The accuracy of dipole sonic logs with implications for synthetic seismograms and wavelet estimation

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ABSTRACT

Sonic logs contain errors due to mud invasion and cycle skipping, and repeat logs may be recorded to validate measurements. For repeated dipole sonic logs, it is interesting to note differences in the (compressional) $P$-wave and (shear) $S$-wave velocities, as well as the resulting differences in reflectivity sequences and synthetic seismograms. For synthetic seismograms with low-frequency wavelets, the differences are often barely perceptible, especially for $P$-wave synthetic traces. When correlating these different synthetic traces with reflected events on real seismic data, our interpretations would often not be affected. However, for the purposes of deconvolution, seismic wavelets are often estimated by using both sonic logs and real seismic data. In some cases, where there are noticeable differences in estimated log-based wavelets, it is advisable to check log-based wavelet estimates using statistical methods, such as minimum phase wavelet estimation. Also in these comparisons of dipole sonic logs, synthetic seismograms and wavelet estimates, we have generally found the repeatability of $P$-wave logs to be superior to that of the shear-wave logs. This is not surprising due to the difficulty of picking shear-wave arrivals compared to $P$-wave first break picks. In summary, repeat measures of dipole sonic logs will be worthwhile for insuring that the $P$-wave synthetic seismograms, shear-wave synthetic seismograms and wavelet estimates are accurate.

INTRODUCTION

It is well known that, like all geophysical measurements, sonic logs contain inaccuracies. Such inaccuracies may be caused by mud invasion, cycle skipping, hole conditions, instrument problems or noisy recording conditions. Inaccuracies with sonic logs may cause the sonic logging experiment to be repeated. In this short note, we examine discrepancies in repeat logs. Through this analysis, we specifically examine:

1. Difference in reflectivity functions.
2. Resulting differences in synthetic seismograms.
3. Resulting differences in log-based wavelet estimation due to reflectivity errors.
5. Comparisons of the above attributes for $P$-wave and shear-wave logs.

In this analysis of sonic log errors and repeatability, we examine a series of sonic logs from Nexen’s Long Lake heavy oil field. From a set of 42 dipole sonic logs, we examined some repeated logs and examined the differences and similarities of these data. In this note, we show some typical examples of $P$-wave and shear-wave logs from this field.

Figure 1 shows two $P$-wave sonic logs from a well between depths of 153.5m and 292.1m, sampled at intervals of 0.1m. We note that the logs are quite similar except at
depths of 155, 172, 192, 252 and 290 m. We plot a "discrepancy" log as the third trace in this display to show the difference between the velocities.

![Sonic log comparison](image)

**FIG. 1.** A comparison of sonic logs from two wells (denoted in the text as log A and log B) spanning a depth range from 153.5-292.1 m. Maximum trace deflection is 3480 m/s. The third trace in the plot is a discrepancy log giving the difference in velocities between wells A and B (log B velocities – log A velocities). Note that the biggest differences are at depths of about 155, 172, 192, 252 and 290 m.

In other words the repeat log (log B) looks similar to well A except for a few locations, specifically at about 1 m (10 sample intervals) at depths of 2, 35, 40, 96, 120 and 137 m from the top of the log.

Our sonic log gives slowness (reciprocal velocity) measurements at regular depths (10 cm intervals) through the subsurface. The seismic data are recorded in time. In order to compare the sonic log data to seismic data, we convert the reciprocal velocity versus depth readings to readings of velocity versus time. We can then compute reflection coefficients, $r_j$, as a function of time by the simple formula at a time sample at an interface between the layers $j$ and $j+1$ as
More correctly, if we convert density versus depth to density versus time, we compute the reflectivity as the contrast in acoustic impedances, \( I_k = \rho_k v_k \)

\[
\begin{align*}
    r_j &= \frac{v_{j+1} - v_j}{v_{j+1} + v_j} \\
    r_j &= \frac{\rho_{j+1} v_{j+1} - \rho_j v_j}{\rho_{j+1} v_{j+1} + \rho_j v_j} = \frac{I_{j+1} - I_j}{I_{j+1} + I_j}
\end{align*}
\]

As expected by the velocity differences versus depth in Figure 1, there will be differences between the reflectivities of logs A and B. In Figure 2 we compare the computations for logs A and B as a function of time. Also shown is a third trace that is the difference in the reflectivities, showing that the comparisons (as one would expect) are similar to Figure 1.

![Figure 2](image)

**FIG. 2.** The \( P \)-wave reflectivity on the left is for log A, and the reflectivity in the middle is for log B. The third trace represents the difference between reflectivities given by reflectivity B-A. The two logs span 223 samples at a sample rate of 0.5 ms or a total time of 111.5 ms.

In Figure 2, there are differences between the reflectivities at the times expected from the velocity Figure 1. Note that these areas of difference are now in time rather than in...
depth and that we are looking essentially at trace amplitudes which are proportional to derivatives of the velocity values in Figure 1. Despite the differences between reflectivities, the semblance between the traces 1 and 2 in Figure 2 is still fairly high with a value of 0.93.

**COMPARISON OF P-WAVE SYNTHETIC SEISMOGRAMS**

For our initial calculations, we shall focus on the $P$-wave interpretation, which is still the main application of synthetic seismograms in interpretation. One of the main purposes of acquiring sonic logs is to compute synthetic seismograms in order to initiate the interpretation of reflections on seismic data. In order to do this, we convolve the reflectivities derived from the sonic log with a seismic wavelet believed to be representative of source wavelets in our seismic recordings. Ideally, we would want these seismic wavelets to have a broad band of frequencies extending to the Nyquist frequency in our recording. In typical recordings, these wavelets will have spectra in the 5–80 Hz range for land recording. In exceptional cases of shallow reflectors, we can have useful frequencies that extend as high as 150–200 Hz. For this particular example, we are dealing with shallow recordings of reflectors shallower than 400 m. Hence, we use a Ricker wavelet with peak frequency of 150 Hz.

Figure 3 shows synthetic seismograms wherein the reflectivities of Figure 2 have been convolved with the minimum phase equivalent of 150 Hz Ricker wavelet. (We choose the minimum-phase equivalent of the Ricker wavelet rather than the symmetric Ricker in order to conduct the wavelet estimation tests in a later section.) The traces in Figure 3 are essentially smoothed versions of the reflectivities of Figure 2. The third trace represents the difference between the synthetic traces of logs B and A. Interestingly (and perhaps not surprisingly), the traces appear even more similar than the reflectivities and have a semblance of 0.985. Considering these small discrepancies between traces and the high similarity (semblance), the question that naturally arises from this comparison is the following. Would the interpretation of seismic data be likely to change if we used the synthetic trace in B rather than the synthetic trace in A? The answer for these particular synthetic seismograms is “probably not”.
FIG. 3. The synthetic trace on the left is for log A, and the trace in the middle is for log B. The third trace represents the difference between the seismic traces by trace B-A. The two traces span 223 samples at a sample rate of 0.5 ms or a total time of 111.5 ms. Accuracy of log-derived wavelets

At this point, we may be questioning whether the repeat log may have been worthwhile or not—least for $P$ waves. Did the second log simply verify the validity of the first log without any significant change to interpreted arrivals? However, before dismissing the repeated log as unnecessary or merely a confirmation of our original log, we may wish to examine other uses of dipole sonic logs such as the interpretation of shear-wave reflections as well as the process of wavelet estimation.

For the process of wavelet estimation, we use a method described by Danielson and Karlsson (1984), and later by Lines and Treitel (1985). In these wavelet estimation methods, we consider the convolutional model of the seismic trace in the frequency domain in which the Fourier transform of the trace, $Y(f)$, is given by the product of the wavelet’s Fourier transform, $W(f)$, and the reflectivity’s Fourier transform, $R(f)$

$$ Y(f) = W(f)R(f). \quad [Y(\omega) = W(\omega)R(\omega)] \quad (3) $$

Generally speaking, the seismic trace is known and we hope to estimate the reflectivity. We often do not explicitly know the wavelet either, but use statistical...
properties of the reflectivity and wavelet to estimate $W(f)$. If we assume that the reflectivity is random, we can obtain the wavelet’s spectrum to be given by the trace’s amplitude spectrum. If we use a minimum phase assumption, the wavelet’s phase spectrum is often obtained by computing the Hilbert transform of the log amplitude spectrum (as originally described by Robinson, 1967). The minimum phase assumption is generally believed to reasonably accurate for impulsive sources such as dynamite or air guns. If sources do not have an impulsive nature, we may wish to use other statistical methods that do not make these assumptions (but different assumptions) such as homomorphic deconvolution, (Ulrych, 1971).

However, if we have reliable sonic logs, we may not need to invoke minimum phase or random reflectivity assumptions. We can compute the wavelet by using the seismic data and the reflectivity computed from the sonic log. We can essentially compute the reflectivity and take its Fourier transform to give $R(f)$ and then divide the trace’s Fourier transform, $Y(f)$, by this value. That is, the wavelet’s Fourier Transform is given by computing:

$$W(f) = \frac{Y(f)}{R(f)} \quad \left[ W(\omega) = \frac{Y(\omega)}{R(\omega)} \right]$$  \hspace{1cm} (4)

Lines and Treitel (1985) gave computational reasons for computing this result in the time domain using Wiener filters, but the results are mathematically equivalent to equation (4). Once the wavelet is estimated by this process, a digital filter designed to deconvolve the wavelet, can then be applied to the entire seismic line. This wavelet deconvolution process does make two fundamental assumptions:

1. The wavelet estimated at the well is basically consistent for the entire seismic line.

2. The reflectivity used in wavelet estimation can be reliably estimated from the well log.

The first assumption is essentially one of source repeatability in the field experiments. The second assumption of reflectivity reliability is one that we can evaluate using repeated logs. The repetition of logs gives us some representation of the log’s accuracy.

For the logs in our example, let us test the accuracy of log-based wavelet estimation by assuming that the second sonic log, log B, is accurate, and that log A has errors. How will this situation affect the wavelet estimate? In order to compare with minimum phase statistical measures, we use traces in Figure 3 which contain the minimum phase equivalent of the 150 Hz Ricker.

Since we are assuming log B to be accurate, we use the second trace in Figure 3b as our data. We then apply equation (4) to estimate $W(f)$ using reflectivities from the two sonic logs. The wavelet estimate derived with the reflectivity from log B should be correct while the estimate with the erroneous log A will show variability. This is exactly the result shown in Figure 4, which compares the correct wavelet, the wavelet derived
from log A, and the wavelet derived from B. As expected, there is slight difference using log A for trace B, but the wavelet estimate using the incorrect log is still a good estimate of the correct wavelet, with a deviation from the true wavelet being 3.58%. The wavelet estimate using the correct log (third wavelet in Figure 4) is virtually identical to the true wavelet, as expected.

![Wavelet estimation diagram](image)

**FIG. 4.** A comparison of actual wavelet and the wavelet estimates using different reflectivity functions in the log-based wavelet estimation. The first is the actual wavelet, which minimum phase equivalent of 150Hz Ricker. Sample interval = 0.5ms. The data trace is the convolution of this wavelet with the reflectivity of log B. The second wavelet is the estimate obtained using log A. The third wavelet is obtained using correct reflectivity from log B and as expected, is virtually identical to the correct wavelet. Although the second wavelet has used the incorrect log values, it is still accurate to within 3.58%.

A simple mathematical error analysis of this situation could be viewed as an alteration of equation 4 in which $R(f)$ is replaced by $R(f) + \varepsilon(f)$, where $\varepsilon(f)$ represents the reflectivity errors in the frequency domain. Hence the wavelet estimate for an erroneous reflectivity would require that equation (4) be revised to give;
\[
\hat{W}(f) = \frac{Y(f)}{R(f) + \varepsilon(f)} \quad \left[ \hat{W}(\omega) = \frac{Y(\omega)}{R(\omega) + \varepsilon(\omega)} \right]
\]

(5)

For the case where reflectivity errors are small, where \(\varepsilon(f)\) is much less than \(R(f)\) in amplitude, we can write:

\[
\hat{W}(f) \equiv \frac{Y(f)}{R(f)} \left(1 - \frac{\varepsilon(f)}{R(f)}\right) \quad \left[ \hat{W}(\omega) \equiv \frac{Y(\omega)}{R(\omega)} \left(1 - \frac{\varepsilon(\omega)}{R(\omega)}\right) \right]
\]

(6)

Hence the accuracy of the wavelet estimate is essentially controlled by the “noise to signal ratio” for the estimated reflectivity. An estimate of this signal-to-noise ratio, \(\varepsilon(f)/R(f)\), can be gauged by examining the repeated log measurements.

**COMPARISONS WITH STATISTICAL WAVELET ESTIMATION**

As previously mentioned, a competitor in wavelet estimation for the log-derived wavelet is the minimum phase wavelet, which does not rely on the same assumptions as the log-derived wavelet, but rather depends on two other assumptions:

1. The reflectivity function is uncorrelated. (That is, its autocorrelation is a delta function and the wavelet’s spectrum equals the trace spectrum multiplied by a constant.)

2. The wavelet is minimum phase.

The minimum-phase wavelet estimates do not require well logs, and can be effective even when used on traces that are somewhat noisy (Kelly and Lines, 1995). In fact, a small amount of white noise can stabilize the estimation process. Figure 5 illustrates the minimum phase estimates derived from the traces’ amplitude spectra. We see that these estimated wavelets are similar to the actual wavelet, but are slightly shifted in time. (Since the reflectivity is not perfectly random, the trace amplitude spectra are not identical to the wavelet spectra and the minimum-phase wavelets are not perfect.)

Nevertheless, the deviation of the minimum-phase wavelet estimates from the correct wavelet estimates is about 3.5%, almost the same using the log-based wavelet estimate. **It is interesting that although both the log-based wavelet estimate and the minimum-phase wavelet estimates have made different assumptions, they have both provided excellent estimates that deviate from the correct wavelet by less than 4%**.

For the cases where one has both seismic data and well log data, the key question regards whether one should use log-based wavelet estimates or statistical methods. A key to answering this question comes from the reliability of the sonic log.
FIG. 5. A comparison of actual wavelet and the minimum phase wavelet estimates using different reflectivity functions in the log-based wavelet estimation. The first is the actual wavelet, which minimum phase equivalent of 150 Hz Ricker. The second and third wavelets are obtained by computing minimum phase wavelets directly from the seismic traces.

REPEATABILITY OF SHEAR-WAVE LOGS

From our present calculations, the repeatability tests for $P$–wave logs have shown generally encouraging results. For the $P$–wave logs, the reflectivities and the synthetic seismogram results are very similar with very high semblance values. The log-based wavelet estimates are very similar to minimum-phase wavelet estimates, both appearing to be very reliable.

However, an important characteristic of dipole sonic log information is the shear-wave information. This is important information since the shear-wave velocities allow for important lithology discrimination between sandstones and shales that would not be found from $P$–wave velocity information alone. The use of $V_p/V_s$ ratios for lithology discrimination have been shown to be useful in at least two heavy oil fields in Western Canada including Plover Lake Field (Lines et al., 2005) and Long Lake Field (Dumitrescu et al., 2009).
As expected in the excellent review article by Close et al. (2009), the estimation of shear-wave velocities from dipole sonic logs is expected to be much more challenging than the estimation of $P$-wave velocities due to the problem of picking shear-wave arrivals. The $P$-wave arrival is obtained from the first break in the seismic wave arrivals in the borehole while the shear-wave arrival is imbedded in the coda of earlier arrivals. The repeatability of shear-wave velocity values would be expected to be much more challenging.

As anticipated, the results for shear-wave repeatability for this area do not look quite as good for $P$-wave repeatability. If we examine the shear-wave results for logs A and B in Figure 6, we note that the velocities are similar but not quite as similar as for the $P$-wave results in Figure 1. The figure shows that the shear-wave logs for A and B similar, with a difference trace that shows 5.14% difference, compared to the $P$-wave velocities in Figure 1 which show a difference of about 1.42% difference.

**FIG. 6.** A comparison of the shear-wave logs from two wells (denoted in the text as log A and log B) spanning a depth range from 153.5-292.1 m. Maximum deflection on traces is 1600 m/s. The third trace in the plot is a discrepancy log giving the difference in velocities between wells A and
B. The difference trace in this case represents an average absolute amplitude difference of 5.14%, which is about 3.5 times greater than for the $P$-wave logs.

If the synthetic traces are computed using $150\text{Hz}$ minimum phase wavelet for the reflectivities of the logs in Figure 6, we obtain shear-wave synthetic traces for the logs as shown in Figure 7. The semblance for the traces in Figure 7 is 0.884 which is somewhat less than the semblance for $P$-wave synthetic traces which was 0.985.

![Figure 7](image.png)

FIG. 7. The synthetic trace on the left is for log A, and the trace on the right is for log B. The two traces span 248\text{ms}. The semblance between the traces is 0.884.

We do notice that there are definite phase shifts between the shear-wave synthetic traces that would give rise to a lower semblance value than for the $P$-wave synthetic traces. We notice these phase shifts in the log-based wavelet estimates as well when we repeat the wavelet estimation procedure used to produce the wavelet. We repeat the same wavelet estimation procedure used to produce Figure 4. Figure 8 shows the actual minimum phase wavelet as well as the wavelet estimates for logs A and B. The wavelet for trace A has a distinctive delay from the actual wavelet and the wavelet for log B. The average fractional difference between the actual wavelet (first wavelet in Figure 8) and
the log-based estimate (second wavelet in Figure 8) is 5.14%, which is slightly worse than the estimates for the $P$-wave synthetic seismic trace.

![Wavelet Comparison](image)

**FIG. 8.** A comparison of actual wavelet and the wavelet estimates using different reflectivity functions in the log-based wavelet estimation for shear waves. The first is the actual wavelet, which minimum phase equivalent of 150Hz Ricker. The actual trace is the convolution of this wavelet with the reflectivity of log B. The second wavelet is the estimate obtained using log A. The third wavelet is obtained using correct reflectivity and as expected, is virtually identical to the correct wavelet.

The repeatability for the shear-wave logs is not as reliable as for the $P$-wave log. This is not unexpected since the detection of $P$-wave arrivals essentially involves picking the first arrivals of waves in a borehole, whereas the shear-wave arrival is immersed in a coda of arrivals. The detection of the shear-wave is more problematic since critically refracted shear waves do not occur if the shear-wave velocity is less than the $P$-wave velocity in the borehole fluid. The shear-wave arrival arises from a flexural wave created by a dipole source. Since the shear-wave arrivals is imbedded in a coda of events following the $P$-wave, its detection and timing is generally more difficult than the timing of the $P$-wave. Therefore it is not surprising that we may see more inconsistencies in the shear-wave arrivals than the $P$-waves. For a detailed explanation of these sonic log phenomena, the reader is referred to an excellent paper by Close et al. (2009).
PROPOSED STRATEGY FOR WAVELET ESTIMATION AND ANALYSIS OF
REPEATED DIPOLE SONIC LOGS

It is interesting in these computational experiments to notice that both the log-based and minimum-phase wavelet estimates were similar and were accurate. Both methods use different sets of assumptions, and in these cases, both sets of assumptions were valid. Similarity of wavelet estimates would lead one to have confidence in the well logs and the wavelet estimates.

However, one can imagine a strategy that could be used if the log-based wavelet estimates for the repeated logs were radically different. Then one would suspect that at least one of the sonic logs would be erroneous. In such cases, which one of the log-based estimates would we choose?

In such cases, one could compare the log-based wavelets to minimum phase wavelet estimates for the seismic data. This data-derived estimate does not depend upon the reliability of the log, but on the validity of a minimum phase source wavelet and a random reflectivity. One can question the validity of those assumptions, but it has been our general experience that these assumptions are reasonably valid for dynamite and air gun sources. Otherwise, the process of spiking deconvolution, which uses these assumptions, would not have enjoyed decades of applications in seismic data processing (Robinson and Treitel, 1980). If a wavelet from the logs more closely resembles the minimum phase wavelet, this wavelet and its corresponding sonic log would tend to have more credibility.

CONCLUSIONS

Sonic logs involve errors in measurements. Hence, the well log experiments are repeated to evaluate their reliability. The resulting logs and computed reflectivities will show differences. Due to the smoothing effect of seismic wavelets, the difference in synthetic seismograms for repeated logs is often less evident than for reflectivities. The sonic log differences will also affect the log-based wavelet estimations for deconvolution. The decision of whether to use log-based wavelet estimation or minimum phase wavelet estimation will depend on the accuracy of the sonic logs and on the wavelet phase. In our experience, we have found the $P$-wave logs to be slightly more repeatable and reliable than the shear-wave information. This is due to the nature of the dipole logging measurements and the ease of timing $P$-wave arrivals compared to shear-wave arrivals.

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REFERENCES


