Split-step Two-way phase-shift time stepping for wavefield propagation

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ABSTRACT

The phase-shift time-stepping equation (PSTS) is a wavefield propagator that allows two-way in time propagation for the acoustic wave equation. PSTS is based an an exact solution to the constant velocity acoustic wave equation. It is adapted to a variable velocity wave equation by a windowed Fourier transform where in each window a constant velocity solution is computed. We consider a correction to the phase-shift time-stepping equation that corrects the wave propagators for variable velocity. The correction is based on a similar Taylor-series expansion used to derive the split-step correction for one-way depth steppers or to derive higher-order in time pseudospectral methods using the modified equation approach or Lax-Wendroff method. The computational properties of the split-step correction to PSTS equation are similar to higher-order in time pseudospectral methods.

INTRODUCTION

Reverse-time migration (RTM) and forward modeling by finite-differenceing the two-way acoustic wave equation (Baysal et al., 1983; McMechan, 1983) are computationally expensive. However with an accurate velocity model they are a very effective methods for migration and modeling. We consider a number of alterative methods to solve the acoustic wave equation in the wavenumber domain. For a summery of solution methods for the wave equation refer to Carcione et al. (2002).

To solve the wave equation, the one-way in depth wavefield continuation methods oper-

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ate typically in the frequency-wavenumber domain by recursively phase shifting the transformed wavefield. The Gazdag phase-shift (Gazdag, 1978) is accurate but can only propagate in a constant velocity medium. The phase-shift plus interpolation (PSPI) (Gazdag and Sguazzero, 1984) algorithm introduces multiple constant reference velocities and interpolates to accomplishing propagation in a heterogenous medium. It is much more numerically intensive than the Gazdag phase shift because it must shuttle between the space and wavenumber domains at every depth step. Additionally, it is limited to media with small lateral velocity variations and is only accurate for small angles of propagation about a preferred direction normally taken to be vertical. Split-step depth stepping (Hardin and Tappert, 1973; Stoffa et al., 1990) corrects for strong lateral velocity variations but is inaccurate at large angles of propagation. To correct for lateral velocity variations and wide angles of propagation pseudoscreen (Jin and Wu, 1999), phasescreen (Wu and Huang, 1992), and split-step Fourier finite difference (Biondi, 2002) have been proposed. For turning waves, using curvilinear coordinates (Shragge, 2008) and multiple directions of propagations (Shan and Biondi, 2008) work well.

Similar to one-way in depth wavefield propagators, two-way in time wavefield propagators are solutions that originated from solving the two-way in time wave equation for the constant velocity acoustic wave equation. The constant velocity integral solutions are adapted to heterogenous medium by replacing the constant velocity in the solution with a variable velocity (Wards et al., 2008). These Fourier-like integrals, however, are too computationally complex to be calculated explicitly and must be approximated. We consider a number of approximations to this integral and compare them to higher-order pseudospectral methods. We call these approximations split-step phase-shift time-stepping.

PSEUDOPSECTRAL METHODS

We first motivate the study of split-step phase-shift time stepping by reviewing higher-order in time pseudospectral methods. Pseudospectral methods are numerically efficient methods to solve the full two-way acoustic wave equation. They compute the spacial Lapla-

cian by using the Fourier transform and as a result they allow larger spactial sampling rates than finite-difference methods and reduce the memory requirements. However boundary conditions cannot be as precisely implemented (Fornberg, 1975, 1987).

The following conventions are used for the forward and inverse Fourier transform of the function $\varphi : \mathbb{R}^2 \to \mathbb{C}$,

$$\hat{\varphi}(\vec{k}) = \mathcal{F}_{\vec{x}\to\vec{k}}(\varphi) = \int_{\mathbb{R}^2} e^{2\pi i \vec{x}\cdot\vec{k}} \varphi(\vec{x}) dx dz, \tag{1}$$

and

$$\varphi(\vec{x}) = \mathcal{F}^{-1}_{\vec{k}\to\vec{x}}(\hat{\varphi}) = \int_{\mathbb{R}^2} e^{-2\pi i \vec{x}\cdot\vec{k}} \hat{\varphi}(\vec{k}) dk_x dk_z, \tag{2}$$

where $\mathbb{R}$ is the real line, $i = \sqrt{-1}$, $\vec{x} = (x, z) \in \mathbb{R}^2$, $\vec{k} = (k_x, k_z) \in \mathbb{R}^2$ is the Fourier domain coordinate conjugate to $\vec{x}$.

The symbols $\mathcal{F}_{\vec{x}\to\vec{k}}$, and $\mathcal{F}^{-1}_{\vec{k}\to\vec{x}}$ are used to denote the forward and inverse Fourier transforms as abstract operators, respectively. Later, the symbols $\mathcal{F}_{\vec{x}\to\vec{k}}$, and $\mathcal{F}^{-1}_{\vec{k}\to\vec{x}}$ are also used to denote the Fourier-like integrals when φ or $\hat{\varphi}$ depend upon $\vec{k}$ and $\vec{x}$ explicitly.

The acoustic constant-density variable-velocity wave equation is

$$\begin{cases} \frac{\partial^2 U}{\partial t^2} = v^2 \left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial z^2} \right) \\ U(0, \vec{x}) = f(\vec{x}) \\ U(-\delta t, \vec{x}) = g(\vec{x}) \end{cases}, \tag{3}$$

where $U(t, \vec{x})$ is the amplitude of the wave at the point $(t, \vec{x})$, x is the lateral coordinate, z is the depth coordinate, t is the time coordinate, $\partial^2 U / \partial t^2$ is, for example, the second-order partial derivative of the wavefield with respect to the time coordinate, and v, is the speed of propagation. Assume $\vec{x} \in \mathbb{R}^2$ and $t \in \mathbb{R}$.

The pseudospectral method uses the Fourier transform over the spacial coordinates to

numerically calculate the Laplacian. The resulting equation is

$$\frac{\partial^2 U}{\partial t^2} = -v^2 \mathcal{F}^{-1}_{\vec{k}\to\vec{x}}[(2\pi|\vec{k}|)^2 \mathcal{F}_{\vec{x}\to\vec{k}}[U]]. \tag{4}$$

The second-time derivative is approximated by the second-order centered finite-difference operator to derive a time-marching algorithm

$$U^{n+1} = 2(U)^n - U^{n-1} - \delta t^2 v^2 \mathcal{F}^{-1}_{\vec{k}\to\vec{x}}[(2\pi|\vec{k}|)^2 \mathcal{F}_{\vec{x}\to\vec{k}}[U]], \tag{5}$$

where the superscripts n refers to the approximation at timestep n.To deduce a higher-order algorithm, a higher-order finite-difference approximation can be used for the second-time derivative of U. However these algorithms are unconditionally unstable (Cohen, 2001). Alternatively, the modified equation approach (Cohen, 2001) can be used. The Taylor series expansion of the second-order time derivative is

$$\left(\frac{\partial^2 U}{\partial t^2}\right)^n = \frac{U^{n+1} - 2U^n + U^{n-1}}{\delta t^2} - \frac{\delta t^2}{12}\left(\frac{\partial^4 U}{\partial t^4}\right)^n + O(\delta t^6) \tag{6}$$

which is derived by adding together the Taylor series expansion of $U(t+\delta t)$ and $U(t-\delta t)$ about t. Substituting equation (6) into the scalar wave equation gives the fourth-order time approximation,

$$U^{n+1} = \delta t^2 v^2 (\Delta U)^n - U^{n-1} + 2U^n + \frac{v^4 \delta t^4}{12}\left(\Delta^2 U\right)^n + \ldots, \tag{7}$$

where ΔU refers to the 2-dimensional Laplacian of the function U and $\Delta^2 U$ is the biharmonic or the Laplacian applied twice to U. If the procedure leading to equation (7) is iterated, then formally (Etgen, 1989; Dablain, 1986; Chen, 2007),

$$U^{n+1} = -U^{n-1} + 2\sum_{k=0}^{\infty} \frac{(\delta t v)^{2k}}{(2k)!}\left(\Delta^k U\right)^n. \tag{8}$$

Taking the Fourier transform of both sides of equation (8) with respect to the spatial coor-

dinates,

$$\begin{aligned}\widehat{U}^{n+1} &= -\widehat{U}^{n-1} + 2\sum_{k=0}^{\infty}\frac{(\delta t v)^{2k}}{(2k)!}\left((-2\pi|\vec{k}|)^{2k}\widehat{U}\right)^{n} \\ &= -\widehat{U}^{n-1} + 2\cos\left(2\pi v|k|\delta t\right)\widehat{U}^{n}. \end{aligned} \tag{9}$$

As an alternative method to approximating $\cos\left(2\pi v|k|\delta t\right)$ with a Taylor series approximation Soubaras and Zhang (2008) uses an equiripple polynomial to implement a higher-order pseudospectral method.

In the case that the velocity depends upon the spatial coordinates $v(\vec{x})$ the modified approach needs to be adjusted to insure convergence (Cohen, 2001). Substituting the variable velocity wave equation into the fourth-order time derivative gives

$$\frac{\partial^4 U}{\partial t^4} = v^4\Delta^2 U + v^2\Delta(v^2)\Delta U + v^2\nabla(v^2)\cdot\nabla(\Delta U), \tag{10}$$

where Δv is the Laplacian applied to the velocity v and ∇ is the gradient operator. The equivalent of equation (7) for variable velocity is

$$U^{n+1} = \delta t^2 v^2(\Delta U)^n - U^{n-1} + 2U^n + \frac{\delta t^4}{12}\left(v^4\Delta^2 U + v^2\Delta(v^2)\Delta U + v^2\nabla(v^2)\cdot\nabla(\Delta U)\right)^n. \tag{11}$$

A fourth-order in time, exponential-order in space, solution of the variable velocity wave equation is then

$$\begin{aligned} U^{n+1} = &- U^{n-1} + 2U^n - \left(\delta t^2 v^2 + \frac{\delta t^4 v^2\Delta(v^2)}{12}\right)\mathcal{F}^{-1}_{\vec{k}\to\vec{x}}\left[(2\pi|\vec{k}|)^2\mathcal{F}_{\vec{x}\to\vec{k}}[U^n]\right] \\ &+ \frac{\delta t^4}{12}\left(v^4\right)\mathcal{F}^{-1}_{\vec{k}\to\vec{x}}\left[(2\pi|\vec{k}|)^4\mathcal{F}_{\vec{x}\to\vec{k}}[U^n]\right] \\ &+ \frac{\delta t^4}{12}\left(v^2\right)\nabla(v^2)\cdot\mathcal{F}^{-1}_{\vec{k}\to\vec{x}}\left[2\pi\vec{k}(2\pi|\vec{k}|)^2\mathcal{F}_{\vec{x}\to\vec{k}}[U^n]\right]. \end{aligned} \tag{12}$$

When the velocity varies slowly or when we propagate the wavefields with smoothed back-

ground velocity the term involving Δv^2 and ∇v^2 can be ignored.

THE PHASE-SHIFT TIME-STEPPING EQUATION

We first derive the phase-shift time stepping equation. Applying the Fourier transform over the spatial dimensions $\vec{x} = (x, z)$ to both sides of equation (3), reduces it to a collection of ordinary differential equations,

$$\begin{cases} \frac{\partial^2 \hat{U}}{\partial t^2} = -(2\pi)^2 v^2 \left(k_x^2 + k_z^2\right) \hat{U} \\ \hat{U}(0, \vec{k}) = \hat{f}(\vec{k}) \\ \hat{U}(-\delta t, \vec{k}) = \hat{g}(\vec{k}) \end{cases} . \tag{13}$$

When $\vec{k} = 0$, the solution is identically zero, if the initial wavefield and its derivative are integrable functions. The resulting exact timestepper at time $t = 0$ of the constant velocity wave equation is

$$U(\delta t, \vec{x}) = -U(-\delta t, \vec{x}) + 2\mathcal{F}^{-1}_{\vec{k}\to\vec{x}}[\cos\left(2\pi v|\vec{k}|\Delta t\right) \mathcal{F}_{\vec{x}\to\vec{k}}[U(0, \vec{x})]]. \tag{14}$$

The fast Fourier transform can be employed because the kernel of the Fourier integral is independent of the spatial coordinate $\vec{x}$.

TIMESTEPPING IN A VARIABLE-VELOCITY MEDIUM

The variable-velocity acoustic wave equation is

$$\frac{\partial^2 U}{\partial t^2} = v^2(x, z) \left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial z^2}\right), \tag{15}$$

where $v(x, z)$ is the spatially dependent velocity. We now adapt equation (14) which propagates an acoustic wavefield exactly in a constant velocity medium to propagate approximately in a variable velocity medium.

As a result, the right hand side of equation (15), $v^2(\vec{x})\,(U_{xx}+U_{zz})$, can be approximated locally near $\vec{x}_0$ by the solution to the frozen equation $v^2(\vec{x}_0)\,(U_{xx}+U_{zz})$. This means that by replacing the constant velocity appearing in the dispersion relation in equation (14) by the variable velocity (i.e. unfreezing the velocity), we have an approximate solution

$$U(\delta t,\vec{x}) = -U(-\delta t,\vec{x}) + 2\mathcal{F}^{-1}_{\vec{k}\to\vec{x}}\left[\cos\left(2\pi v(\vec{x})|\vec{k}|\delta t\right)\mathcal{F}_{\vec{x}\to\vec{k}}\left[U(0,\vec{x})\right]\right]. \qquad (16)$$

This is the freezing-unfreezing argument that appears in the literature in the context of hyperbolic and elliptic partial differential equations e.g., (p. 230-231, Stein, 1993). Such solutions are often called locally homogeneous approximations (e.g., Ma and Margrave, 2008) and they approximate the solution to the variable velocity wave equation by the solution locally from the constant velocity wave equation.

Equation (12) is the fourth-order time pseudospectral method, comparing this to equation (16) with the cosine replaced by its fourth-order approximation we see that it does not contain the term fourth-order in time containing $\Delta(v^2)$. Therefore the fourth-order pseudospectral method may be higher order method than equation (16).

SPLIT-STEP TIME STEPPING

A Taylor series can be used to approximate the variable velocity cosine operator about the reference velocity v_0. For some velocity functions this neighborhood can be extended to the entire domain of computation if a small error in the velocity is accatable. The power series expansion about the point v_0 with the variation $\delta v = v(x) - v_0$ for the function

$\cos(2\pi v(\vec{x})|\vec{k}|\Delta t)$ is

$$\begin{aligned}\cos(2\pi v(\vec{x})|\vec{k}|\Delta t) &= \cos(2\pi v_{ref}|\vec{k}|\delta t) \\ &- \sin(2\pi v_{ref}|\vec{k}|\delta t)\delta v(\vec{x})2\pi|\vec{k}|\Delta t \\ &- \frac{1}{2}\cos(2\pi v_{ref}|\vec{k}|\delta t)\left[\delta v(\vec{x})2\pi|\vec{k}|\Delta t\right]^2 \\ &+ H.O.T. \end{aligned} \tag{17}$$

where H.O.T. denotes higher order terms. Substituting the Taylor series expansion (17) into equation (16) gives the second order splitstep correction

$$\begin{aligned}U(\delta t,\vec{x}) \simeq &- U(-\delta t,\vec{x}) + 2\mathcal{F}^{-1}_{\vec{k}\to\vec{x}}\left[\cos(2\pi v_{ref}|\vec{k}|\delta t)\mathcal{F}_{\vec{x}\to\vec{k}}\left[U(0,\vec{x})\right]\right] \\ &- 2\pi\delta v(\vec{x})\Delta t\mathcal{F}^{-1}_{\vec{k}\to\vec{x}}\left[|\vec{k}|\sin\left(2\pi v(\vec{x})|\vec{k}|\delta t\right)\mathcal{F}_{\vec{x}\to\vec{k}}\left[U(0,\vec{x})\right]\right] \\ &- \frac{1}{2}(2\pi\delta v(\vec{x})\Delta t)^2\mathcal{F}^{-1}_{\vec{k}\to\vec{x}}\left[|\vec{k}|^2\cos\left(2\pi v(\vec{x})|\vec{k}|\delta t\right)\mathcal{F}_{\vec{x}\to\vec{k}}\left[U(0,\vec{x})\right]\right]. \end{aligned} \tag{18}$$

Higher-order algorithms can similarly be derived by taking a higher-order approximation in the Taylor series expansion in equation (17).

MULTIPLE REFERENCE VELOCITIES

For large velocity variations δv, the split-step correction can become inaccurate and unstable. To eliminate this problem a windowed Fourier transform can be used with a number of reference velocities about which a smaller split-step correction is needed.

Window functions are used to separate the wavefield into N regions each of which is propagated with a reference velocity. In our method these regions need not be simply connected. The wavefield in each region is then propagated with the corresponding reference velocity and split-step correction. We call a set of windowing functions that sum up to one a partition of unity. The windowing functions are taken to be positive and if they are smooth they suppress reflections. To constuct the POU $\{\Omega_1(\vec{x}),\ldots,\Omega_N(\vec{x})\}$ given a set of reference velocities $\{v_1,\ldots,v_N\}$, we first construct a simpler POU based on discontinuous

indicator functions $\{I_1(\vec{x}), \ldots, I_N(\vec{x})\}$ where

$$I_n(\vec{x}) = \begin{cases} 1 & : \quad |v_n - v(\vec{x})| \text{ is a minimum for } j \in [1, N] \\ 0 & : \quad \text{otherwise} \end{cases} \tag{19}$$

where the meaning of the right-hand side is that the indicator function is only unity when $|v_n - v(\vec{x})|$ assumes a minimum for the n^{th} reference velocity, and is otherwise zero.

The POU is used to window the wavefield into regions at each timestep and the combination of windowing and Fourier transformation results in the Gabor approximation to equation (14) given by

$$U(\delta t, \vec{x}) = -U(-\delta t, \vec{x}) + 2 \sum_{n=1}^{N} \Omega_n(\vec{x}) \mathcal{F}^{-1}_{\vec{k} \to \vec{x}} \left[\cos\left(2\pi v_n |\vec{k}| \delta t\right) \mathcal{F}_{\vec{x} \to \vec{k}} \left[U(0, \vec{x}) \right] \right], \tag{20}$$

where $\Omega_n(\vec{x})$ is the windowing functions, and v_n is the reference velocity used for propagation in the nth window. Define the multiplier

$$M_m^{v_n} = \begin{cases} (-1)^{m/2} (2\pi |\vec{k}| \delta t)^m \cos(2\pi v_n |\vec{k}| \delta t) & m \text{ even} \\ (-1)^{(m+1)/2} (2\pi |\vec{k}| \delta t)^m \sin(2\pi v_n |\vec{k}| \delta t) & m \text{ odd} \end{cases} . \tag{21}$$

So that the wavefield propagator for N windows of order M is

$$U(\delta t, \vec{x}) = -U(-\delta t, \vec{x}) + 2 \sum_{n=0}^{N} \Omega_n \sum_{m=0}^{M} \frac{(\delta v_n(\vec{x}))^m}{m!} \mathcal{F}^{-1}_{\vec{k} \to \vec{x}} \left[M_m^{v_n} \mathcal{F}_{\vec{x} \to \vec{k}} \left[U(0, \vec{x}) \right] \right]. \tag{22}$$

The split-step correction equation (22) requires significantly fewer reference velocities than using equation (20). As well, equation (20) is capable of modeling continuous velocity gradients.

SAMPLING ISSUES

Ideally it would be desired to timestep at the Nyquist sampling rate of the seismic data,

$$\Delta t_{nyq} = \frac{1}{2f_{max}} \tag{23}$$

where f_{max} is the maximum signal frequency. Reverse time migration methods however often require finer sampling. For the PSTS equation, the smallest wavelength must be sampled at least twice,

$$\Delta x < \frac{V_{min}}{2f_{max}}, \tag{24}$$

where V_{min} is the minimum velocity of the model and δx is the gridspacing in the x and z directions. The timestep must satisfy (Wards et al., 2007)

$$\delta t < \frac{\delta x}{\sqrt{2}V_{max}}. \tag{25}$$

For processing the seismic survey with $[V_{min}, V_{max}] = [1500m/s, 5500m/s]$ and $f_{max} = 50Hz$, the sampling requirements are $\delta x < 15m$, $\delta t < 0.0015s$, and $t_{nyq} = 0.01s$.

To minimize computation time, a multi-radix FFT is used. The computation domain is padded to the next integer with a large number of prime factors. A pad is necessary to prevent wraparound of the FFT and to enforce a free surface boundary condition.

NUMERICAL EXAMPLES

We compare pseudospectral methods to split-step PSTS methods by looking at some snapshots at a particular time of a forward propagated wavelet through a portion of the BP data set in Figure 1. The wavelet is inject at the center of the model. The BP data set Billette and Brandsberg-Dahl (2005) contains a number of saltdomes and is commonly used to test velocity picking algorithms, wide angle depth continuation algorithms, and reverse time migration algorithms. The BP data set contains a rigorous saltdome embedded in

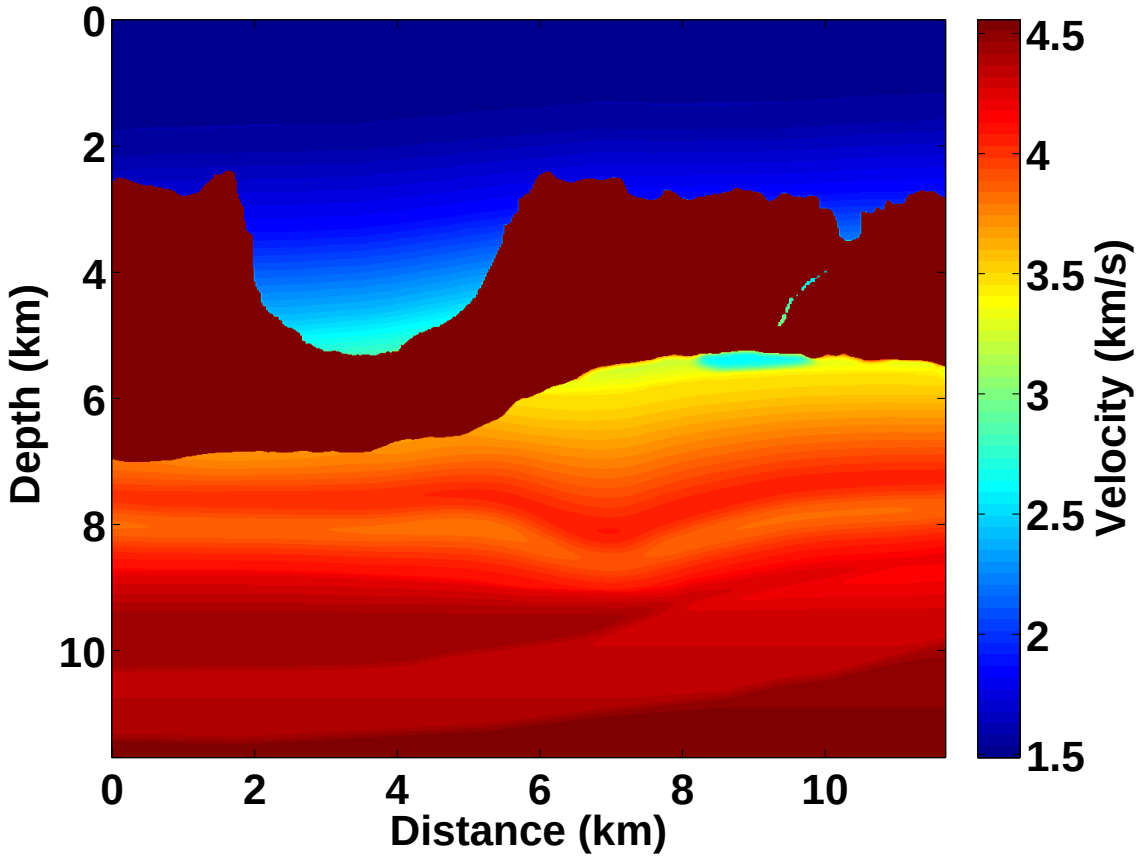

FIG. 1. A section of the BP data set showing the rigours salt dome.

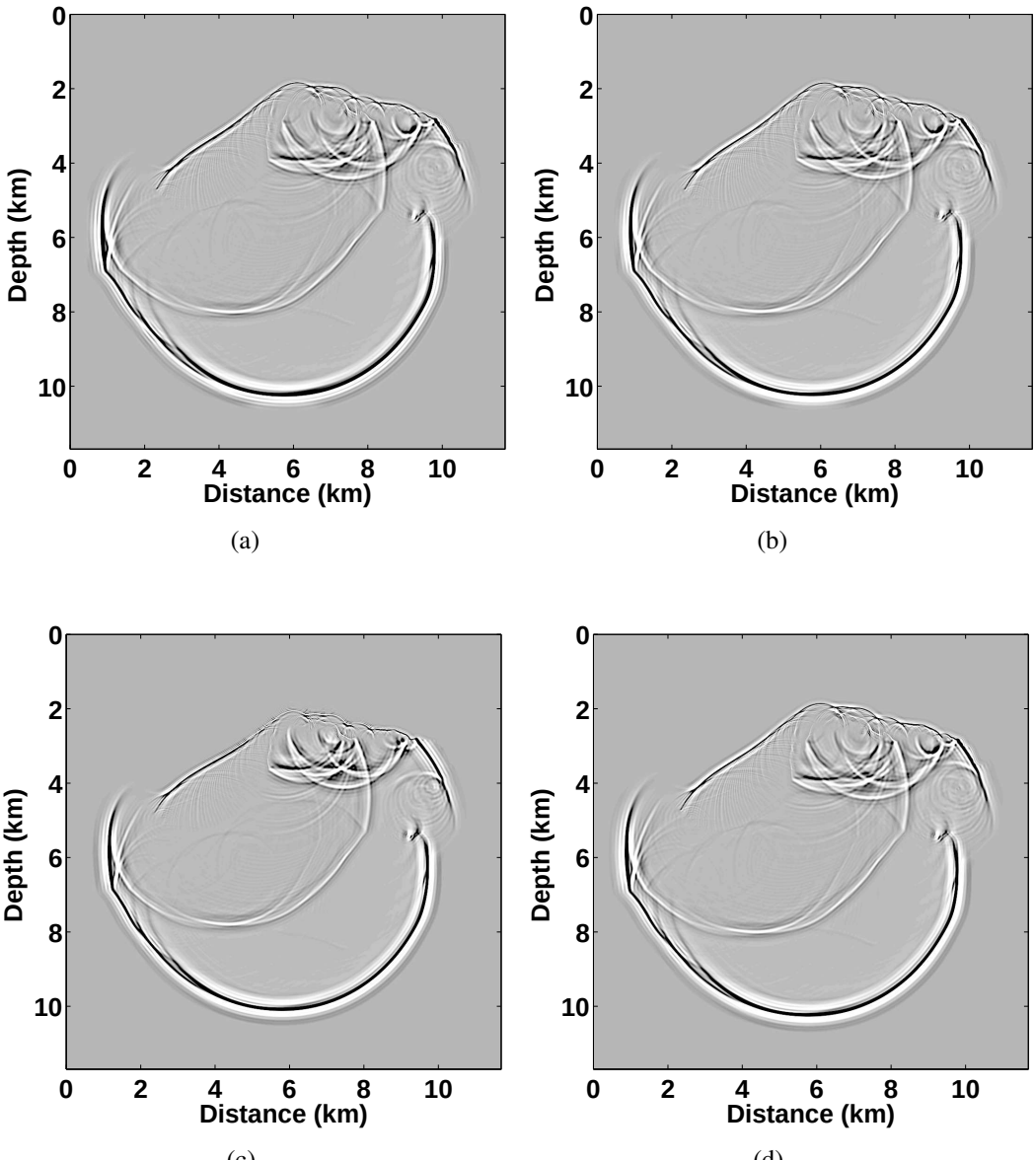

FIG. 2. (a) Second-order pseudo-spectral method. (b) Forth-order pseudo-spectral method. (c) First order split-step PSTS with one window. (d) Second order split-step PSTS with one window.

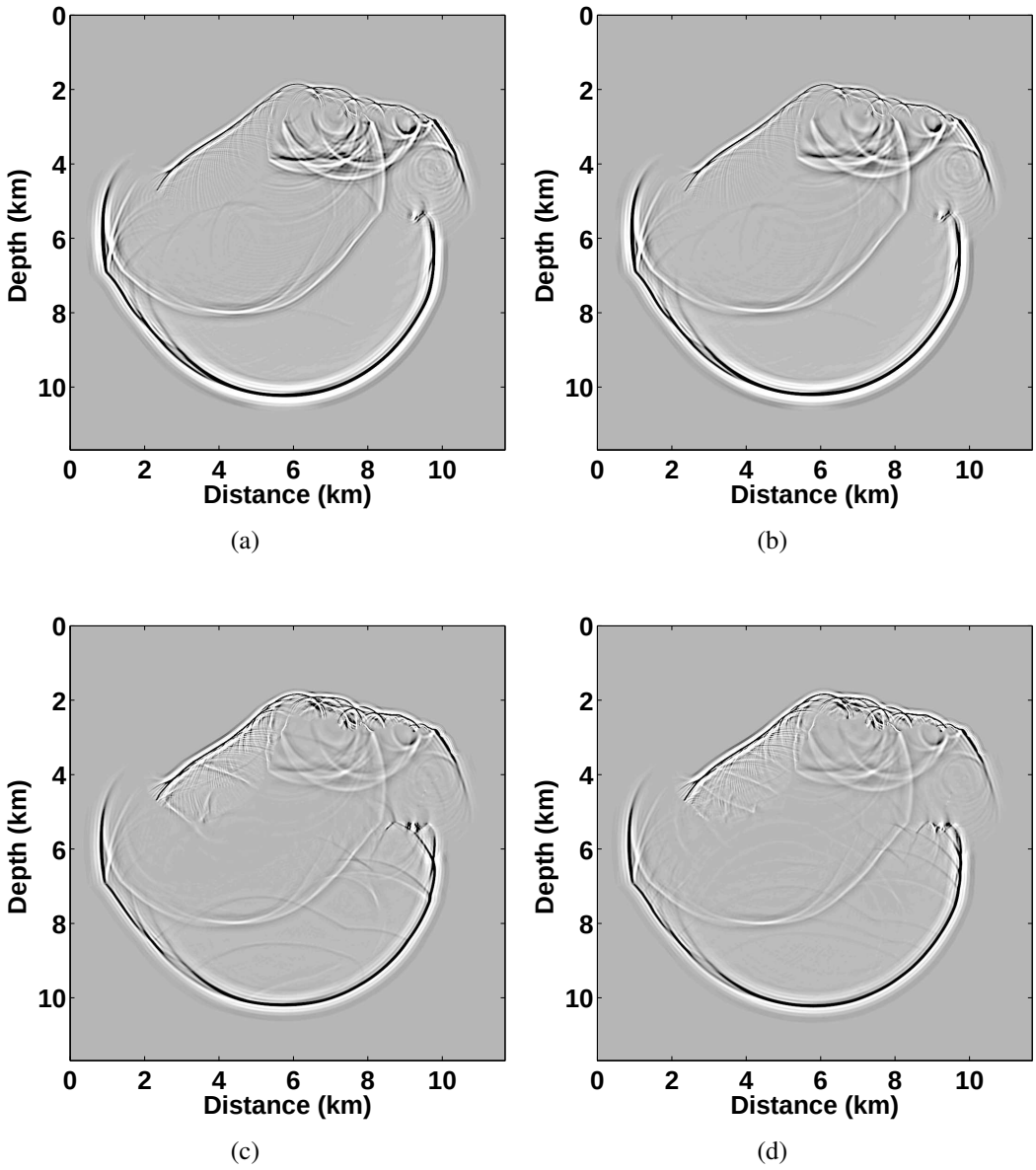

FIG. 3. (a) Forth-order pseudo-spectral method with correction term from equation 11. (b) First-order split-step PSTS with three windows. (c) The PSTS equation with no split-step correction using 10 evenly spaced reference velocities. (d) Similar to (c) but using 20 reference velocities.

algorithm	Relative time	time step (ms)	grid spacing	number of FT
PSTS 10 vels	5	1.5	12.5	11
PSTS 20 vels	10	1.5	12.5	21
pseudo 2nd order	0.8	1.2	12.5	3
pseudo 4th order	1.0	1.5	12.5	3
$+\Delta(c^2)\Delta(U)$	1.0	1.5	12.5	3
splitstep 1st order	0.9	1.5	12.5	3
splitstep 2nd order	1.3	1.5	12.5	4
splitstep 1st order 3 windows	4.5	1.5	12.5	7

Table 1. Relative computation time and timestep size used to make Figure 2 and Figure 3

a background sediment whose velocity smoothly increasing with depth. Figure 2(a) is the snapshot using second-order pseudospectral method derived in equation (5). The method is computationally efficient but contains unacceptable dispersion. Figure 2(b) is the snapshot using fourth-order pseudospectral method derived in equation (5). There is no observable dispersion. Figure 2(c) is the first order split-step correction. Although the model does not contain much dispersion there are large kinematic errors due to the low order of the approximation. Figure 2(d) is the second order split-step correction. The kinematics are much better than in Figure 2(c) but there is more dispersion than the fourth-order pseudospectral method. Figure 3(a) is the first-order split-step correction using three reference velocities. The kinematics are much improved over using one reference velocity. Figure 3(b) is the fourth order pseudospectral method with the correction term in equation (11). The correction term had little noticeable effect. Figure 3(d) is the PSTS approximation using 10 reference velocities. There are noticeable kinematic errors through there is little dispersion. Figure 3(d) use 20 reference velocities but does not significantly improve the poor image of Figure 3(c). Table 1 contains the relative computation times of all the methods. All of the computations where computed a the same spacial sampling rate. For a few methods a smaller timestep was used to ensure stability.

For a further application we compare a forward modeled shot using the second-order and fourth-order split-step PSTS equation and the fourth-order in time pseudospectral method. An automatic gain control was applied to both shot records. Both fourth-order methods are of comparable quality. The second-order split-step PSTS method suffers from

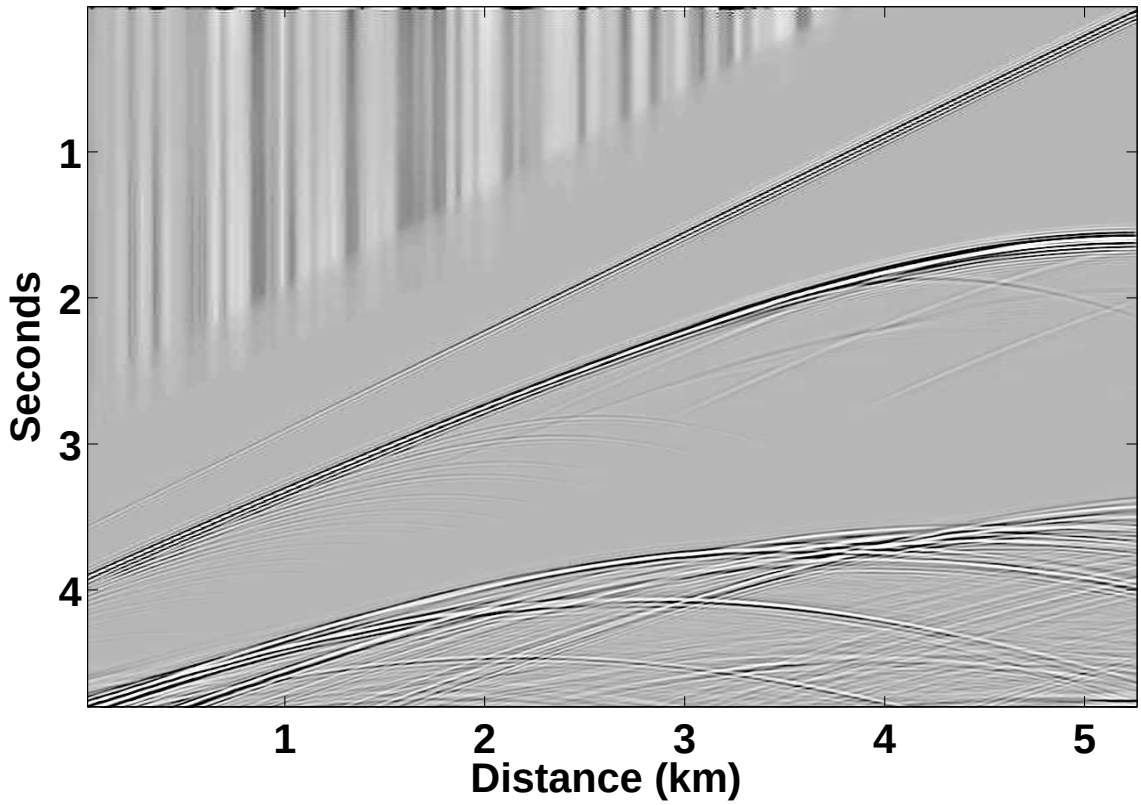

FIG. 4. This shotrecord is generated using a fourth-order pseudospectral method with the BP dataset ignoring density variations.

grid dispersion. This can be reduced by making the timestep smaller. However it is numerically more efficient to use a higher-order method.

CONCLUSION

We presented a new method to approximate equation (16) which solves the acoustic wave equation. It is similar to higher-order in time pseudospectral methods based on the modified equation approach or the Lax-Wendroff method. This approximation scheme can be used for acoustic modeling or reverse time migration. We also presented a windowing scheme where a lower order approximations can be used about multiple reference velocities. However, due to the large number fast Fourier transforms (FFTs) needed to execute the windowing scheme it is much more computationally efficient to use a higher-order method.

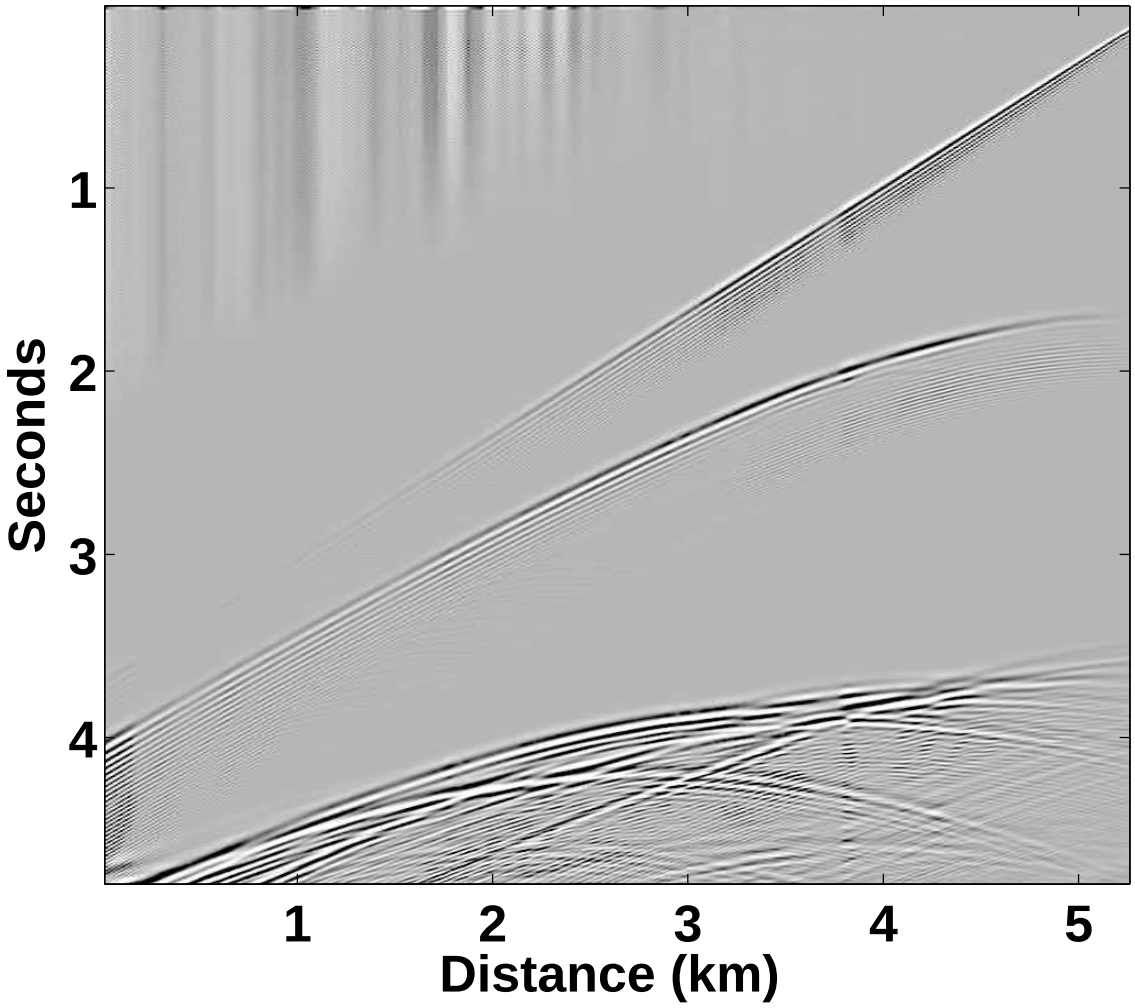

FIG. 5. This shotrecord is generated using the second-order split-step PSTS equation with the BP dataset ignoring density variations.

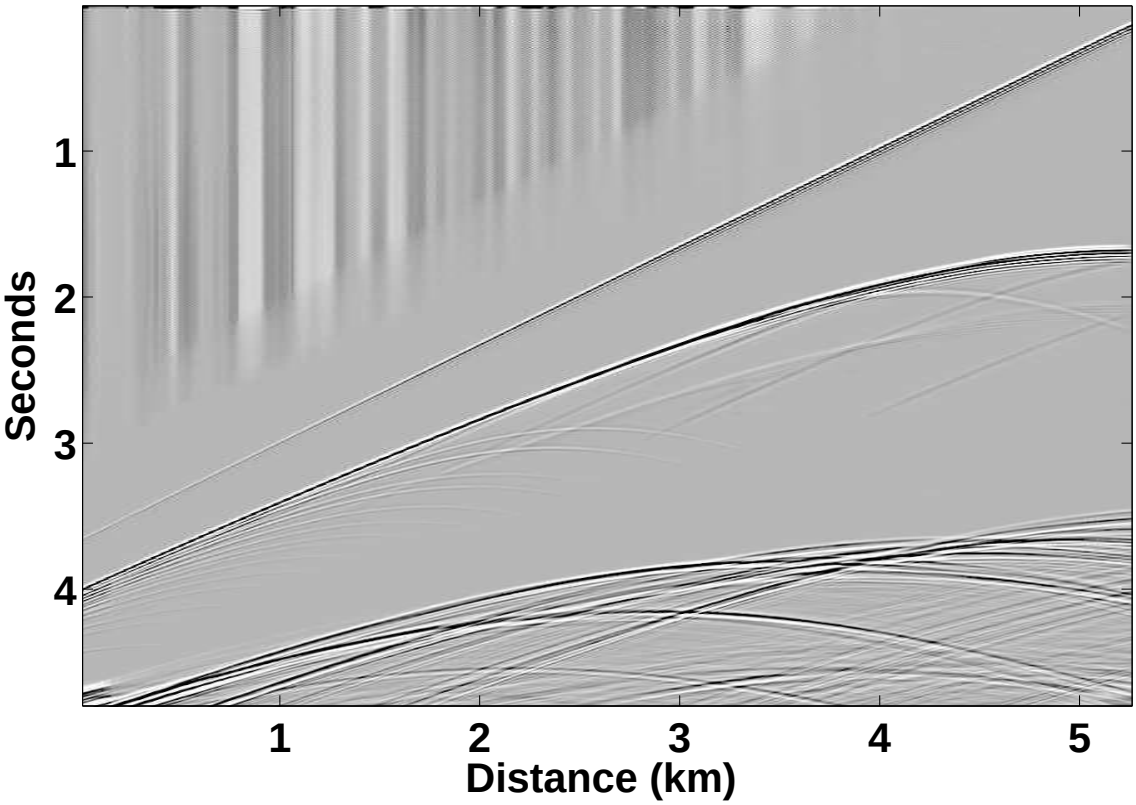

FIG. 6. This shotrecord is generated using the forth-order split-step PSTS equation with the BP dataset ignoring density variations.

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KEY: split

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