Deconvolution after migration

John C. Bancroft, Thais Guirigay, and Helen Isaac

ABSTRACT

Deconvolution is a process that is normally applied before migration. However, there are data conditions where deconvolution should be performed after migration. These situations are reviewed with a final conclusion that deconvolution after migration should be applied and be tested, especially in areas with little structure, such as a sedimentary basin.

INTRODUCTION

Review of frequency changes with dip angle

Consider a geophysical setting where a wavelet is stationary and assumed to be defined by the source, and the raypaths are normal to the reflectors. The reflection energy is plotted below the source/receiver location to form a seismic section, even when coming from a dipping reflector as illustrated in Figure 1a. The reflection events on the section (b) will contain the same wavelet, independent of the dip and will have the same frequency content. After migration, these wavelets are "rotated" back to the geological location. It is apparent that deconvolution, to improve resolution, should be applied to the wavelets while they are in the time section.
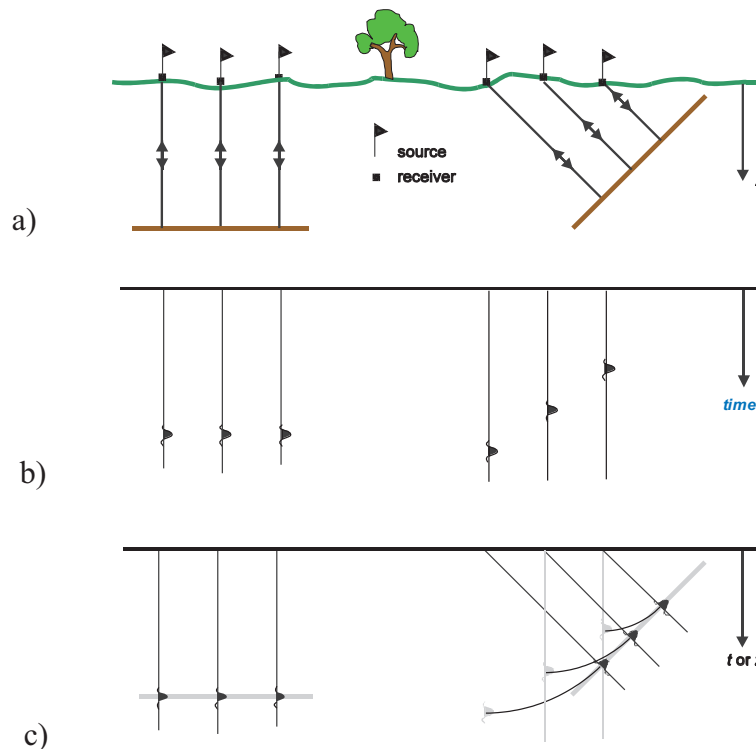

FIG. 1. Cartoon of a) raypaths, b) recorded traveltimes and wavelets, and c) wavelets migrated back to the reflector.

The accompanying cartoon figure below shows numerical data of two events, one horizontal and the other dipping, before and after migration. Before migration, the wavelets in Figure 2a are all the same, independent of the dip of the reflector, as inferred by Figure 1. After migration in (b), the wavelets are rotated as indicated by the red arrows, but the actual wavelet on the traces of the dipping events are stretched or have a lower frequency. The gray and black circles identify the distance between the wavelet minimums. In part (a) the black circles represent the width or resolution of the wavelets on the vertical traces and are all the same, while the gray circle shows the resolution normal to the dipping event, which appears to have a higher resolution. In part (b), the black dots represent the resolution of the events when measured normal to the reflectors and the gray circle the wavelets on the vertical traces showing that the wavelet has been stretched in time. The lower frequency is a natural part of the migration process that prevents aliasing of the dipping event, but does preserve the resolution of the event when measured in the normal direction to the event.
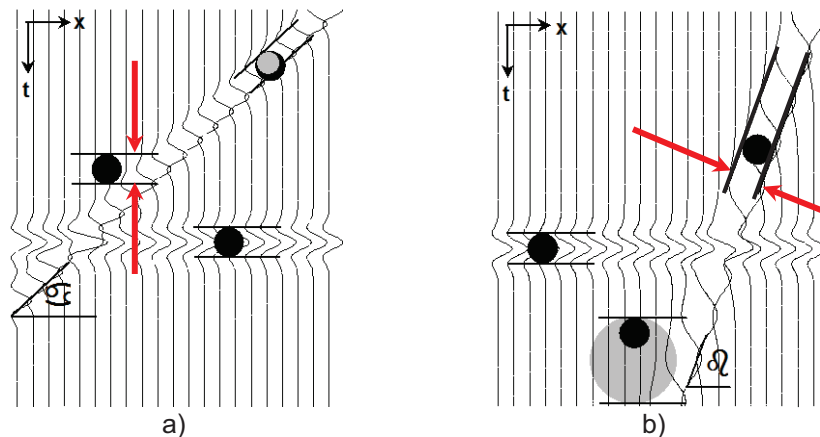

FIG. 2 Numerical seismic event a) before migration and b) after migration..

It appears that deconvolution after migration would increase the frequency of the dipping wavelet and cause it to be aliased. A steep event on the flank of a reflector may not be resolved correctly. Hence, the cry became…

<center>"don't deconvolve after migration."</center>

That is a basic reason for not applying deconvolution after migration. There are other reasons, one that I particularly like was that migration algorithm that formed and image with a cut and paste method. The traces were filtered to smooth the intersection areas so they would not be visible. However, deconvolution restored the intersection areas, so deconvolution after migration was banned.

AN ALTERNATE VIEW OF MIGRATION

Migration is a transpose process. I illustrate this concept by reviewing modelling and migration using diffractions and semi-circles.

When modelling, the "forward process" spreads diffractions on the seismic sections with amplitudes that are proportional to the reflectivity. When migrating, the "reverse

process" spreads energy along semi-circles, with amplitudes that are proportional to the seismic data. It can be shown that the transpose of a "diffraction matrix" is a "semi-circular matrix". The processes use the impulse responses and may be referred to a matched filtering or a transpose process. In contrast, true inversion is not a transpose process. Simplistic examples of the diffraction matrix and its transpose are shown in an appendix.

I will illustrate true inversion using a pseudo form of linear algebra. I will use matrices to define a 4D family of diffractions, a 4D family of semi-circles, a 2D reflectivity structure, and a 2D seismic section. However, the mathematics for least square inversion requires only one 2D matrix and two vectors. Claerbout has shown that we can convert the 4D matrix into a 2D matrix and the 2D section into a single vector by concatenating traces. Using the linear algebra should then be OK.

Consider the forward process of modelling as

$$\mathbf{Dr} = \mathbf{s}, \tag{1}$$

where $\mathbf{D}$ is a diffraction matrix, $\mathbf{r}$ a reflectivity structure and $\mathbf{s}$ the modelled seismic section. A true inversion would be

$$\mathbf{r} = \mathbf{D^{-1}s}, \tag{2}$$

providing D is invertable. It is usually not invertible so we make use of the least square method to get an estimate of the reflectivity from

$$\breve{\mathbf{r}} = \left(\mathbf{D^T D}\right)^{-1} \mathbf{D^T s}. \tag{3}$$

We become comfortable by assuming the $\mathbf{D^T D}$ matrix is diagonally dominant, and approximate it with the identity matrix $\mathbf{I}$, which has the convenient inverse that equals $\mathbf{I}$, i.e.,

$$\left(\mathbf{D^T D}\right)^{-1} \approx \left(\mathbf{I}\right)^{-1} = \mathbf{I}. \tag{4}$$

This allows us to write an alternate estimate for the reflectivity

$$\tilde{\mathbf{r}} = \mathbf{D^T s}, \tag{5}$$

where our "inversion" process has been approximated by a transpose process that we call migration. (Claerbout (1992) described a long time ago that many processes in exploration geophysics that we think they are inverse processes are really transpose processes.)

What have we lost by dropping the $\mathbf{D^T D}$ part of the inversion? I contend that this is an essential part of our processing and will demonstrate this point by using least squares migration (LSM). But let me back up and use more detail in out modelling process.

I conveniently left out any mention of a wavelet in out modelling. This is quite normal as in out modelling and migration we use single valued diffractions and semi-

circles as it is more convenient, faster, and does a reasonable job. The wavelet was assumed to be part of the reflectivity structure.

Let us assume that we have a wavelet matrix $\mathbf{W}$ that can be multiplied with the diffraction matrix $\mathbf{D}$ to put wavelets on the diffractions, i.e.,

$$\mathbf{WGr} = \mathbf{s} . \qquad (6)$$

Now our reflectivity matrix $\mathbf{r}$ can really be a high frequency representation of the true reflectivity and we have the wavelet with the diffraction as it should be. The least squares solution is now

$$\breve{\mathbf{r}} = \left(\mathbf{D}^T\mathbf{W}^T\mathbf{WD}\right)^{-1}\mathbf{D}^T\mathbf{W}^T\mathbf{s} . \qquad (7)$$

Removing the inversion part and going back to the transpose solution we get

$$\tilde{\mathbf{r}} = \mathbf{D}^T\mathbf{W}^T\mathbf{s} . \qquad (8)$$

This is quite interesting as we are now implying that we need to "convolve or correlate" the seismic data with another wavelet (actually it is the same wavelet) and that sounds like lowering the frequency content again. That is correct, and if we did do that, we would end up with a zero-phase wavelet, typical of a true matched filter. If we ignore the wavelet matrix, as in equation (5) then we do have a higher frequency migration with the same wavelet (though we still have an aliasing problem). "But wait, there is more". We left out the inversion part again. Let me rewrite equation (7) again but in a form for some kind of dimensional analysis, i.e.,

$$\mathbf{r} = \frac{D^T W^T s}{D^T W^T W D} = \frac{s}{WD} \approx \frac{D^T s}{W} , \qquad (9)$$

where, on the left, we have two wavelets in the numerator and denominator. We end up with our conventional migration $\mathbf{D}^T\mathbf{s}$ that still requires some inverse action with the wavelets, i.e. deconvolution to recover the reflectivity. I am trying to show that a full inversion also applies an inversion (or deconvolution) of the seismic wavelet in $\mathbf{s}$

I illustrate with this concept with a true inversion that uses a full least squares migration (LSM). The full LSM is extremely computationally intensive and usually only simple models are shown. Figure 3 shows a reflectivity structure, a migration from seismic modelled on the structure, and a corresponding least LSM. Notice the wavelet remain with the migration, but has been substantially removed in the LSM.

These results look and are impressive, but the modelling and inversion process did not contain noise, which enabled the high frequency content of the wavelet to accurately reconstruct the reflectivity. I contend that the same resolution in Figure 3c could have been achieved with a deconvolution to the migrated section in Figure 3b.

I am not negating the power of LSM as it has many useful applications, especially in its ability to recover missing data, but only to justify using deconvolution after migration.
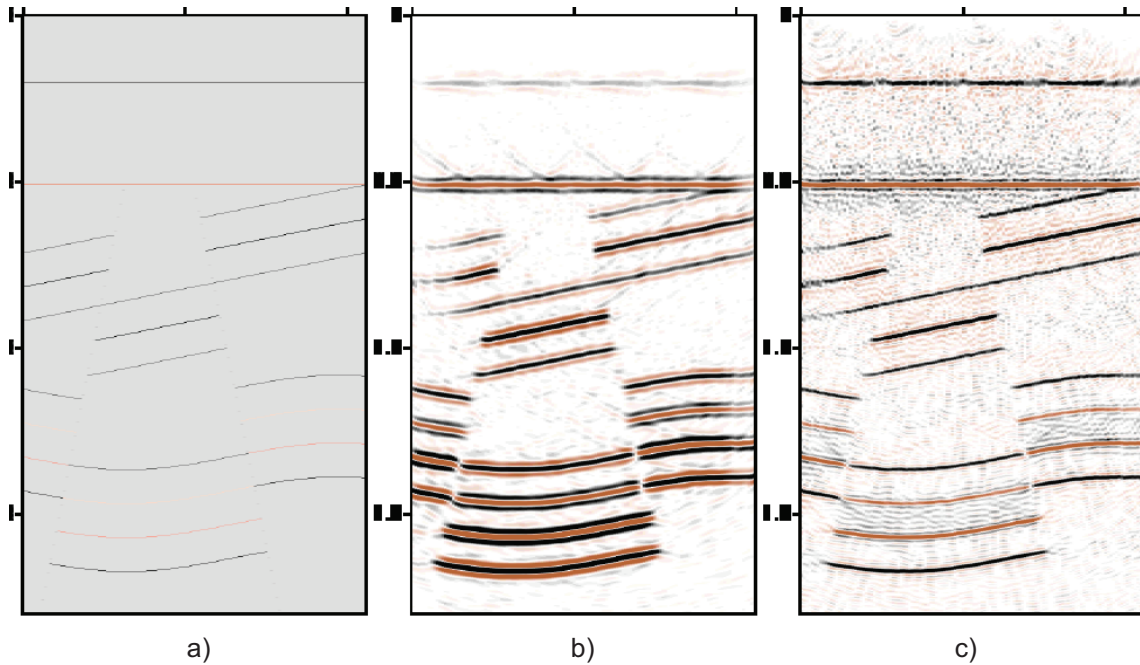
FIG. 3 Illustration of a) a reflectivity structure, b) a migration, and c) a least squares migration.

A SECOND ARGUMENT FOR DECONVOLUTION AFTER MIGRATION

Deconvolution essentially tries to flatten the amplitude spectrum. However, when we flatten the spectrum we flatten the signal and the noise. We generally consider the bandwidth of the signal, or reflection energy, to be the area on an amplitude spectrum where the signal is greater than the noise, i.e., when the signal to noise ratio (SNR) is greater than one, i.e., SNR > 1. This is illustrated in Figure 4 which contains an exaggerated cartoon sketch of the amplitude spectrum of seismic data and three levels of noise. The first noise level represents the noise level in the raw data, the second is the reduced noise after stacking, and the third is the noise level after migration. Each time we reduce noise, we increase the frequency where the signal to noise ratio is greater than one, i.e., from F_r to F_s and then F_m. Bandpass filters remove energy when the SNR < 1.

Migration *should* reduce noise and increases the high frequency content that is above a SNR of one. I use the word *should* ,because some migrations retain noise:
1. deliberately for appearance purposes,
2. the algorithm can't remove the noise, or
3. because an antialiasing filter was not used.
In these cases a deconvolution after migration may not improve the result.

Dips on a seismic section (before migration) are limited to less than 45 degrees. Dipping energy above 45 degrees is noise, and should be removed by the migration algorithm. This is done by the antialiasing filter in a Kirchhoff migration, and a natural part of the frequency domain algorithms. However some algorithms, such as finite difference, cannot remove the noise and it may be spread over the entire section. In these latter cases, a dip filter should be applied to the seismic data to remove noise above dips of 45 degrees.
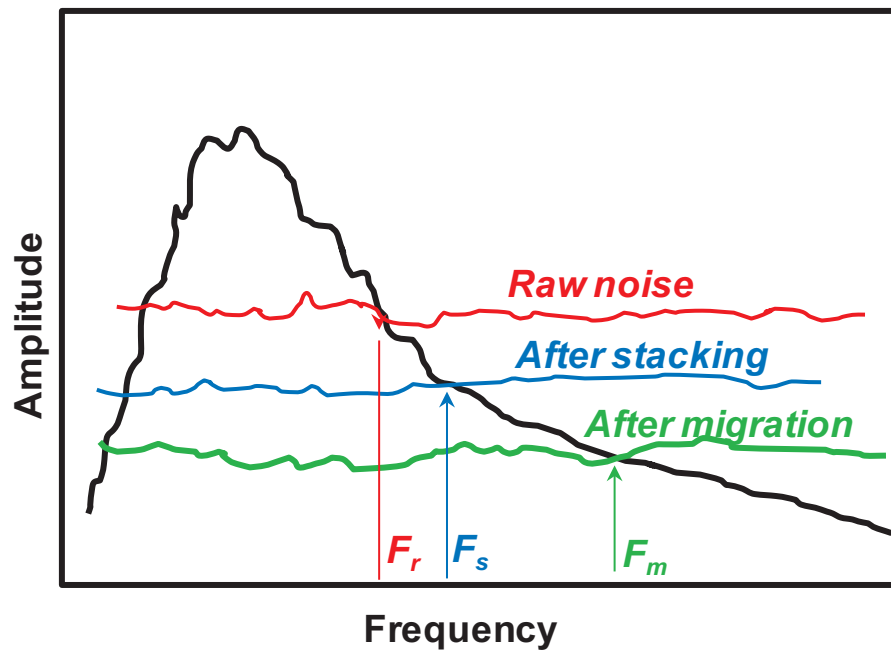
FIG. 4 Cartoon illustrating the increase bandwidth as the noise level is reduce after stacking and then after migration.
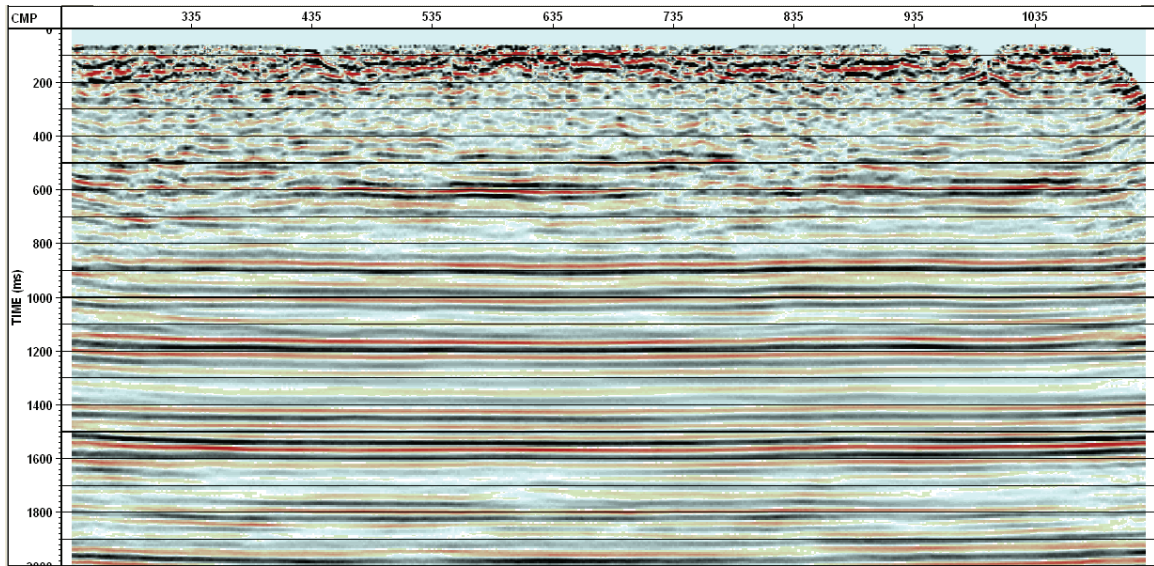
AN EXAMPLE OF DECONVOLUTION AFTER MIGRATION

A noisy 2D seismic line was chosen from the Hussar project that used a low-dwell sweep from 1 to 100 Hz, into the vertical component of a 3C phone. The data was processed to a flat datum at the central elevation. Deconvolution, gain recovery, and statics were applied to the data prestack data. The data was then prestack time migrated using the Equivalent Offset method (EOM). The migrated section is shown in Figure 5a and the migration followed by a spiking deconvolution is shown in Figure 5b. This data had a 10-15-45-60 bandpass filter applied before EOM to simulate low frequency data. The increase in resolution with deconvolution after migration is evident.

The amplitude spectra of the Hussar data are compared in Figure 6, with (a) the prestack migration, and (b) after deconvolution. Note the flatter amplitudes of the deconvolved section in the energy band of the seismic data that was constrained by the filter. The sharp cut off of the high frequencies of the seismic data in Figure 6a and the flattening of the amplitude spectrum to this frequency in (b) suggests that the source frequency was set too low.

COMMENT 1

A third section is missing in the above comparisons. Missing is the post stack migration which would have contained deconvolution to the stacked section before migration. But that is not a desirable product and we now recognize that a prestack migration is superior to a post stack migration. That however is the case for all prestack migrations and further emphasizes the need for a deconvolution after migration.

a)
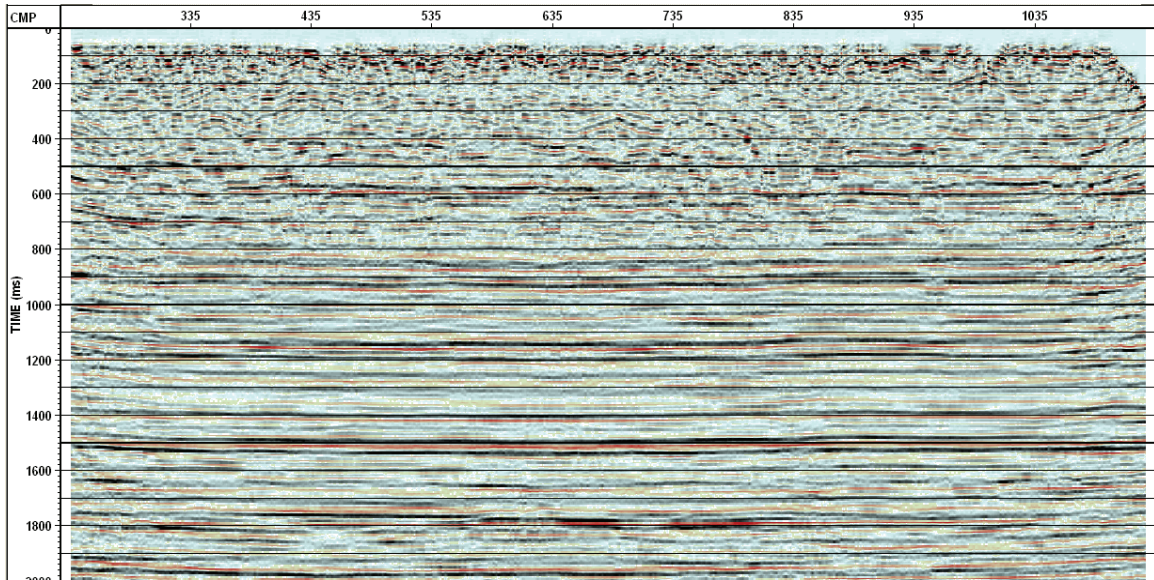
b)

FIG. 5 Seismic sections: a) a prestack migrated section and b) deconvolution to (a).

COMMENT 2

What has happened to the argument that a deconvolution after migration will harm the lower frequencies that define steeply dipping events? I believe that it depends on how we do the deconvolution.
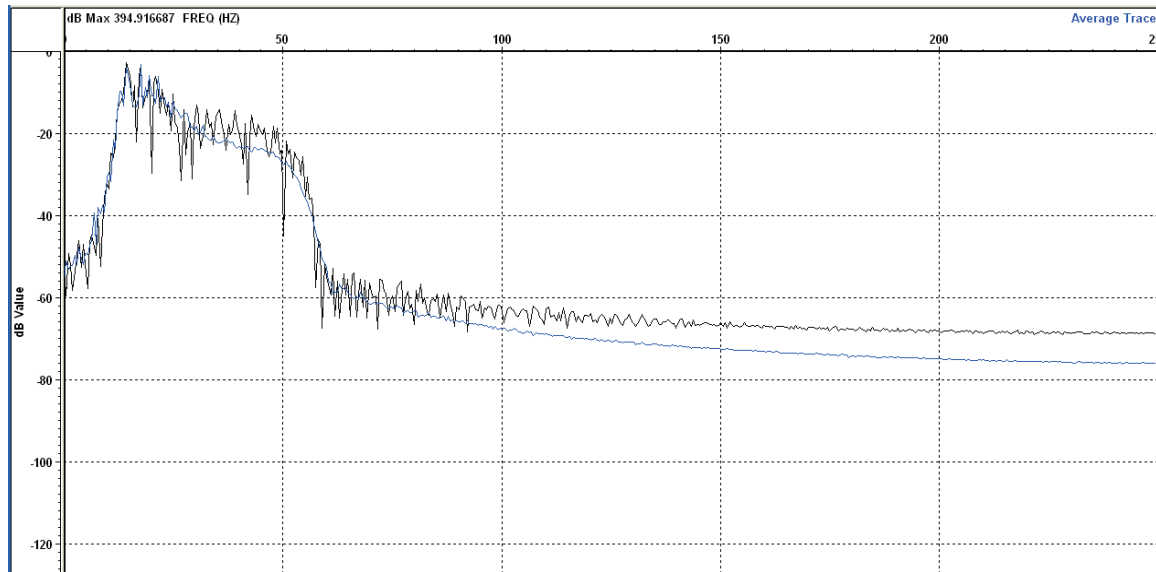
a)
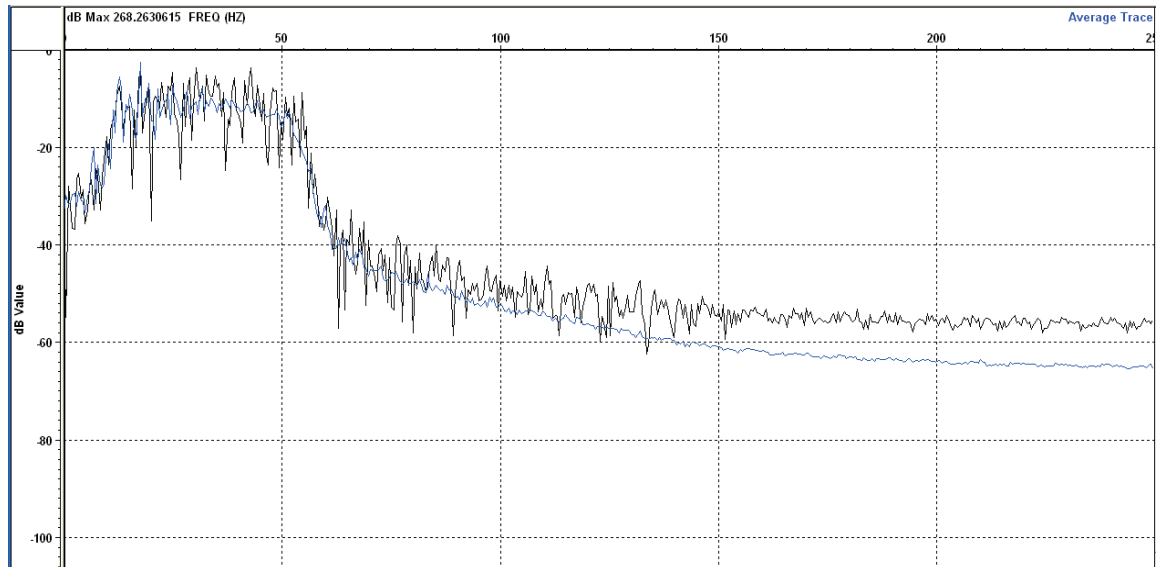
b)

FIG. 6 Amplitude spectra of a) the prestack migrated data, and b) the migrated data with deconvolution.

If the data is from a sedimentary basin where the structure is basically horizontal, then a trace deconvolution is reasonable. But remember that horizontal events include amplitude variations that do contain dipping energy.

Consider Figure 7 that shows FK spectra of seismic data before and after migration for data with very steep dips. The origin is centered at the bottom. On the left in (a) we see the seismic data contained in a blue triangle that is bound by the "maximum frequency" and the maximum dip of 45 degrees. The trace interval for gathering seismic data is based on the maximum usable frequency, so the spatial Nyquist wavenumber is equal to the maximum frequency, assuming a normalized velocity of one. After migration in (b), the energy moves down to a maximum dip of 90 degrees, with the maximum frequency following a semi-circle, as illustrated in Figure 7b.
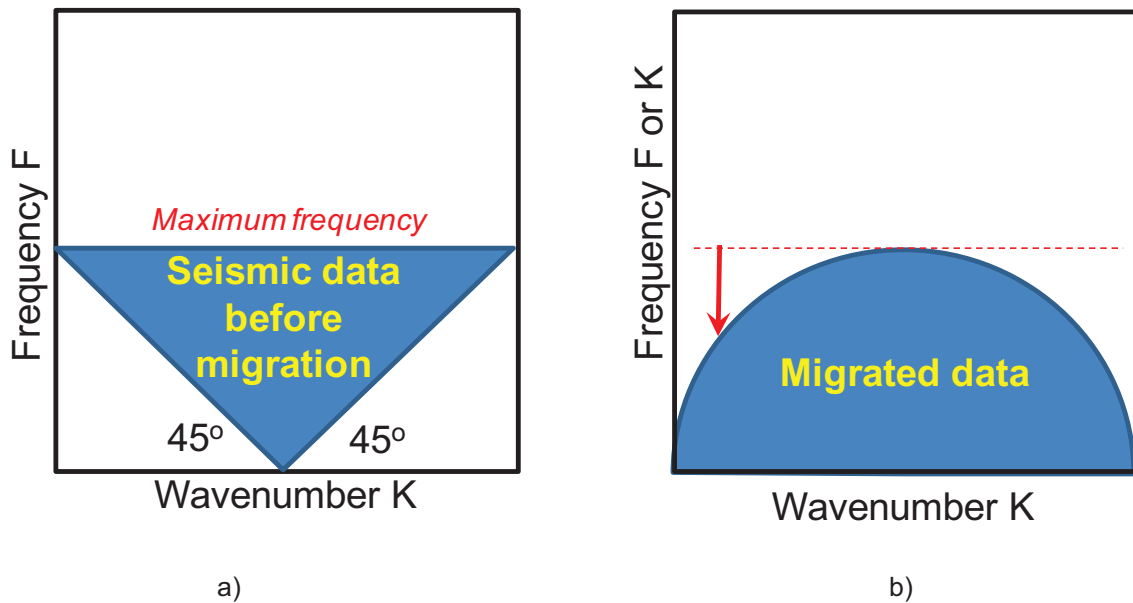
FIG. 7 FK displays of seismic data a) before migration, and b) after migration.

Structured data

Now consider Figure 8 that contains the same information as in Figure 7, but now includes two lines that identify the SNR before migration and after migration. These lines may not have the shape or location as indicated. Processing and deconvolution moves the location of the *Prestack SNR = 1* to *Migrated SNR = 1*.

We may need to re-evaluate the value of the maximum frequency especially if we can recover greater bandwidths with a prestack migration followed by deconvolution. This in turn may reduce the trace interval used during acquisition.

There may be a need to develop a 2D deconvolution that is applied after a prestack migration for highly structured data.

An aggressive deconvolution may be applied to the data before prestack migration, with a high cut filter to allow the processing within the migration to lower the SNR. However, the shape of the spectral shaping operator will still be biased by the spectral envelope of the noise.
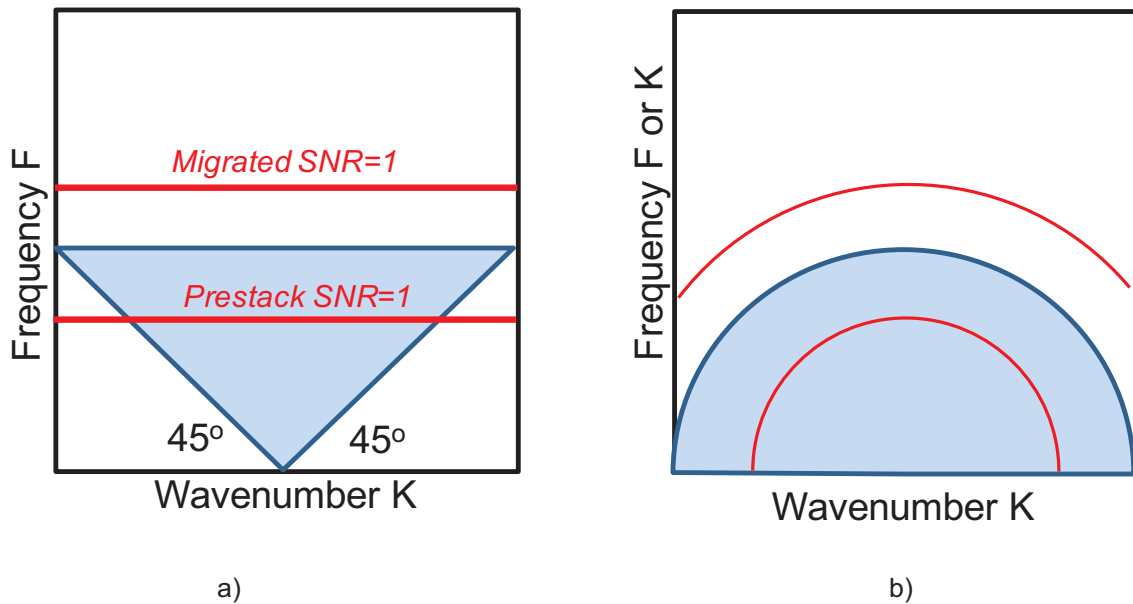
FIG. 8 FK displays of seismic data a) before migration, and b) after migration, illustrating changes in frequency content due to changing signal to noise levels.

CONCLUSIONS

The value of applying deconvolution after a prestack migration is presented and discussed. Deconvolution should be applied based on the least squares migration concept, and on the spectral envelope of signal plus noise.

Some migration algorithms attenuate noise, others do not. An FK filter for dips beyond 45 degrees of dip should be used.

Data before a prestack migration do not have the advantage of a poststack migration that has deconvolution applied to the stacked section before migration. Deconvolution after a prestack migration becomes even more important.

The higher frequencies obtained with deconvolution after migration may require re-evaluation of the trace interval used in acquiring seismic data.

A special 2D deconvolution may need to be developed for highly structured data 2D data.

ACKNOWLEDGMENTS

REFERENCES

Claerbout, J. F., 1992, Earth Sounding Analysis: Processing versus Inversion, Blackwell

APPENDIX 1

The diffraction matrix in 4D form $\mathbf{G}_{4D}$ is shown below in Figure A1. Each square represents a diffraction from a reflectivity structure that has 3 depth sample (j = 1 to 3) in 5 spatial locations, (k = 1 to K). Each square contains a diffraction with 4 samples (i = 1 to 4) in 5 traces (j = 1 to 5). The diffraction matrix is dimensioned as $\mathbf{G}_{4d}(I,J,K,L)$, with each element as $\mathbf{G}_{4D}(i,j,k,l)$. The 4D transpose is shown in Figure A2

$$
\begin{bmatrix} 1 & . & . & . & . \\ . & 1 & . & . & . \\ . & . & 1 & . & . \\ . & . & . & 1 & . \end{bmatrix}
\begin{bmatrix} . & 1 & . & . & . \\ 1 & . & 1 & . & . \\ . & . & . & 1 & . \\ . & . & . & . & 1 \end{bmatrix}
\begin{bmatrix} . & . & 1 & . & . \\ . & 1 & . & 1 & . \\ 1 & . & . & . & 1 \\ . & . & . & . & . \end{bmatrix}
\begin{bmatrix} . & . & . & 1 & . \\ . & . & 1 & . & 1 \\ . & 1 & . & . & . \\ 1 & . & . & . & . \end{bmatrix}
\begin{bmatrix} . & . & . & . & 1 \\ . & . & . & 1 & . \\ . & . & 1 & . & . \\ . & 1 & . & . & . \end{bmatrix}
$$

$$
\begin{bmatrix} . & . & . & . & . \\ 1 & 1 & . & . & . \\ . & . & 1 & . & . \\ . & . & . & 1 & . \end{bmatrix}
\begin{bmatrix} . & . & . & . & . \\ 1 & 1 & 1 & . & . \\ . & . & . & 1 & . \\ . & . & . & . & 1 \end{bmatrix}
\begin{bmatrix} . & . & . & . & . \\ . & 1 & 1 & 1 & . \\ 1 & . & . & . & 1 \\ . & . & . & . & . \end{bmatrix}
\begin{bmatrix} . & . & . & . & . \\ . & . & 1 & 1 & 1 \\ . & 1 & . & . & . \\ 1 & . & . & . & . \end{bmatrix}
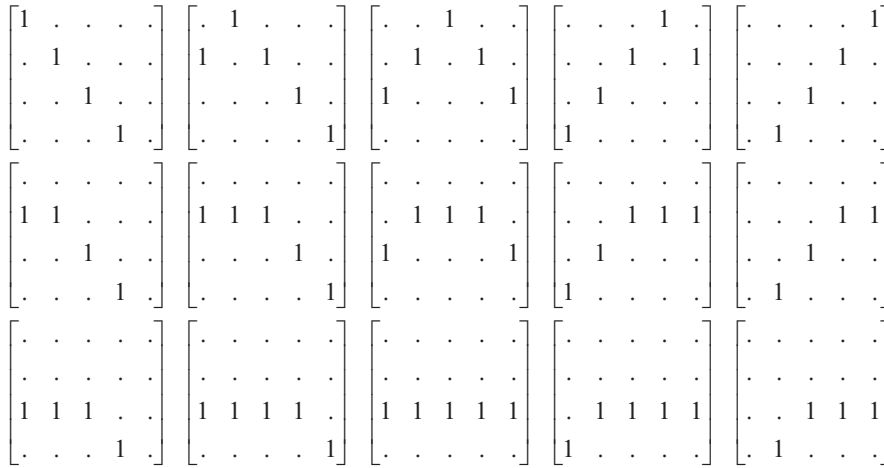\begin{bmatrix} . & . & . & . & . \\ . & . & . & 1 & 1 \\ . & . & 1 & . & . \\ . & 1 & . & . & . \end{bmatrix}
$$

$$
\begin{bmatrix} . & . & . & . & . \\ . & . & . & . & . \\ 1 & 1 & 1 & . & . \\ . & . & . & 1 & . \end{bmatrix}
\begin{bmatrix} . & . & . & . & . \\ . & . & . & . & . \\ 1 & 1 & 1 & 1 & . \\ . & . & . & . & 1 \end{bmatrix}
\begin{bmatrix} . & . & . & . & . \\ . & . & . & . & . \\ 1 & 1 & 1 & 1 & 1 \\ . & . & . & . & . \end{bmatrix}
\begin{bmatrix} . & . & . & . & . \\ . & . & . & . & . \\ . & 1 & 1 & 1 & 1 \\ 1 & . & . & . & . \end{bmatrix}
\begin{bmatrix} . & . & . & . & . \\ . & . & . & . & . \\ . & . & 1 & 1 & 1 \\ . & 1 & . & . & . \end{bmatrix}
$$

FIG. A1 The 4D diffraction matrix

$$
\begin{bmatrix} 1 & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \end{bmatrix}
\begin{bmatrix} . & 1 & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \end{bmatrix}
\begin{bmatrix} . & . & 1 & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \end{bmatrix}
\begin{bmatrix} . & . & . & 1 & . \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \end{bmatrix}
\begin{bmatrix} . & . & . & . & 1 \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \end{bmatrix}
$$

$$
\begin{bmatrix} . & 1 & . & . & . \\ 1 & 1 & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \end{bmatrix}
\begin{bmatrix} 1 & . & 1 & . & . \\ 1 & 1 & 1 & . & . \\ . & . & . & . & . \\ . & . & . & . & . \end{bmatrix}
\begin{bmatrix} . & 1 & . & 1 & . \\ . & 1 & 1 & 1 & . \\ . & . & . & . & . \\ . & . & . & . & . \end{bmatrix}
\begin{bmatrix} . & . & 1 & . & 1 \\ . & . & 1 & 1 & 1 \\ . & . & . & . & . \\ . & . & . & . & . \end{bmatrix}
\begin{bmatrix} . & . & . & 1 & . \\ . & . & . & 1 & 1 \\ . & . & . & . & . \\ . & . & . & . & . \end{bmatrix}
$$

$$
\begin{bmatrix} . & . & 1 & . & . \\ . & . & 1 & . & . \\ 1 & 1 & 1 & . & . \\ . & . & . & . & . \end{bmatrix}
\begin{bmatrix} . & . & . & 1 & . \\ . & . & . & 1 & . \\ 1 & 1 & 1 & 1 & . \\ . & . & . & . & . \end{bmatrix}
\begin{bmatrix} 1 & . & . & . & 1 \\ 1 & . & . & . & 1 \\ 1 & 1 & 1 & 1 & 1 \\ . & . & . & . & . \end{bmatrix}
\begin{bmatrix} . & 1 & . & . & . \\ . & 1 & . & . & . \\ . & 1 & 1 & 1 & 1 \\ . & . & . & . & . \end{bmatrix}
\begin{bmatrix} . & . & 1 & . & . \\ . & . & 1 & . & . \\ . & . & 1 & 1 & 1 \\ . & . & . & . & . \end{bmatrix}
$$

$$
\begin{bmatrix} . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & 1 & . \\ . & . & 1 & . & . \end{bmatrix}
\begin{bmatrix} . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & 1 \\ . & . & . & 1 & . \end{bmatrix}
\begin{bmatrix} . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ 1 & . & . & . & 1 \end{bmatrix}
\begin{bmatrix} . & . & . & . & . \\ . & . & . & . & . \\ 1 & . & . & . & . \\ . & 1 & . & . & . \end{bmatrix}
\begin{bmatrix} . & . & . & . & . \\ . & . & . & . & . \\ . & 1 & . & . & . \\ . & . & 1 & . & . \end{bmatrix}
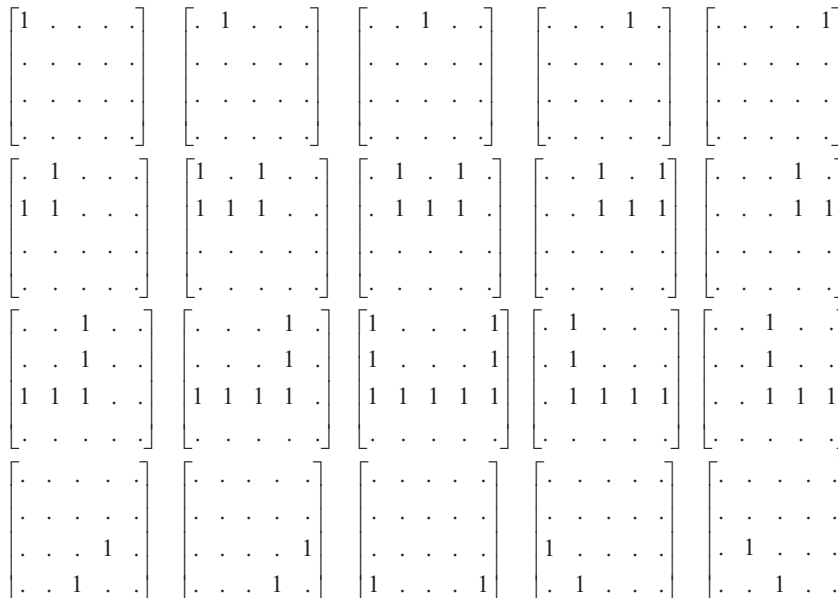$$

FIG. A2 The 4D transpose of the diffraction matrix

APPENDIX 2 "LATE BREAKING NEWS"

As we went to press, some very interesting test came back that we think should be included in this report.

We applied two additional filters to the CSP gathers before processing. These additional filters, 1-2-124-140 and 1-2-200-240 contain much greater bandwidths and significantly more resolution after the deconvolution. The first one was expected as it included more of the sweep spectrum, but the third filter surprised us as it appeared to contain still more signal. We applied the third filter because we did not see a significant drop in the amplitude spectrum as expected beyond the maximum sweep frequency of 100 Hz. We don't know if we are dealing with artifacts or if we have possible included higher frequencies from harmonics that could be caused by distortions to the sweep. There may be better explanations that we are not aware of due to the rush nature of including this information.

The three panels in Figure A3 contain the EOM migrations for the three filters as indicated. The area of the sections that are displayed in these panels are from CMP 335 to 935, and from 800 to 2,000 ms, as defined on Figure 5a. The windows are laterally squeezed for an easier comparison.

The panels in Figure A4 contain the deconvolutions, displayed with a similar window. Amplitude spectra are shown in Figure A5 for the three different filters, with (a) the EOM data using the complete migrated section, (b) the complete window of the deconvolved EOM, and (c) the partial window described above.
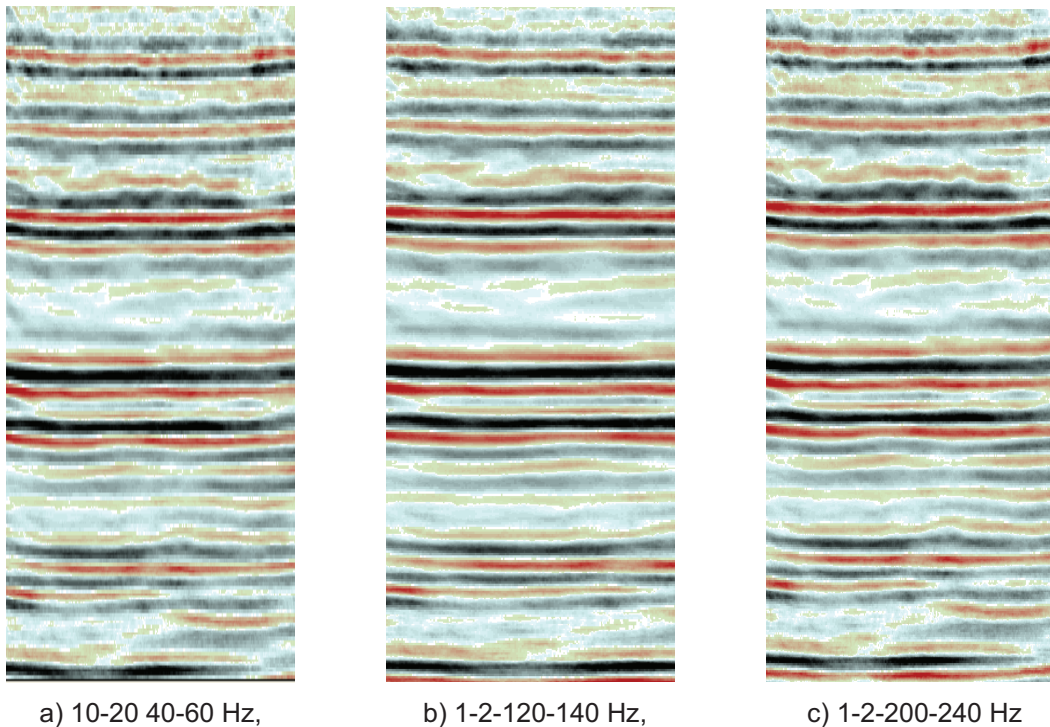

a) 10-20 40-60 Hz,   b) 1-2-120-140 Hz,   c) 1-2-200-240 Hz

FIG. A3 Three panels produced by different bandpass filters that were applied at input to EO migration.
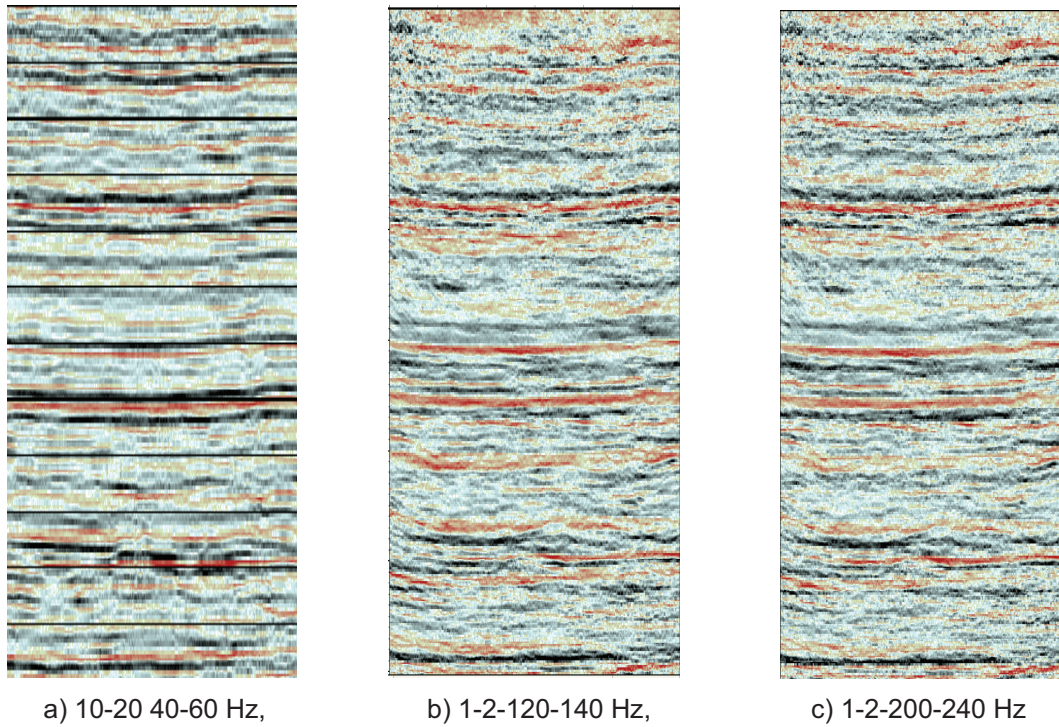
| a) 10-20 40-60 Hz, | b) 1-2-120-140 Hz, | c) 1-2-200-240 Hz |

FIG. A4 Three panels produced by deconvolution after EOM. The input data was filtered using the parameters indicated below each panel.
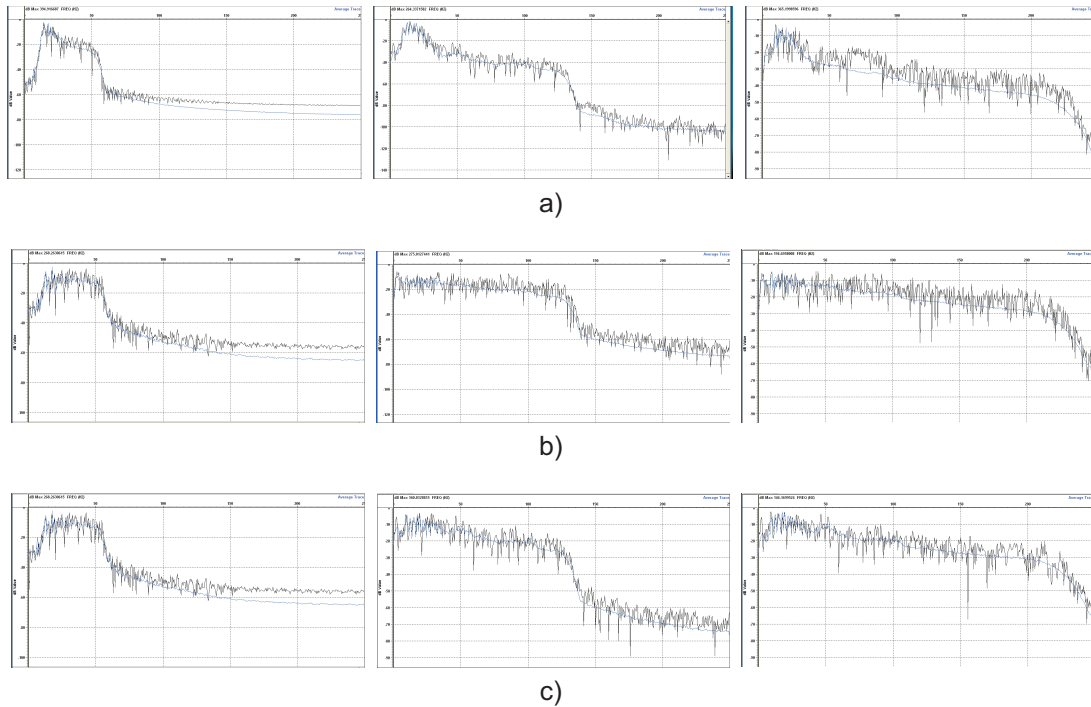

a)

b)

c)

FIG. A5 Amplitude spectra of the EOM using three different filters for a) the entire migration, b) deconvolution of the entire migration, and c) the window (above) of the deconvolution.,

Any ideas?