting the tangent circle. The shortest clock-time can be subtracted from all the receiver clock-times to define the clock-time at one receiver to be zero. This may be use to create the smaller circles, or to reduce the computational overhead. Using a receiver clock-time of zero will result in a negative source clock-time as illustrated in Figure 4 where a negative source clock-time of t = -3.9 is used to make the shortest clock-time of the first receiver to be zero.

The effect of modifying the location of the receivers is illustrated in Figure 4b in which the second receiver is lowered slightly (1.4, -0.20) producing a significant change in the solutions.

Similar solutions are illustrated in Figure 5 in which the receivers lie in a circular path and the source is located close to the center of that circular path. The differences between the travel times are small, and when the shortest clock-time is set to zero, the radii of the other two clock-times are also small. The two solutions represented by the cyan and magenta circles show the first solution is correct, but the center of the magenta circle represented by the magenta "x" will be reasonably close to that of the first solution, and would probably lie within a designated range of possible solutions. Note, however, that the first solution is still the correct solution and would probably be selected.

There is one analytic indicator that shows the first solution to be correct. Figure 5b shows the estimated parameters with the source times and the estimated location. The first solution shows the correct location and source clock-time. The second solution shows the source location to be close to the actual source as plotted by the magenta "x", but the source clock-time is positive at $t = +3.8497$.

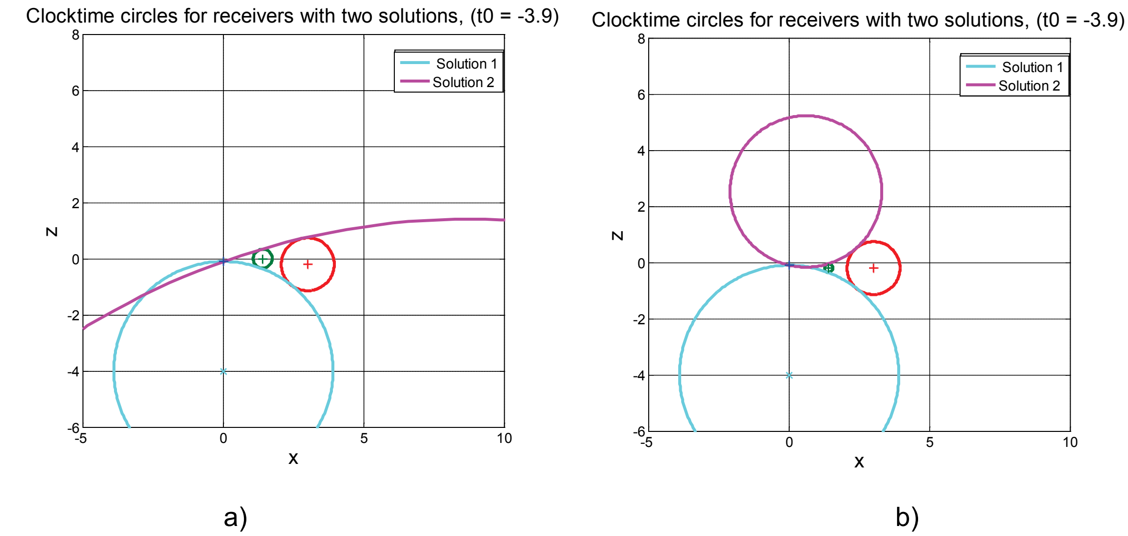
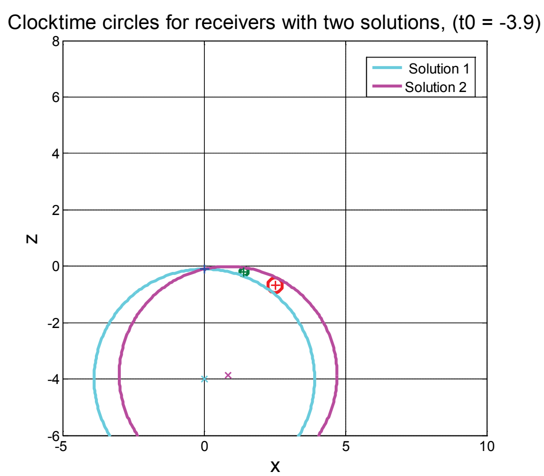


FIG. 4 The receiver clock-time is reduced to zero in a) by defining the source time to be equal to the negative delta-time. The location of the second receiver in b) is lowered to (1.4, -0.20).

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The defined valuess are
t0 =-3.9    x0 =0    z0 =-4
t1 =0    t2 =0.14969    t3 =0.26

The Solution 1 results are
t01=-3.9    x01=3.2507e-015    z01=-4
t11=1.3323e-015    t21=0.14969    t31=0.26

The Solution 2 results are
t02=3.8497    x02=0.83882    z02=-3.8572
t12=7.6995    t22=7.5498    t32=7.4395
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FIG. 5 Special case where the receivers are located on a circular path and the source is close to the center of that circle. The output text highlights the two estimated clock-times.

By ensuring the minimum receiver clock-time to be zero, the clock-time of the source is forced to be negative. A positive clock-time indicates a false solution.

Identifying the best solution

We now have three indicators for the correct solution:

1. the first solution
2. the geometry of the system, and
3. a unique negative source time that assumes the minimum receiver clock-time is zero.

Only the third indicator is reliable, and that only applies if the two solutions have a positive and negative source clock-time.

The second solution may be valid if an estimated location is unreasonable such as an event occurring above the surface of the Earth. In that case the correct solution can be identified. In other cases, the incorrect solution may lie beyond a reasonable range of the correct solutions. However, I am always aware that there may be two solutions that are close enough to lie within a defined range. These cases can be rare, and when combined with other measurements, may be negated.

Another possibility is that there is no solution, as in the case where all four receivers are on the same plane, or are in a linear array. These cases can be identified when the input to the one square root in the solution is a negative value.

Voting system

I use a vote system to determine which is the correct solution. The system adds a value to indicate if that solution is possible and subtracts the same value if it is not possible. For example, if the estimated source time is negative, then it gets a positive vote. If the other solution is also negative, it gets the same positive vote. A comparison of votes may produce a definite answer. The value of the vote may vary for different tests. For example, the first solution gets a small vote of 1 and the second solution gets a vote of zero. If all other factors are the same, then the final decision will be for the first solution. A negative source time gets a much large value.

An area of expected source locations is defined by a rectangular parallelepiped with specified dimensions for the x, y, and z components. Positive votes count if the estimated source component is within the specified range, and negated if outside that range. Multiple ranges can be used that have different vote values. Estimated source locations that are closer to a fraccing location may have a stronger vote than a more distant one.

The voting system will give a maximum negative number if attempting to compute the square-root of a negative value and will bypass all other tests. This condition should be established when the source locations are defined. There are alternate solutions available when this test fails. These may be receivers on a planar square grid at the surface, or the receivers may be collinear in a well.

CONCLUSIONS

Two Apollonius solutions are possible when locating the source of a microseismic event. Determining which answer is correct, or which solution is best can be difficult. A voting system was implemented that summed values that depend upon a number of tests. These test may involve the estimated source clock-time or the proximity of the estimated source location to a targeted area.

ACKNOWLEDGEMENTS

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SOFTWARE

The software was created in MATLAB.
`2009-MATLAB\TravelTimeMicroseismic\`
`TestVelToMinLocationErrorUsing5Pts.m.` and `TT3DfunJCB.m`

REFERENCES

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