Determination of anelastic reflectivity: how to extract seismic AVF information

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ABSTRACT

Strongly dispersive reflection coefficients associated with highly absorptive, hydrocarbon charged targets, have been observed in seismic data. A frequency by frequency method (AVF) for determining Q of a highly absorptive target from measurements of the dispersive reflection coefficient is reviewed in this paper. In order to implement the AVF technique to invert for Q, it is necessary that we have a method of estimating the local spectrum of the reflection coefficient. We develop a method of implementing AVF inversion by using a calibrated, fast, non-redundant S-transform (FST) algorithm to estimate the local spectrum of dispersive reflection coefficients. We test the effectiveness of the FST for estimating the spectrum of reflection coefficients by comparing with the analytic reflection coefficient as calculated by a fast Fourier transform. Using forward modeling to generate synthetic traces for a single absorptive reflection coefficient we observe accurate results of the AVF inversion, using the FST, for a range of Q values.

INTRODUCTION

Absorption is the progressive decay of the highest frequencies of a seismic pulse as it travels through Earth material(O'Doherty and Anstey, 1971). The effect is that the peak amplitude decays and the pulse becomes broader(O'Doherty and Anstey, 1971). The quality factor, Q, is a measure of how absorptive an Earth material is, a low Q value corresponds to a highly absorptive material. The ability to determine Q presents an important problem in exploration seismology because absorption is closely related to rock fluid properties such as viscosity, porosity and fluid saturation(Quan and Harris, 1997; Vasheghani and Lines, 2009). Absorption is not easy to measure in the field as it is difficult to isolate the effect of absorption on the seismic pulse from other attenuation mechanisms (Sheriff and Geldart, 1995) such as geometrical spreading and scattering. However, methods of determining Q exist such as the centroid frequency method which uses the shift in average frequency between two points to determine Q (Quan and Harris, 1997).

It has been observed that strong absorptive reflection coefficients, caused by reservoirs with very low Q, cause frequency dependent seismic anomalies(Odebeatu et al., 2006). In this paper a frequency-by-frequency (AVF) method of inverting for Q from absorptive reflections is reviewed and a method of implementing this technique using a fast S-transform algorithm (FST) is proposed. The method is tested using synthetic traces modeled with a single absorptive reflection. The ability of the FST to estimate the spectrum of the absorptive reflection coefficients is compared with the analytic result as calculated by a fast Fourier transform.

MOTIVATION

If a plane wave is incident upon a planar boundary, oriented perpendicularly to z, and separating two media then the reflection coefficient will be given by

$$R = \frac{k_{z_i} - k_{z_{i+1}}}{k_{z_i} + k_{z_{i+1}}}, \tag{1}$$

where R is the reflection coefficient, k_{z_i} is the vertical wavenumber for the layer above the interface and $k_{z_{i+1}}$ is the vertical wavenumber for the layer below the interface. In order to model absorptive reflection coefficients, we use an expression for wavenumber which includes a model for nearly constant Q described by Aki and Richards (2002) as

$$k = \frac{\omega}{c}\left(1 + \frac{i}{2Q} - \frac{\log(\omega/\omega_r)}{\pi Q}\right). \tag{2}$$

Here c is the seismic velocity, Q is quality factor, ω is the frequency component, and ω_r is a reference frequency. For waves at normal incidence the expression for wavenumber (k) given in (2) can be used as the vertical wavenumber (k_z) and implemented in equation (1). Hence, the equation which models anelastic absorptive reflection coefficients in one dimension, by plugging equation (2) into equation (1) is given by

$$R(\omega) = \frac{\left(\frac{1}{c_i}(1 + \frac{i}{2Q_i} - \frac{\log(\omega/\omega_r)}{\pi Q_i}) - \frac{1}{c_{i+1}}(1 + \frac{i}{2Q_{i+1}} - \frac{\log(\omega/\omega_r)}{\pi Q_{i+1}})\right)}{\left(\frac{1}{c_i}(1 + \frac{i}{2Q_i} - \frac{\log(\omega/\omega_r)}{\pi Q_i}) + \frac{1}{c_{i+1}}(1 + \frac{i}{2Q_{i+1}} - \frac{\log(\omega/\omega_r)}{\pi Q_{i+1}})\right)}, \tag{3}$$

Where the reflection coefficient, $R(\omega)$, is now complex and a function of frequency. Equation (3) is the general form for the normal incidence reflection coefficient in anelastic media. Consider the situation where an overburden which has very high Q is overlaying a target with a small value of Q, then equation (3) becomes

$$R(\omega) = \frac{\left(\frac{1}{c_i} - \frac{1}{c_{i+1}}(1 + \frac{i}{2Q_{i+1}} - \frac{\log(\omega/\omega_r)}{\pi Q_{i+1}})\right)}{\left(\frac{1}{c_i} + \frac{1}{c_{i+1}}(1 + \frac{i}{2Q_{i+1}} - \frac{\log(\omega/\omega_r)}{\pi Q_{i+1}})\right)}, \tag{4}$$

now if we make the following substitutions (from Innanen and Lira, 2008) $F(\omega) = \frac{i}{2} - \frac{1}{\pi}log(\omega/\omega_r)$, $a_C = 1 - \frac{c_i^2}{c_{i+1}^2}$ and $a_Q = \frac{1}{Q_{i+1}}$. We can expand equation (4), and linearize (assuming small a_Q) to obtain the following expression

$$R(\omega) = -\frac{1}{2}a_Q F(\omega) + \frac{1}{4}a_C, \tag{5}$$

where a_Q and a_C are the perturbation parameters in Q and acoustic seismic velocity respectively. It is also worth noting that $F(\omega)$ is a known function. Equation (5) defines the forward problem of calculating $R(\omega)$ given a_C and a_Q. The inverse problem, referred to as AVF, is to determine a_Q or a_C from measurements of $R(\omega)$. In this paper, we wish to obtain reliable measurements of $R(\omega)$ in order to determine a_Q. Equation (5) contains two unknowns, a_C and a_Q. If we can determine $R(\omega)$ for two different frequencies, ω_1 and ω_2 , then we can take the difference between $R(\omega_1)$ and $R(\omega_2)$ and obtain an expression for a_Q given by

$$a_Q = -2\left(\frac{R(\omega_2) - R(\omega_1)}{F(\omega_2) - F(\omega_1)}\right). \tag{6}$$

PROBLEM

Equation (6) provides an expression for inverting for a_Q provided we have an estimate of $R(\omega)$ for at least two different frequencies. The problem is how to determine $R(\omega)$ from recorded seismic data. It is necessary that we be able to estimate the local spectra of seismic events. We have a calibrated fast S-transform algorithm (FST) which our testing indicates offers high fidelity estimates of local spectra (see Bird et al., 2010).

In this paper, we define $\tilde{R}(\omega)$ as the estimate of the spectrum of the true reflection coefficient, $R(\omega)$, which we obtain using the calibrated FST algorithm. We use forward modeling codes to generate synthetic traces with a single absorptive reflection and then implement the FST to estimate the local spectrum of the absorptive reflection. Finally, we invert for a_Q using a slightly modified version of equation (6). For synthetic traces with a single reflection, the true spectrum of the reflection coefficient, $R(\omega)$ may be obtained by using a fast Fourier transform (FFT). For our synthetic traces with a single absorptive reflection, we compare the spectrum of the reflection coefficient as estimated by the FST ($\tilde{R}(\omega)$), with the true spectrum as calculated by an FFT.

METHOD

We start by generating a synthetic trace with a single absorptive reflection coefficient (see Figure 1). Figure 1(a) is a plot of the synthetic trace, modeled using a plane wave incident upon a planar boundary separating an acoustic medium overlaying a medium with very low Q. Figure 1(b) is a plot of the synthetic trace in the S-domain. The black line in Figure 1(c) is the local spectrum of the reflection ($\tilde{R}(\omega)$) which is obtained by extracting the profile of the reflection through the S-domain. The blue line in Figure 1(c) is a plot of the true spectrum, $R(\omega)$, calculated directly from equation (3). Note that $\tilde{R}(\omega)$ in Figure 1(c) is not a continuous spectrum; it has a blocky appearance due to the tiling associated with the FST algorithm (see Bird et al., 2010). Notice that the spectrum estimated by the FST is a very good approximation to the average of the true spectrum across the frequency band of each "tile".

Consider the $\tilde{R}$ value in Figure 1(c) associated with $f = 31Hz$ to $f = 62Hz$. It is an average of the true spectrum over that frequency range. We can relate the $\tilde{R}$ values to the $R(\omega)$ values by the equation

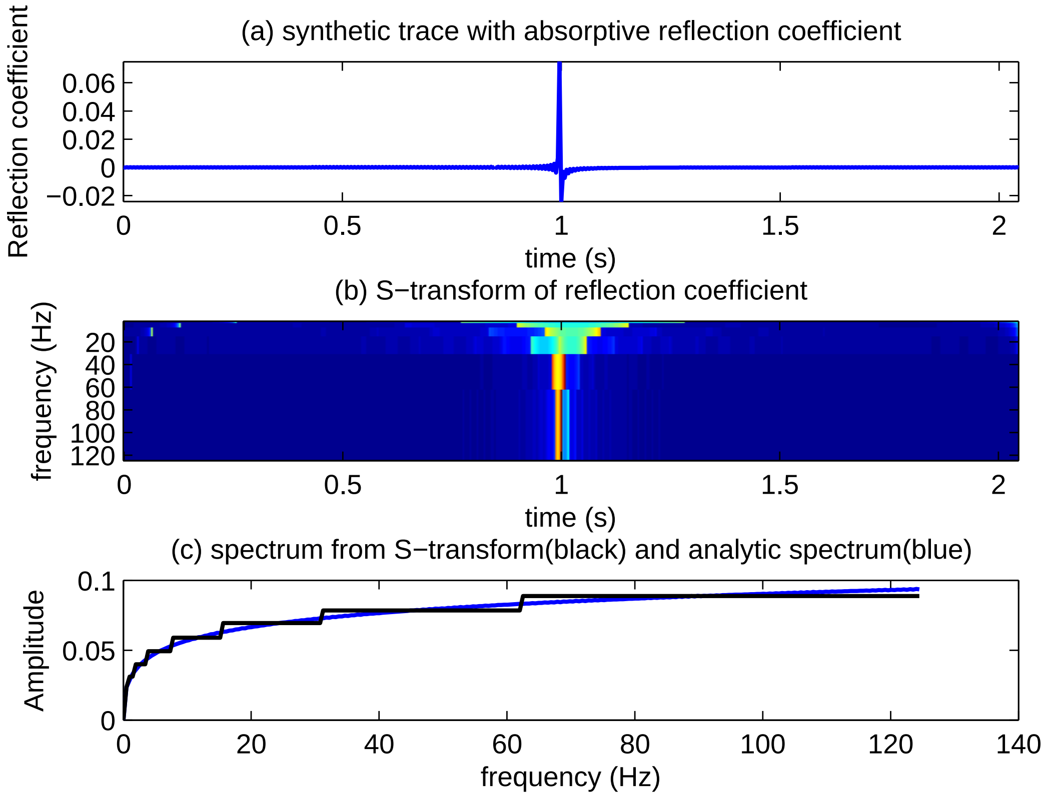

FIG. 1. How to extract a frequency-dependent reflection coefficient from a seismic trace. In (a) the single primary reflection generated by a contrast from acoustic to highly attenuative media is plotted in the time domain; in (b) the calibrated fast S-transform is carried out on the trace in (a), identifying the location of the event and estimating it's spectrum; in (c) the amplitudes picked from the S-transform(in black) is compared with the analytic reflection coefficient(in blue).

$$\tilde{R}_{M1} = \frac{\sum_{i=M1s}^{M1e} R(\omega_i)}{(M1e - M1s)}, \tag{7}$$

where $\tilde{R}_{M1}$ is the average of $R(\omega)$ from frequency sample $M1s$ to $M1e$. We now define $\tilde{F}_{M1}$ as the sum of the known $F(\omega)$ function over the same frequency samples

$$\tilde{F}_{M1} = \sum_{i=M1s}^{M1e} F(\omega_i), \tag{8}$$

Now consider $\tilde{R}_{M2}$ which is the average of $R(\omega)$ from frequency samples $M2s$ to $M2e$ and $\tilde{F}_{M2}$ which is the sum of the known $F(\omega)$ function over the same frequency samples as $\tilde{R}_{M2}$. Now, let $M_1 = (M1e - M1s)$ and $M_2 = (M2e - M2s)$ and define Z such that $Z \cdot M_2 = M_1$. Then we can rewrite equation (6) in terms of $\tilde{R}_{M1}$, $\tilde{R}_{M2}$, $\tilde{F}_{M1}$, and $\tilde{F}_{M2}$

$$a_Q = -2\left(\frac{M_1\tilde{R}_{M1} - ZM_2\tilde{R}_{M2}}{\tilde{F}_{M1} - Z\tilde{F}_{M2}}\right). \tag{9}$$

With equation (9), we may invert for a_Q after using the FST to estimate the local spectrum of absorptive reflection coefficients.

RESULTS

We implement equation (9) to invert for Q by generating synthetic seismic traces with a single absorptive reflection, and using the FST to estimate the spectrum of the reflection. The traces are generated for a two-layer, single interface model in which Q is very high above the interface and very low below the interface. We use the FST to estimate the spectrum of the reflection and then implement equation (9) to invert for Q. A number of traces were modeled in which Q below the interface ranged from 200 to 1, the inversion was performed and compared with the actual value for accuracy. This is shown in Figure's 2 through 4, where the red line is the inverted Q value and the black line is the actual Q value used to model the reflection. From these figures, it appears that the inversion works best for Q values ranging from about 15 to 50 but works satisfactorily until Q drops below 8. This is highlighted in table 1, which shows the difference between the inverted and actual Q values.

As can be seen from Figures 2 through 4 and Table 1 the inversion works satisfactorily until Q drops below 8 and then the difference between the inverted Q value and the actual value becomes quite large. The failure of the inversion at very low Q is not due to an inability of the FST to estimate the spectrum of the reflection, rather it is because we made the assumption that a_Q was small in order to linearize equation (4). Thus, when Q is very low, the assumption of a small a_Q is not satisfied and the linearized expression for a_Q is no longer valid.

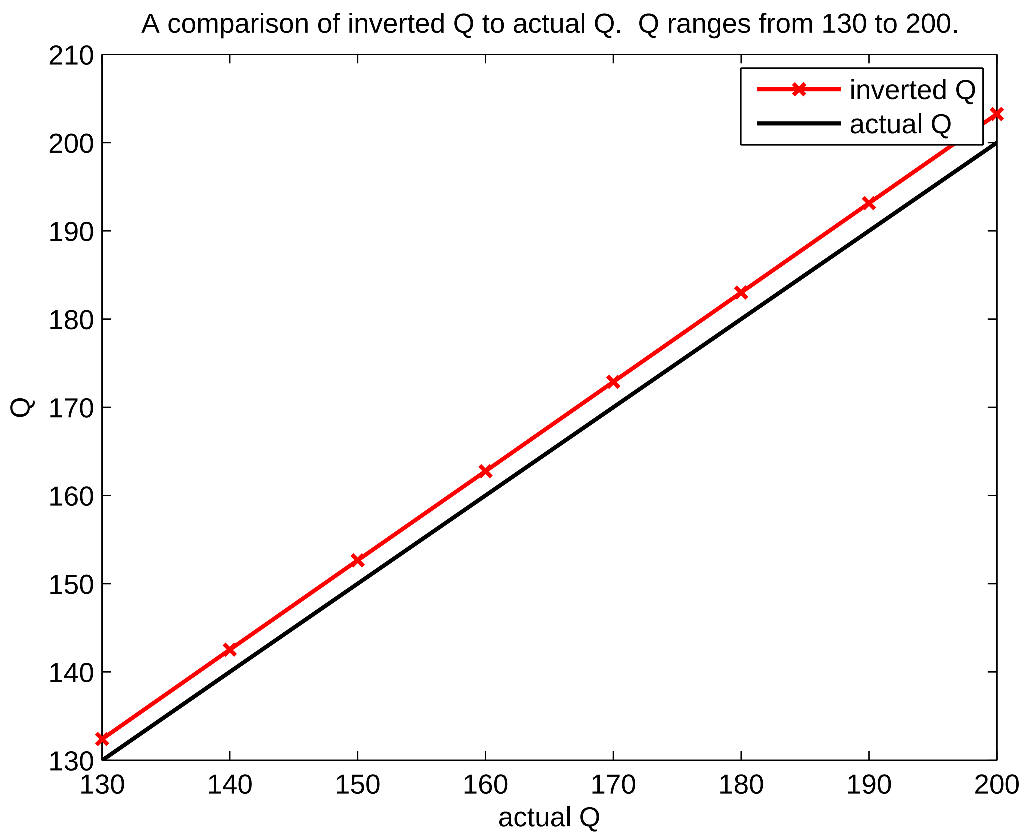

FIG. 2. Determination of Q using linear AVF inversion. The value of Q used to model the single primary reflections is plotted in black and the value of Q determined by the linear AVF inversion is plotted in red.

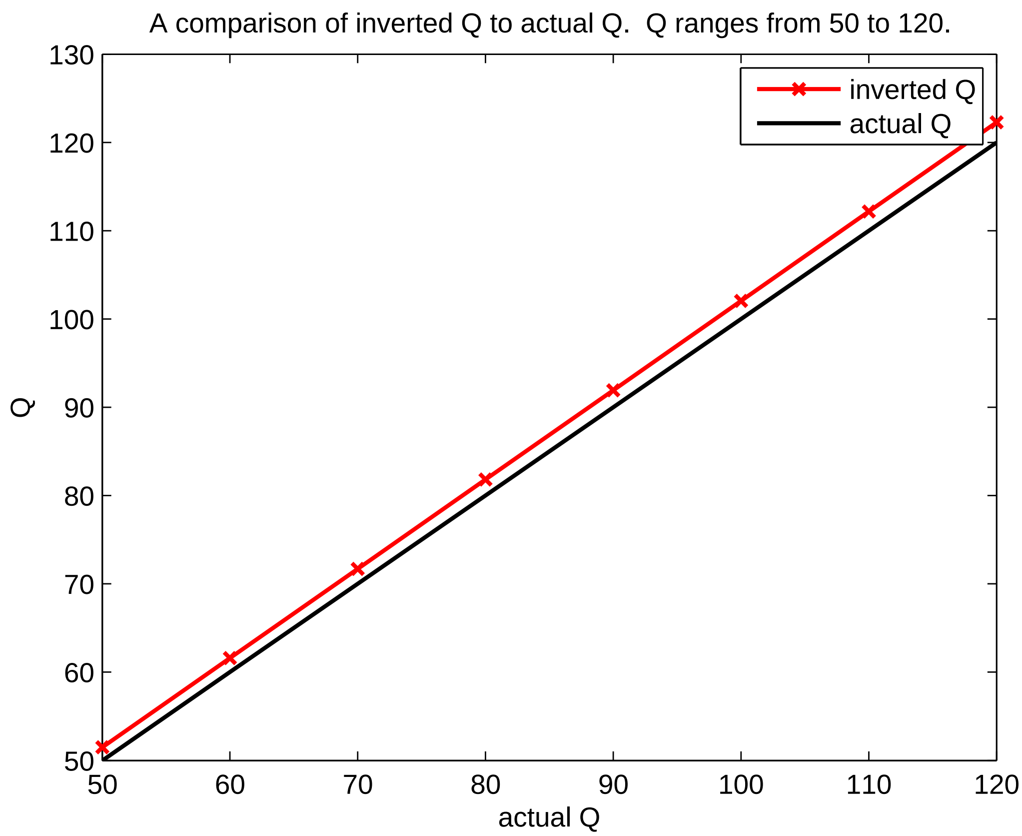

FIG. 3. Determination of Q using linear AVF inversion. The value of Q used to model the single primary reflections is plotted in black and the value of Q determined by the linear AVF inversion is plotted in red.

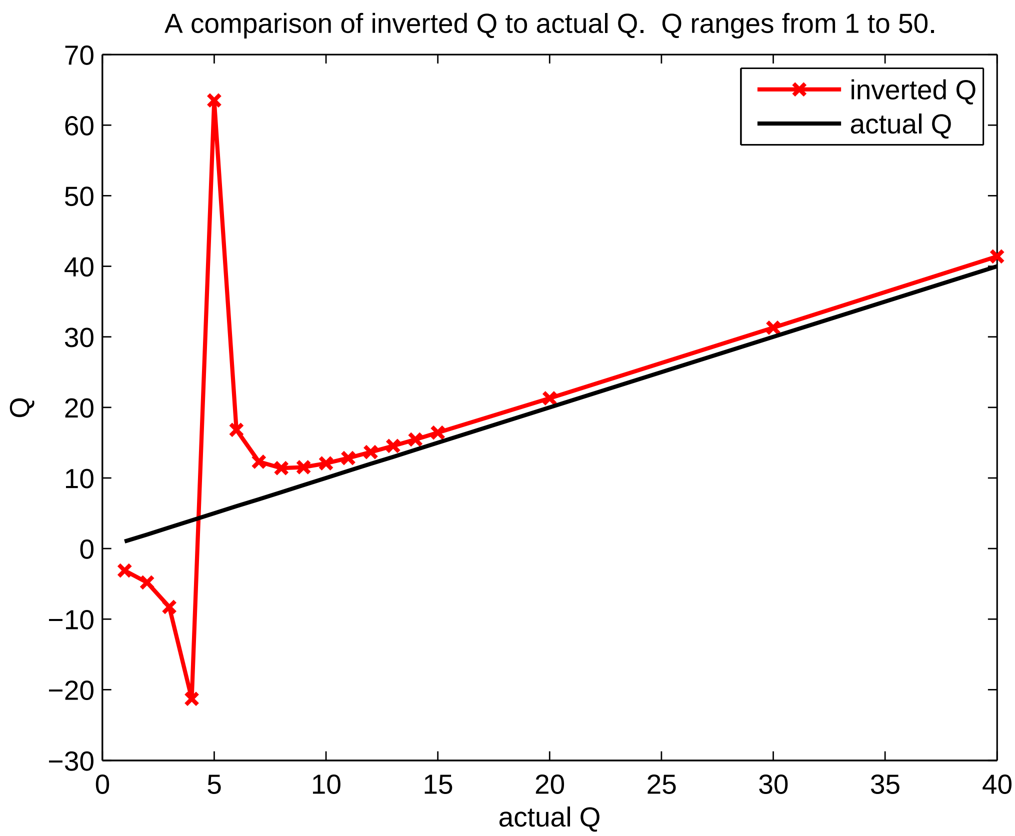

FIG. 4. Determination of Q using linear AVF inversion. The value of Q used to model the single primary reflections is plotted in black and the value of Q determined by the linear AVF inversion is plotted in red.

Table 1. Comparison of inverted Q value to actual Q and the misfit

Actual Q	Inverted Q	difference(absolute value)
50	51.4	1.48
40	41.3	1.38
30	31.3	1.30
20	21.2	1.29
15	16.4	1.41
14	15.4	1.47
13	14.5	1.54
12	13.6	1.65
11	12.8	1.82
10	12.0	2.09
9	11.5	2.54
8	11.4	3.4
7	12.3	5.3
6	16.8	10.82
5	63.4	58.47

CONCLUSION

We conclude that the calibrated FST is a promising time-frequency decomposition tool which may be used to extract AVF information. The FST was shown to produce a very good approximation to the average of the spectrum of the analytic reflection coefficients. For a situation where an acoustic overburden overlays a highly attenuative target, which has been observed for some gas charged reservoirs(Odebeatu et al., 2006), an expression for inverting for Q using the FST to estimate the spectrum of the reflection was presented. Synthetic traces for a single absorptive reflection were generated and Q was inverted for and it was shown that the inverted values were in close agreement with the true values, except for very low $Q(Q < 8)$ when the assumptions made in order to linearize equation (4) were no longer satisfied.

Moving forward we will extend this method of extracting AVF information to traces with numerous reflection events. In another paper we discussed the ability of the FST to estimate the spectrum of local events as a function of proximity to other events (Bird et al., 2010). The more proximal events are to each other, the greater the frequency range that their individual spectra interfere with each other. Therefore, it is important that we only use values of $\tilde{R}$ which fall outside this zone of interference when performing AVF inversion.

We will also extend our method to non-normal incidence reflections, reflections modeled using a source wavelet, and finally we will attempt our method on field data. Further, we can apply a non-linear correction to equation (5) and improve the accuracy of the inversion.

REFERENCES

Aki, K., and Richards, P. G., 2002, Quantitative Seismology: University Science Books, 2nd edn.

Bird, C., Naghizadeh, M., and Innanen, K., 2010, Amplitude calibration of a fast S-transform: CREWES Sponsor's Meeting 2010, , No. 4, 1–12.

Innanen, K., and Lira, J. E., 2008, Direct non-linear Q-compensation of primaries in layered media: theory and synthetic examples: SEG Las Vegas 2008 Annual Meeting, 2957–2962.

Odebeatu, E., Zhang, J., Chapman, M., and Li, X.-Y., 2006, Application of spectral decomposition to detection of dispersion anomalies associated with gas saturation: The Leading Edge, **February**, 206–210.

O'Doherty, R. F., and Anstey, N. A., 1971, Reflections on amplitudes: Geophysical Prospecting, **19**, 430–458.

Quan, Y., and Harris, J. M., 1997, Seismic tomography using the frequency shift method: Geophysics, **62**, No. 3, 895–905.

Sheriff, R., and Geldart, L., 1995, Exploration Seismology: Cambridge University Press, 2nd edn.

Vasheghani, F., and Lines, L. R., 2009, Viscosity and Q in heavy-oil reservoir characterization: The Leading Edge, **July**, 856–860.