Comparisons between 2D and 3D finite-difference models of near source surface waves

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ABSTRACT

Comparisons are made between nearly identical models represented in two and three dimensions. Two comparisons show that where the geological model and the acquisition are well represented in two dimensions the results are identical except for the expected difference in amplitude attenuation with distance. Also shown is a case where acquisition in only two dimensions may be adequate for reflection data, but not for simulation of surface noise.

INTRODUCTION

Rayleigh waves may be modelled very effectively in two dimensions once they have propagated a significant distance from their source. It is not so obvious that near their source, Rayleigh waves behave in the same simple manner. This is suggested by the fact that in seismic exploration records, Rayleigh waves do not usually appear as simple confined events, but as a whole zone of wave action that overwhelms our desired reflection data. Could it be that near their source the physics enables Rayleigh waves to interact with themselves to create a noise cone? The theory would likely be difficult, but models should show some indication of the effect if it exists. Hence, the tests which follow.

VALID SURFACE MODELS IN TWO DIMENSIONS

The 2D and 3D finite difference models built for comparison were extremely simple: constant velocity with an explosive source appropriate for two or three dimensions, and the same models with the vertical source appropriate for both two and three dimensions.

The results shown below are a clear vindication of the use of two dimensional models extending right from the source point. Most of the differences between the two model types can be qualitatively explained as the amplitude differences between cylindrical and spherical models.

Observations on theoretical amplitudes

Rayleigh waves are true surface waves. They do not transfer energy away from the surface, but only along the surface. In a two dimensional mode, or cylindrical mode, they do not fall off with distance at all. In a three dimensional (spherical) mode they fall off as $r^{-1/2}$ in amplitude.

First arrivals can represent body waves because they are the wave edges of body waves, and they spread deeper as they propagate. In a two dimensional mode they fall off as $r^{-1}$ in energy, or $r^{-1/2}$ in amplitude. In a three dimensional mode they fall off as $r^{-2}$ in energy, or $r^{-1}$ in amplitude.
The amplitude relationships are then quite simple. If the decay rates may be considered as three orders \( (1, r^{-1/2}, r^{-1}) \) then the first arrivals may be considered one order higher than the surface waves, and the three dimensional modelling may be considered to be one order higher than two dimensional modelling. After considering how the record as a whole will be rescaled as the decay rates change, the amplitudes seem to fit in the orders given here.

**Boundary reflections in three dimensions**

An interesting side effect of three dimensional models is that the influence of edge effects is a little less. Clayton Engquist conditions have been used on the right and bottom edges of all the models to approximate transparent boundaries. The reflection from the far edge of Figure 1 is a remnant boundary reflection. The equivalent reflection cannot be seen in Figure 2 because of the greater amplitude decay of the 3D model. On the other hand, the body wave reflection in Figure 2 is a little stronger.

In the Figures which follow, \( U \) represents particle displacement, and it is followed by a displacement direction of X, Y, or Z.

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**FIG. 1.** An explosive source for a model in two dimensions. The source is on the left edge at \( X = 0 \), and at a depth of 8 metres. The upper event is the direct arrival and the lower event is the Rayleigh wave (ground-roll).
FIG. 2. An explosive source for a model in three dimensions. As with Figure 1, the source is on the left at 8 metres depth. The events are similar to the two dimensional model but the amplitudes drop off more strongly with offset, as expected.

FIG. 3. A surface vertical source for a model in two dimensions. The Rayleigh wave starts right at time zero, and the first arrivals branch off the Rayleigh wave.
WHEN SURFACE MODELS IN THREE DIMENSIONS ARE NECESSARY

If models are necessary to study the effects of surface conditions on a particular seismic line they might have to be 3D models. This is because if anomalies must be modelled, they are not likely to be in the plane of the line. The simple example presented here models a significant anomaly in the X direction which persists for all values of Y. This means that in the X direction it is a model in two dimensions, but in the Y or any other direction it is a model in three dimensions.

The significant anomaly in this case is a step decrease of the surface layer thickness from 15 metres to 5 metres at an offset of 55 metres. The symmetry about the Y axis which this model assumes then effectively places another equal step at X = -55 metres. The surface layer has a P-wave velocity of 1000 metres per second over a layer of 1500 metres per second. Figure 5 shows the $U_x$ displacement on this X line which could have been adequately modelled in two dimensions. The thin low velocity layer past 55 metres cannot easily transmit the 25 Hertz Rayleigh wave, and so it is partially reflected back to the origin.

Figure 6 shows a $U_y$ line acquired in the Y direction, parallel to the discontinuity in the surface layer. The direct arrival of an ordinary Rayleigh wave may be seen just below the green velocity line, but all the energy below this event has propagated as ground-roll after being reflected back at an angle from the discontinuity in the X direction.
FIG. 5. This display shows the data acquired in the X-direction, perpendicular to the discontinuity at 55 metres. A model in two dimensions would have been sufficient here.

FIG. 6. This display shows the data collected in the Y direction, parallel to the discontinuity. The upper Rayleigh wave, reaching 350 ms at 250 metres is a direct arrival. The events below this are sideswipe from the discontinuity in X.
FIG. 7. The $U_z$ component of the record in Figure 6. The noise here peaks below 200 ms and is perfectly hyperbolic. It is very reminiscent of the noise pattern seen on many seismic lines.

FIG. 8. A snapshot of displacements at the surface at a time of 280 ms after the shot. This surface noise has been trapped between the near surface discontinuity at 55 m in X and an identical (assumed) discontinuity at -55 m. Acquisition along the Y axis in this case will have noise which cannot be modelled within the plane of the line.
Figure 7 shows the $U_z$ (vertical) component corresponding to the record of Figure 6. The noise event is perfectly hyperbolic, and very reminiscent of the noise seen on many seismic lines. It is difficult to imagine a mechanism that would result in this pattern on the open prairie, unless it might be caused by the compaction of the equipment shooting the line.

Figure 8 is a snapshot of the displacements in the vicinity of the source point at a time 280 ms after the initiation of the pulse. It represents one quarter of the total wave field, which is assumed to be symmetric about the X and Y axes. The modelling program assumes that the discontinuity of the surface layer at 55 metres in X continues parallel to the Y axis, and the symmetry assumes a similar discontinuity at -55 metres in X.

The Rayleigh waves may be seen to be concentrated and directed between the two discontinuities as they expand from the source point. Models in two dimensions like this may be required for accurate results in surface layers.

**CONCLUSIONS**

The models indicate that a vibrator source does not on its own create a cone of noise spreading from the shotpoint. Other mechanisms will have to be found to explain this type of phenomenon on so many seismic records.

Surface wave models in two dimensions are perfectly adequate where conditions may be accurately modelled in two dimensions. This will be the case where the more abrupt discontinuities are more perpendicular to the line direction. The slightly different change in amplitude with offset from reality to these models is not likely to cause any problems.

Surface wave models in three dimensions might be required where it is necessary to understand the details of complex noise conditions.

It would be interesting to do a careful comparison of the propagation of Rayleigh waves on uncompacted and compacted prairie fields.

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