Reflectivity modeling for stratified anelastic media

Peng Cheng and Gary F. Margrave

ABSTRACT

The reflectivity method is widely used for the computation of synthetic seismograms for layered media due to its capacity of modeling all kinds of wave propagation and attenuation for a given model with sufficient accuracy and relatively low computation cost. This paper gives a brief introduction of the reflectivity method, and then demonstrates that reflectivity method can give accurate and realistic modeling results for stratified anelastic media.

INTRODUCTION

The reflectivity method is a wave-number or slowness integration method, which computes the response of a model in the frequency-wavenumber domain and automatically includes contributions from all possible rays within the reflecting zone. The reflectivity method was originally developed by Fuches and Müller (1971). Their pioneer work was followed by Kennett (1975, 1979, 1980), Kind (1976), Stephen (1977), Kennett and Kerry (1979), Kennett and Illingworth (1981), Fryer (1981), Kennett and Clark (1983). In reflectivity modeling, the wavenumber or slowness integration is calculated by a matrix or propagator techniques, which mainly deals with the computation of the reflection and transmission coefficients for plane waves, incident on a plane surface or a stack of homogenous layers. The coefficients for an interface are given analytically according to the Zoeppritz equations, and those for a stack of layers are derived by a recursive algorithm proposed by Kennett (1975). Some practical aspects of the slowness integration calculation such as windowing and aliasing in the frequency domain were discussed by Mallick and Frazer (1987). For a stratified earth model, the reflectivity method decomposes the propagating waves into downgoing waves and upgoing waves, and waves can be decoupled into P-SV and SH wave. So, the reflection, transmission and conversion of all wave modes can be fully described. In addition, the attenuation effect of anelastic media can be incorporated conveniently in the frequency domain (Kennett, 1975; Sipkin et al, 1978; O'Neil and Hill, 1979). Generally, the reflectivity method can generate realistic seismogram for layered media.

This paper is an update of that of Ma et al (2004). The matlab code for the reflectivity method is a file named "reflectivity.m" in the directory "reflectivity" of CREWES toolbox, which replaces the old one. Our main goal is to evaluate and verify the modeling results in terms of correct events, amplitudes and incorporation of attenuation. This paper is organized as follows: the first part introduces the theory of reflectivity modeling. Then, some earth models are used to test the reflectivity method. Finally, some basic conclusions are drawn from results of the test.

BASIC THEORY OF REFLECTIVIY MODELING

The implementation of the reflectivity method in this paper is an update of the work of Ma et al (2004), which follows the algorithm described by Müller (1985).

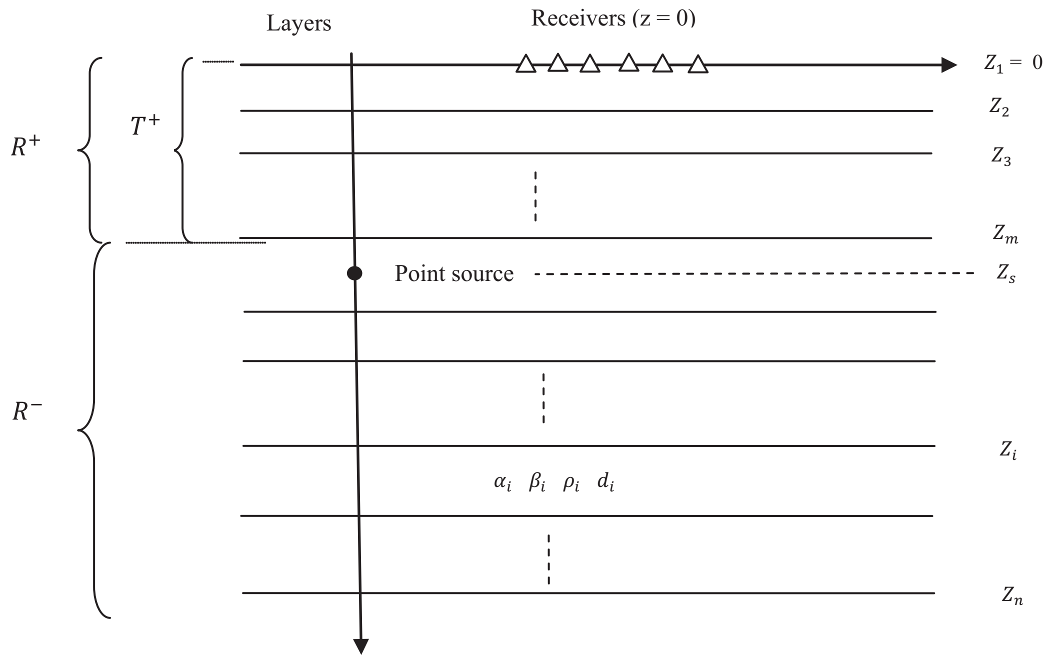

Figure 1 Layered media. α_i- p wave velocity; β_i - S wave velocity; ρ_i – density; d_i - thickness of the *i*-th layer; Z_i - depth to the surface. R^+ - total reflection coefficient matrix corresponds to the upper half space above source location; R^- - total reflection coefficient matrix corresponds to the lower half space below the source position; T^+ - total transmission coefficient matrix corresponds to the upper half space.

For a layered earth model shown in figure 1, suppose that the receivers are on the surface (Z=0); the far field wavefield in frequency domain for P-SV waves can be formulated as (Müller, 1985)

$$\begin{pmatrix} \mathrm{u_r} \\ \mathrm{u_z} \end{pmatrix} = \frac{\omega}{4\pi\rho_\mathrm{m}} \sum_{i=1}^{2} \epsilon_i \int_0^\infty J_i U T^+ \left[\mathrm{I} - \mathrm{R^-R^+}\right]^{-1} (\mathrm{S_i^U} + \mathrm{R^- S_i^D}) du, \quad (1)$$

where $\mathrm{u_r}$, $\mathrm{u_z}$ are the radial and vertical components of displacement respectively; ω is the angular frequency; ρ_m is the density of the layer within which the point source locates; ϵ_i is related to the magnitude of the point source; J_i(i=0,1) is a 2 x 2 matrix related to Bessel function of the first kind; the arguments of Bessel functions are $u\omega$r, where r is the radial component in a cylindrical coordinate system; $\mathrm{R^+}$ is the total reflection coefficient matrix of the upper half space between $z_1 = 0$ and z_m; $\mathrm{R^-}$ is the total reflection coefficient matrix of the lower half space below z_m; $\mathrm{T^+}$ is the total transmission coefficient between $z_1 = 0$ and z_m; $\mathrm{S_i^U}$ and $\mathrm{S_i^D}$ are the source amplitude vectors for upgoing and downgoing waves respectively. A tutorial by Müller (1985) gives the explicit formulas of the above terms (Eqns. (46), (47), (66), (71), (72), (81) and (82) of that paper), and describes a recursive algorithm to calculate $\mathrm{R^+}$, $\mathrm{R^-}$ and $\mathrm{T^+}$ in detail.

Here we give a conceptual explanation of equation (2). The point source ϵ_i is decomposed into upgoing waves $\mathrm{S_i^U}$ and downgoing waves $\mathrm{S_i^D}$; $\mathrm{S_i^U} + \mathrm{R^- S_i^D}$ denotes the total upgoing source wavefield since the downgoing wave is reflected to change the propagating direction; the reverberation of the total upgoing source wavefield between upper half space and lower half space is taken into account by $[I - R^{-}R^{+}]^{-1}$ to obtain a complete upgoing wavefield; then, the complete upgoing wavefield is shifted to the receivers at surface by the transmission coefficient matrix T^{+}; finally, the wavefield in the space-time domain can be obtained by inverse Fourier transform of equation (2).

NUMERICAL TEST

A two layer earth model was used to evaluate the reflectivity method. The physical parameters of the layered media are shown in table 1. The point source at depth z = 5m radiates both P and S waves, and has a minimum phase wavelet with a dominant frequency of 40Hz, and the receivers are located at the surface. The reflectivity modeling results for P-SV waves are shown in figure 1 and 2. We can see that all the events are modeled including direct P and S wave, reflected PP, PS/SP, SS waves and multiples. For the reflectivity method, we can choose to model the primary reflection events only, and the results are shown in figure 3 and figure 4. Since a two layer model is used and source location is close to surface, the PS and SP waves have the same nearly the same travel time. In figure 3 and figure 4, each trace has three events corresponding to PP, PS/SP, SS waves respectively. We can see that reflectivity method can model all kinds of waves with flexibility.

Table 1 A two layer earth model

Layer index	P wave velocity (m/sec)	S wave velocity (m/sec)	Density (kg/m^3)	Thickness (km)	Q_P	Q_S
1	2500	1600	1800	1	10000	10000
2	3200	1800	2100	2	10000	10000

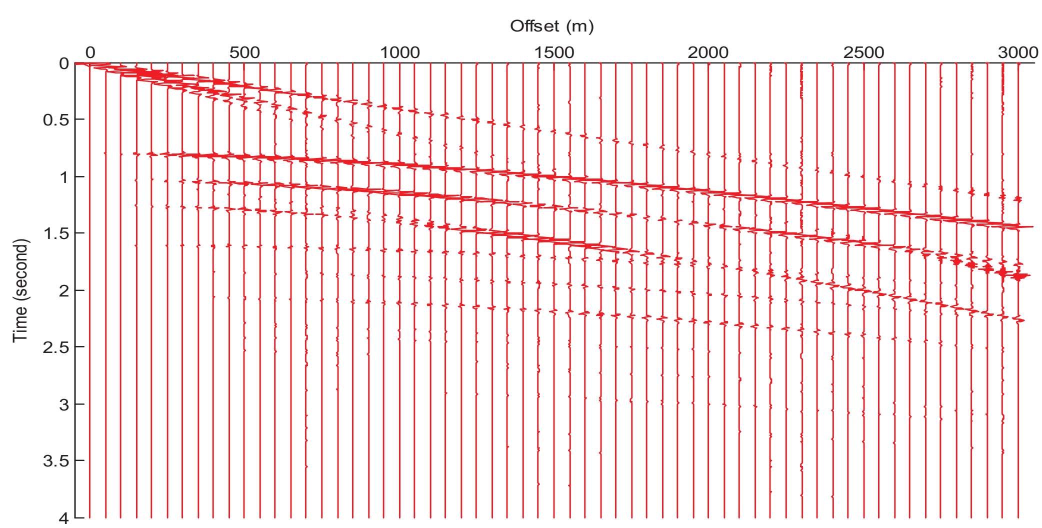

Figure 1. Vertical component of P-SV waves for the two-layer model shown in table 1.

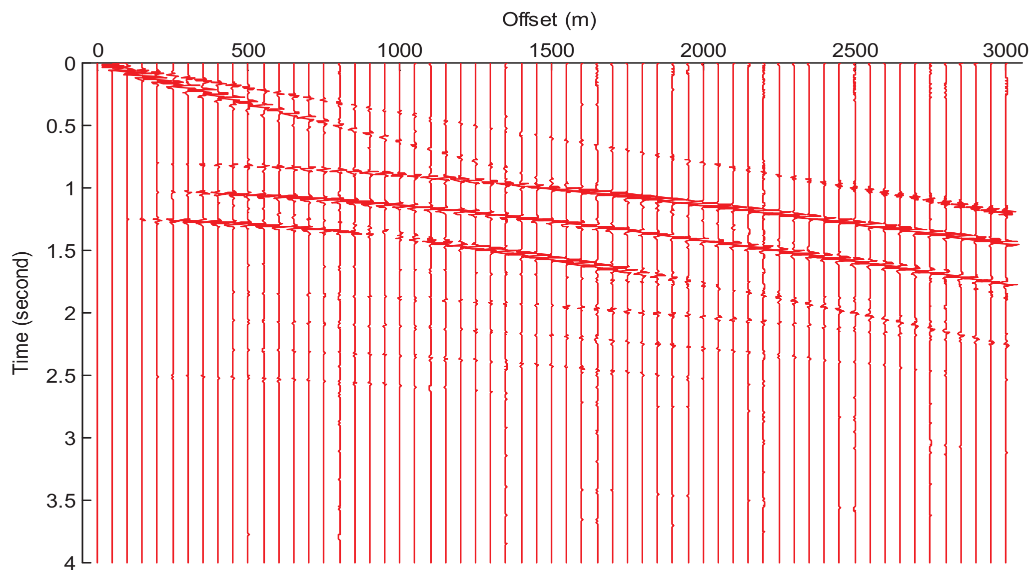

Figure 2. Radial component of P- SV waves for the two-layer model shown in table 1.

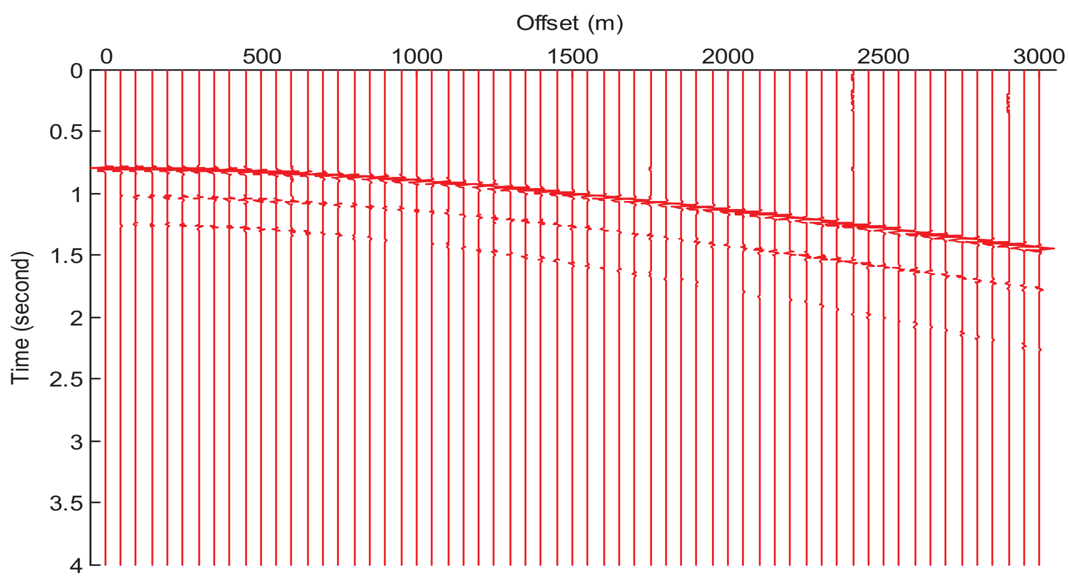

Figure 3. Vertical components of primary reflection events for the earth model shown in table 1.

To evaluate the amplitudes of the events for the reflectivity method, we used the vertical components of the primary reflection events shown in figure 3 as an example to derive the reflection coefficients for PP and SS waves. First, a one layer model was used to estimate the amplitudes for incident waves that experience the same geometric spreading effect as the reflected waves. This one layer model has the same physical parameters as the first layer of the earth model shown in table 1, while the point source is shifted to the image point (Z = 1995m) with respect to the interface in the two layer model and the receivers are remained at the same positions. The vertical components of the direct arrival waves for this one layer model are shown in figure 5. Figure 6 shows the amplitudes of the direct P waves in figure 5 and the amplitudes of the reflected PP waves in figure 4, from which the reflection coefficients for the PP waves can be derived.

From figure 7, we can see that the derived PP reflection coefficients match the Zoeppritz ones perfectly before the critical incident angle is reached at an offset around 1600m. Similarly, the reflection coefficients for SS waves are derived and shown in figure 8, which match theoretical results as well. We do not expect a match near or beyond the critical angle because the reflectivity method calculates spherical reflection coefficients through an integration over plane waves which the Zoeppritz equations are for a single plane wave.

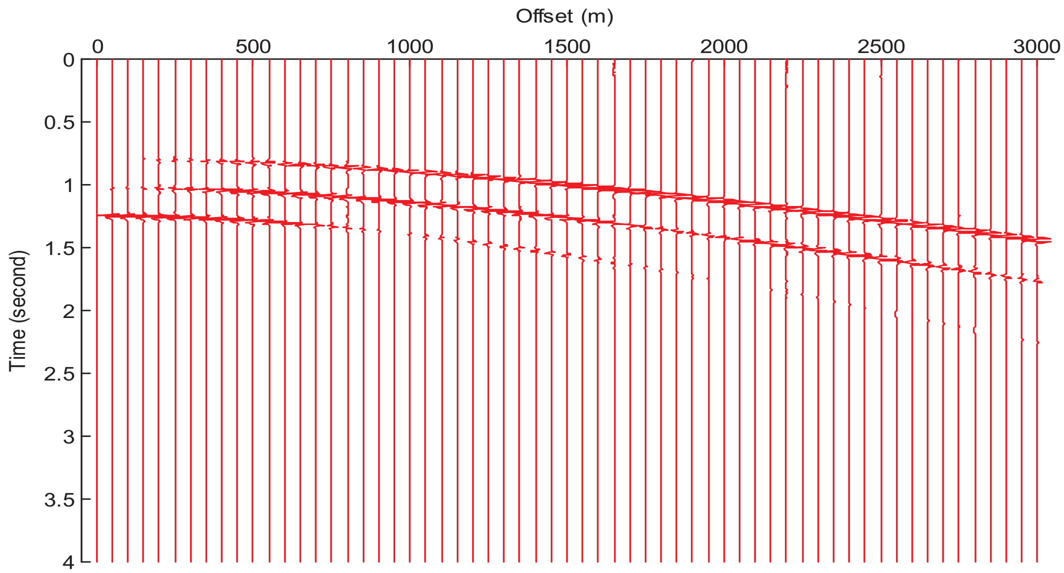

Figure 4. Radial components of primary reflection events for the earth model shown in table 1.

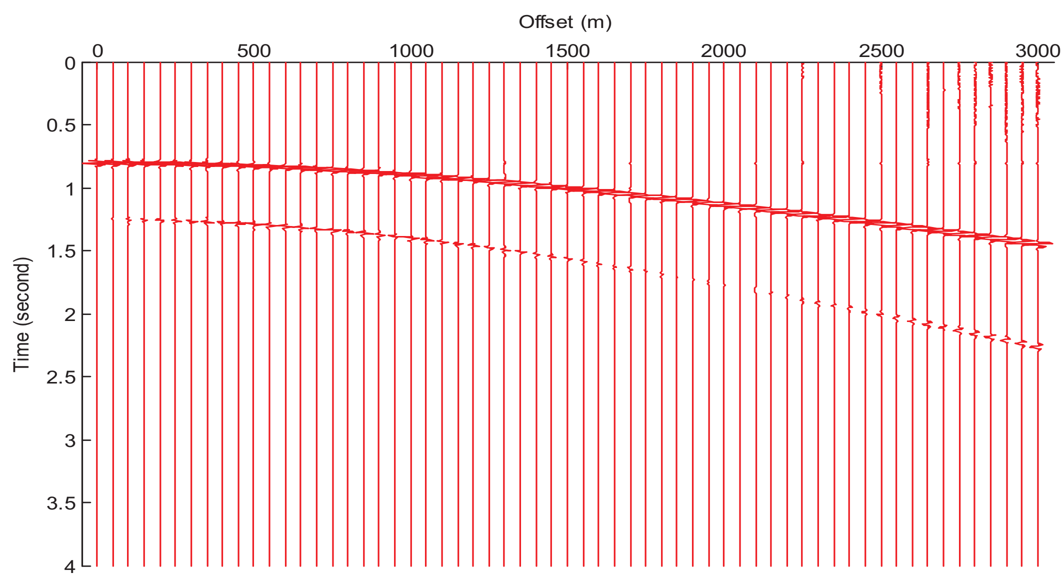

Figure 5 Vertical components of the direct arrivals of a homogenous media with the physical parameters of the layer 1 shown in table 1 and a point source at depth Z = 1995m.

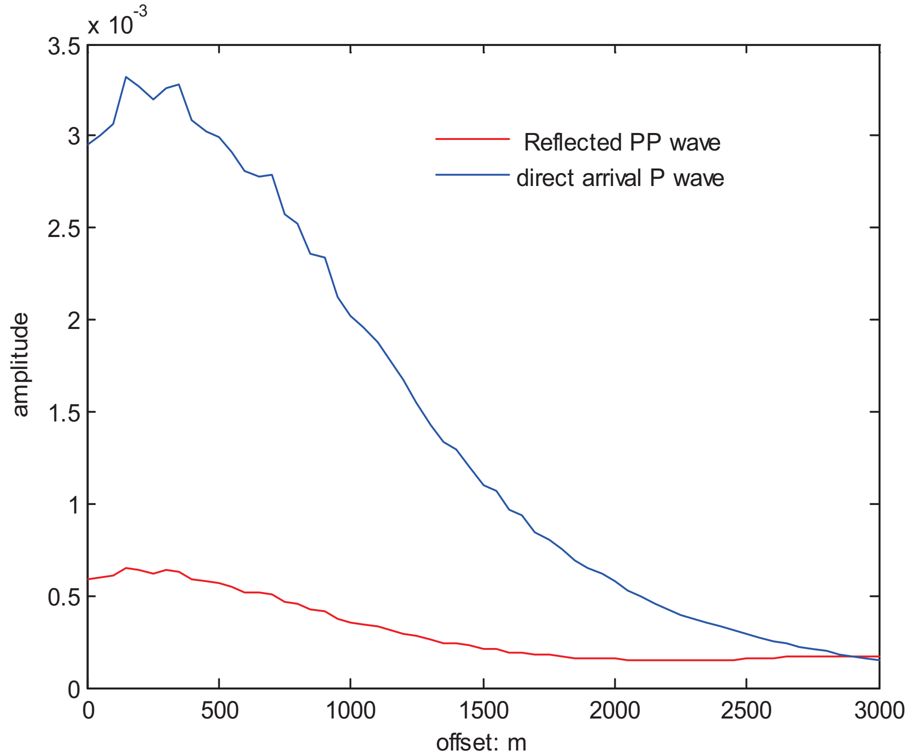

Figure 6. Amplitudes of the direct P waves and reflected PP waves.

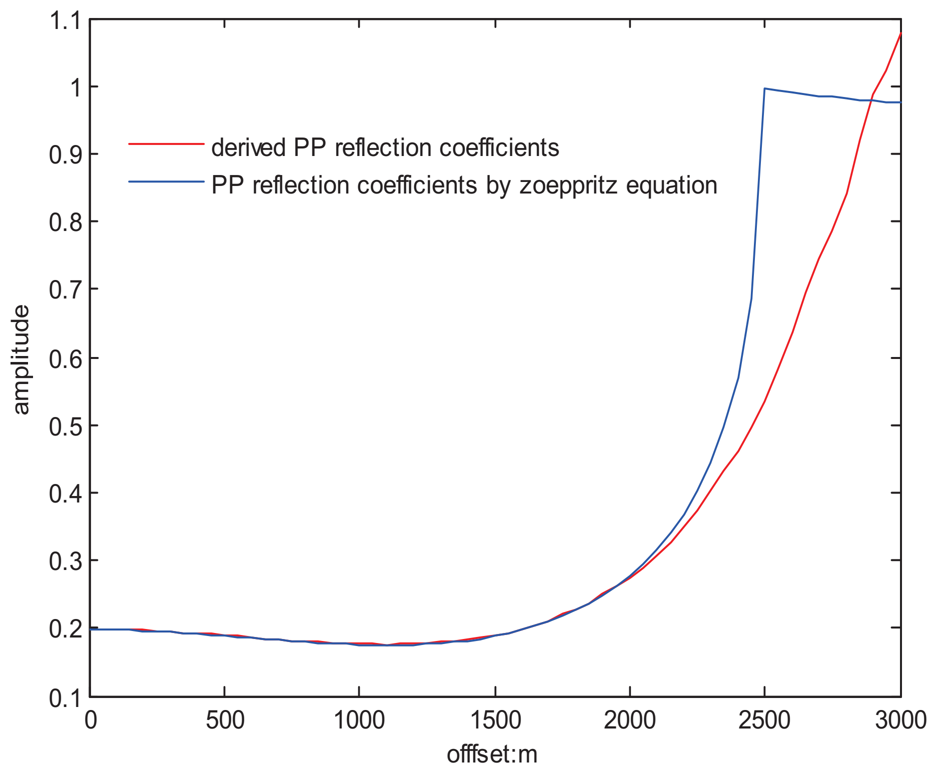

Figure 7. Comparison of PP reflection coefficients.

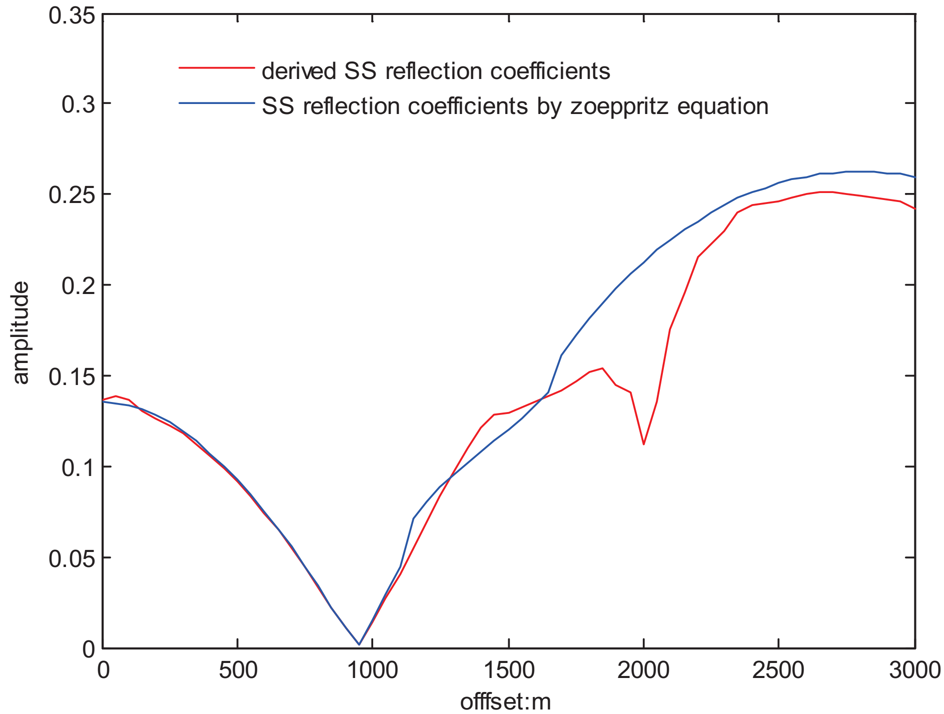

Figure 8. Comparison of SS reflection coefficients.

In order to evaluate the incorporation of Q attenuation of earth for the method, we use a two layer model, as shown in table 2, to test the reflectivity modeling, which has the same physical parameters, source location, source wavelet and receiver stations as the two layer model shown in table 1 except the Q values for P wave and S wave. The vertical components of the primary reflection components are shown in figure 9 corresponding to the Q attenuation free case shown in figure 3. Using the trace with an offset of 1000m as an example, a comparison between modeling with Q attenuation and without Q attenuation is demonstrated by figure 10. The amplitude decay due to Q attenuation is obvious. Figure 11 shows the amplitude spectra of the PP events in figure 10. Then, the spectral ratio method is employed to estimate Q_P (figure 12). The estimated Q_P =83.8 is consistent with the Q model. Similarly, from the SS events shown in figure 10, Qs is estimated to be 61.6 (figure 13), which is very close to theoretical value as well. We can see that the reflectivity method can incorporate the Q attenuation with sufficient accuracy.

Table 2 a two layer earth model

Layer index	P wave velocity (m/sec)	S wave velocity (m/sec)	Density (kg/m^3)	Thickness (km)	Q_P	Q_S
1	2500	1600	1800	1	80	60
2	3200	1800	2100	2	80	60

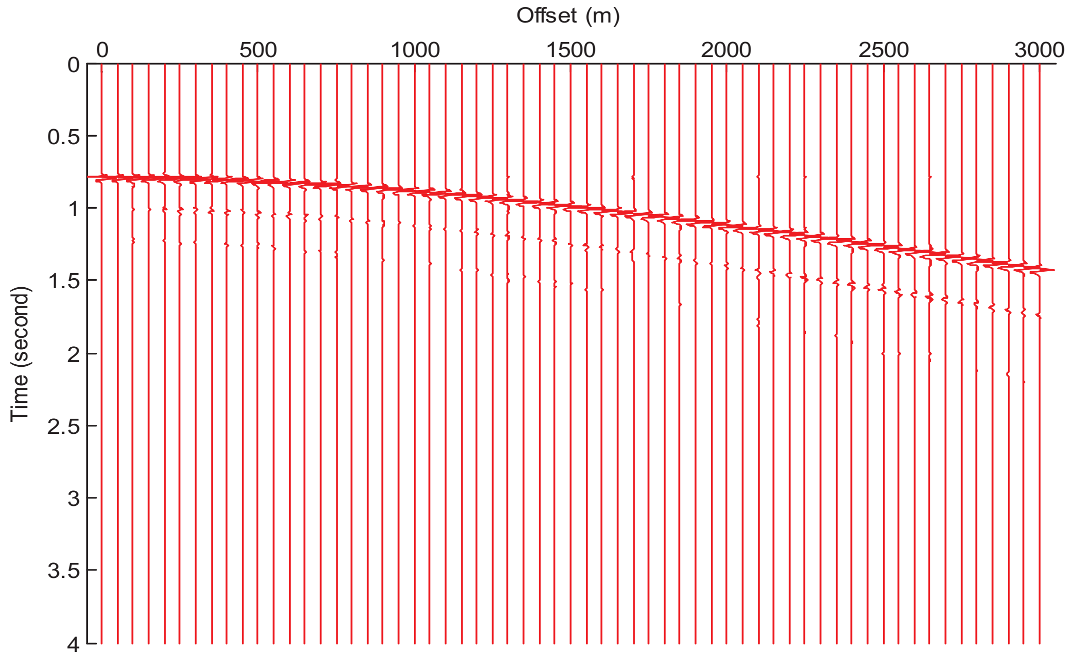

Figure 9 Vertical components of primary reflection events for the earth model shown in table 2.

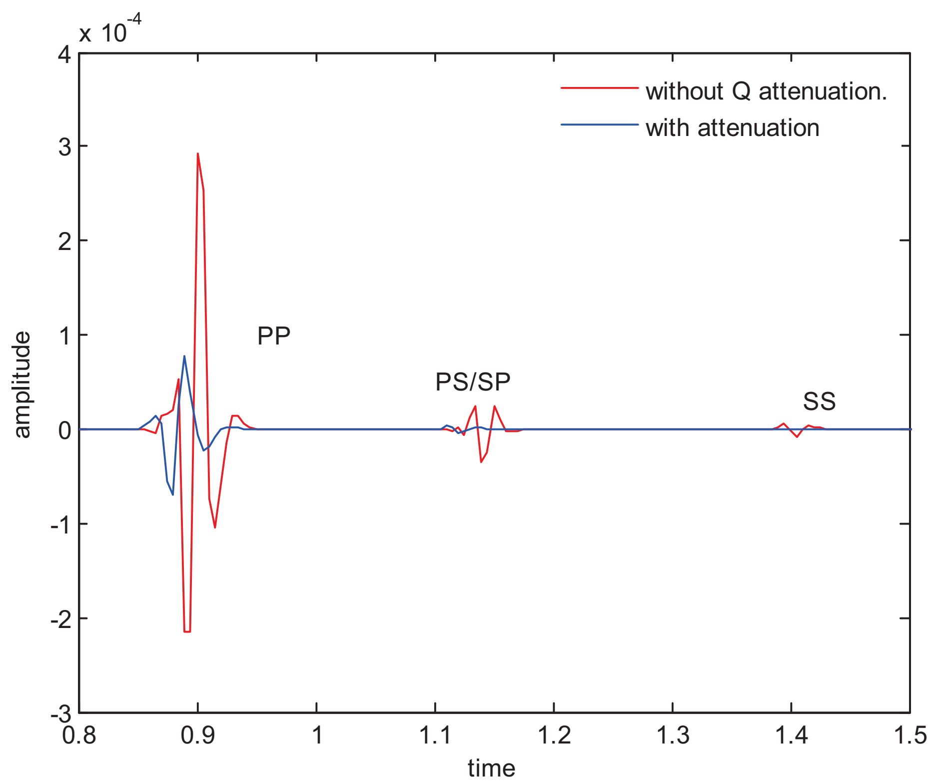

Figure 10. Comparison of the trace with an offset of 1000m shown in figure 3 and figure 9.

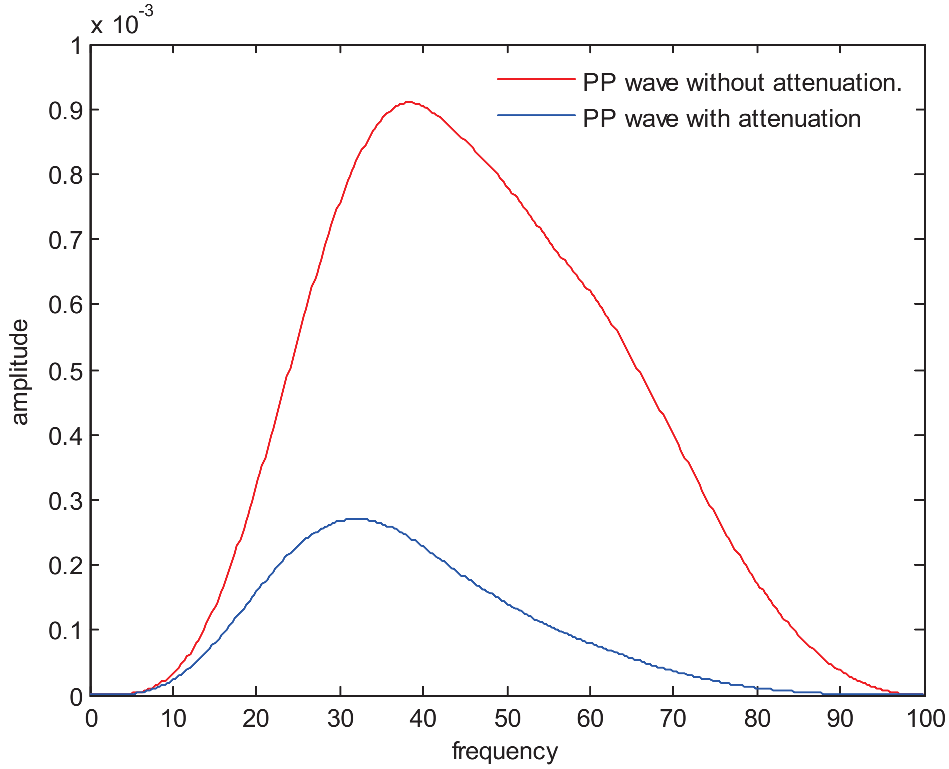

Figure 11 Amplitude spectra of the PP events shown in figure 10.

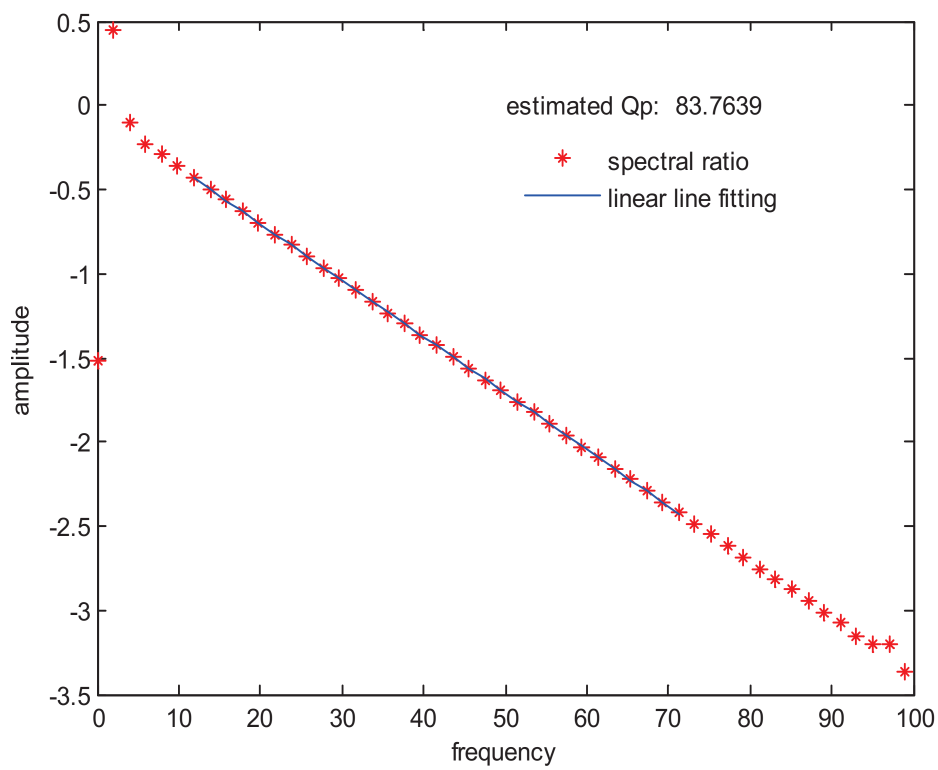

Figure 12 . Q_P estimation using spectral ratio method for the PP events shown in figure 10.

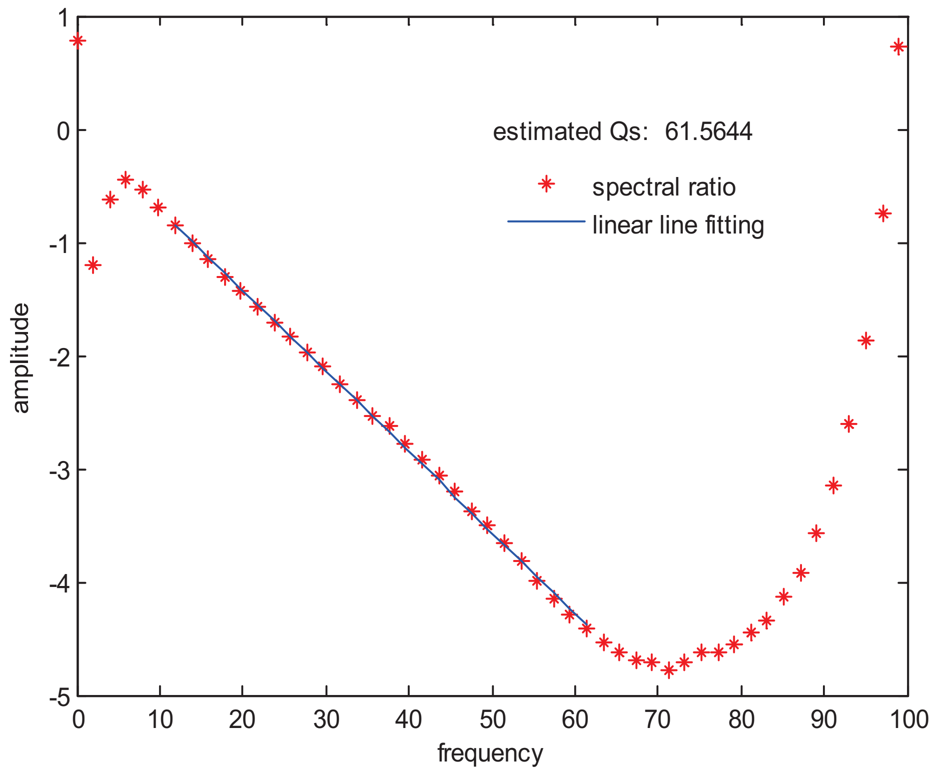

Figure 12. Q_s estimation using spectral ratio method for the SS events shown in figure 10.

CONCLUSION

The reflectivity method is very useful for the seismic modeling of stratified media. It can model all kinds of waves and address the geometric spreading and Q attenuation properly, which makes the modeling result realistic and gives sufficient information for layered earth model.

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