Differencing of time-lapse survey data using a projection onto convex sets algorithm

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ABSTRACT

Irregular spatial sampling in time-lapse surveys can hamper the efforts to obtain optimal difference sections due to very sparse common spatial samples in the baseline and monitor surveys. We exploit the similarities between baseline and monitor surveys via a Fourier-based reconstruction methods called projection onto convex sets (POCS) to obtain a reliable difference section. Simultaneous handling of baseline and monitor surveys provides a robust indicator of dominant harmonics in the difference section for a given frequency component. Synthetic and modeled data examples demonstrate the viability of the approach when baseline and monitoring data sets have so few as 30% common trace locations.

INTRODUCTION

Time-lapse seismic surveys have become an industry standard in exploration seismology. It consists of an operation to acquire and process multiple seismic surveys, repeated at the same location over a period of time (Lumley, 2001). It can be utilized for various purposes such as reservoir monitoring, CO2 sequestration, and environmental studies. The main problem with processing time-lapse seismic surveys lays in the fact that multiple surveys can not be acquired with the same exact geometry. Therefore, efficient processing methods are necessary in order to account for the mismatch between the baseline and monitor surveys.

Imaging and migration of time-lapse surveys are the next step of identifying the subsurface changes. This task can be carried out using joint imaging of baseline and monitor surveys as well as imaging of the difference section. The joint inversion provides the possibility of using various regularization terms but it imposes very high computational costs. On the other hand, migrating the difference section is computationally less demanding but it is difficult to create a reliable differences section from real data sets due to the irregular spatial sampling and presence of statics in seismic sections. This requires finding an optimal way of matching the baseline and monitor surveys to obtain a reliable difference section (Zabihi 2010). Also, maximally complete difference data sets will be of significant importance in implantation of time-lapse imaging based on scattering/perturbation theory, which is expressed in terms of linear and nonlinear operations on these data differences (Innanen et al., 2011).

Fourier reconstruction and interpolation methods has attained spacial attention in seismic data processing community in recent years. Projection onto convex sets (POCS) (Abma and Kabir, 2005), minimum weighted norm interpolation (MWNI) (Liu and Sacchi, 2004) and anti-leakage Fourier transform (ALFT) (Xu et al., 2005) are few to name from many. These techniques rely on imposing sparseness constraints on the Fourier representation of the irregularly sampled seismic data in an attempt to recover seismic data on a regularly sampled spatial grid (Naghizadeh and Sacchi, 2010). In time-lapse seismic surveys there are a great deal of similarities between the baseline and monitor surveys which can be effectively utilized for reconstruction purposes. This becomes very important if the baseline and monitor surveys have irregular spatial sampling as well as different geometries. Utilizing the similarities between baseline and monitor surveys becomes even more crucial for the reconstruction of the difference section since only a small percentage of traces will be available in the difference section.

In this paper, we will utilize the POCS method for robust recovery of the difference section between irregularly sampled baseline and monitor surveys of time-lapse data. First, we will explain the ordinary POCS algorithm for seismic data reconstruction. Then we will introduce an adaptive strategy to utilize both baseline and monitor surveys to built a robust sparsity measure for the POCS reconstruction of time-lapse difference section. Synthetic example and a finite difference modeled time-lapse survey are used to examine the performance of the proposed method.

THEORY

Time-lapse surveys subtraction

We suppose that $\mathbf{d}_b$ and $\mathbf{d}_m$ show the spatial samples of baseline and monitor surveys at a given frequency in the frequency-space (*f*-*x*) domain, respectively. For simplicity we drop the frequency dependency in deriving the formula. Both baseline and monitor surveys have N samples with the same spatial locations. Let's also assume that for the observed baseline and monitor surveys, $\mathbf{d}_b^{obs}$ and $\mathbf{d}_m^{obs}$, some of the spatial samples are randomly replaced with zero indicating missing samples. The ideal and observed baseline survey are related by

$$\mathbf{d}_b^{obs} = \mathbf{T}_b \mathbf{d}_b, \tag{1}$$

and similarly for the ideal and observed monitor survey we have

$$\mathbf{d}_m^{obs} = \mathbf{T}_m \mathbf{d}_m, \tag{2}$$

where $\mathbf{T}_b$ and $\mathbf{T}_m$ are the baseline and monitor sampling matrices, respectively. Notice that sampling matrices are squared matrices with size $N \times N$ which in the case of no missing samples reduce to identity matrices. For missing samples the corespondent elements of the main diagonal of sampling matrix has zero values.

We investigate the structure of sampling matrix $\mathbf{T}_b$ using a simple example. Assume that the desired baseline signal $\mathbf{d}_b$ has 5 samples but only 3, say $\{d_2, d_4, d_5\}$, are available. Equation 1 can be written as

$$\begin{pmatrix} 0 \\ d_2 \\ 0 \\ d_4 \\ d_5 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \end{pmatrix}. \tag{3}$$

The difference section between the observed baseline and monitor surveys can only be restored at the spatial locations that have available samples in both surveys. The sampling matrix of subtracted section is obtained by

$$\mathbf{T}_s = \mathbf{T}_b \mathbf{T}_m. \tag{4}$$

Therefore, the observed subtracted section is related to the desired subtracted section by

$$\mathbf{d}_s^{obs} = \mathbf{T}_s \mathbf{d}_s. \tag{5}$$

In this article we aim to retrieve an optimal estimate of $\mathbf{d}_s$ using $\mathbf{d}_b^{obs}$ and $\mathbf{d}_m^{obs}$. To achieve this goal we use the method of projection onto convex sets (POCS) (Abma and Kabir, 2005).

Projection onto convex sets (POCS)

The POCS algorithm is a Fourier based data reconstruction method for the signals with a sparse number of harmonics. Let's explain POCS method by implementing it to recover the baseline survey. The POCS algorithm for baseline survey reconstruction can be summarized as

$$\begin{array}{l}
\text{Initialization} \\
\quad \mathbf{d}_b^0 = \mathbf{d}_b^{obs} \\
\text{For} k = 1, 2, 3 \ldots \\
\qquad \mathbf{d}_b^k = \mathbf{d}_b^{obs} + (\mathbf{I} - \mathbf{T}_b)(\mathbf{F}^H \boldsymbol{\Gamma}^k \mathbf{F} \mathbf{d}_b^{k-1}), \\
\text{End}
\end{array} \tag{6}$$

where $\mathbf{F}$ and $\mathbf{F}^H$ are the forward and inverse Fourier transforms and $\boldsymbol{\Gamma}^{\mathbf{k}}$ is the thresholding function which enforces samples with absolute values below the threshold values to be zero. Notice that the term $\mathbf{I} - \mathbf{T}$ as an operator which only keeps the values of the missing spatial samples. In each iteration the values of missing spatial samples is updated while the values of the available spatial samples re-interlaced from observed data.

The thresholding function $\boldsymbol{\Gamma}^{\mathbf{k}}$ might be varied by the iteration number k, namely, by choosing different thresholding values for each iteration. The thresholding function performs as follow

$$\boldsymbol{\Gamma}(\mathbf{D}) = \begin{cases} D(i) & |D(i)| > \lambda_k, \\ 0 & |D(i)| \leq \lambda_k, \end{cases} \tag{7}$$

Where $\mathbf{D}$ represent the Fourier representation of of the data $\mathbf{D}$. It is recommended to choose higher threshold values at the start iterations and decrease its value for the last ones.

POCS time-lapse survey subtraction

In order to expand the POCS method to time-lapse survey subtraction, we introduce a hybrid algorithms to be applied for baseline, monitor and subtracted sections, simultaneously. The procedure is summarized as below

$$
\begin{array}{ll}
 & \text{Initialization} \\
1 & \mathbf{d}_b^0 = \mathbf{d}_b^{obs} + [(\mathbf{I} - \mathbf{T}_b)\mathbf{T}_m]\mathbf{d}_m^{obs}, \\
2 & \mathbf{d}_m^0 = \mathbf{d}_m^{obs} + [(\mathbf{I} - \mathbf{T}_m)\mathbf{T}_b]\mathbf{d}_b^{obs}, \\
3 & \mathbf{T}_s = \mathbf{T}_b\mathbf{T}_m, \\
4 & \mathbf{d}_s^{obs} = \mathbf{T}_s(\mathbf{d}_m^{obs} - \mathbf{d}_b^{obs}), \\
5 & \mathbf{d}_s^0 = \mathbf{d}_s^{obs}, \\
 & \text{For} \quad k = 1, 2, 3 \ldots \\
6 & \qquad \mathbf{M}_b^k = \boldsymbol{\Upsilon}_{\lambda_k}(\mathbf{F}\mathbf{d}_b^{k-1}), \\
7 & \qquad \mathbf{M}_m^k = \boldsymbol{\Upsilon}_{\lambda_k}(\mathbf{F}\mathbf{d}_m^{k-1}), \\
8 & \qquad \mathbf{d}_b^k = \mathbf{d}_b^{obs} + (\mathbf{I} - \mathbf{T}_b)(\mathbf{F}^H\mathbf{M}_b^k\mathbf{d}_b^{k-1}), \\
9 & \qquad \mathbf{d}_m^k = \mathbf{d}_m^{obs} + (\mathbf{I} - \mathbf{T}_m)(\mathbf{F}^H\mathbf{M}_m^k\mathbf{d}_m^{k-1}), \\
10 & \qquad \mathbf{M}_s^k = \boldsymbol{\Upsilon}_0(\mathbf{M}_b^k + \mathbf{M}_m^k), \\
11 & \qquad \mathbf{d}_s^{k-1} = \mathbf{d}_s^{k-1} + (\mathbf{I} - \mathbf{T}_s)(\mathbf{d}_m^k - \mathbf{d}_b^k), \\
12 & \qquad \mathbf{d}_s^k = \mathbf{d}_s^{obs} + (\mathbf{I} - \mathbf{T}_s)(\mathbf{F}^H\mathbf{M}_s^k\mathbf{d}_s^{k-1}), \\
 & \text{End}
\end{array}
\tag{8}
$$

where $\boldsymbol{\Upsilon}_{\lambda_k}$ is a mask building function defined as

$$
\boldsymbol{\Upsilon}(\mathbf{D}) = \begin{cases} 1 & |D(i)| > \lambda_k, \\ 0 & |D(i)| \leq \lambda_k, \end{cases} \tag{9}
$$

The matrices $\mathbf{M}_b$, $\mathbf{M}_m$, and $\mathbf{M}_s$ are the mask function correspondent to baseline, monitor and difference data, respectively. In lines 1 and 2 of the algorithm for the initial estimates of baseline and monitor surveys we replace the missing samples in each survey by the available ones from another survey. This operation results in utilizing any coherent information between baseline and monitor surveys. Lines 3-5 of the algorithm identifies the common samples of the baseline and monitor survey and replaces their difference in the difference section vector. In lines 6 and 7 the mask functions for baseline and monitor surveys are created and are used in lines 8 and 9 to recover updated samples of baseline and monitor surveys, respectively. In line 10 we combine the baseline and monitor mask functions to create the mask function for difference section. Finally, in line 11 we obtain the latest difference section based on the updated baseline and monitor surveys and in line 12 impose the thresholding criteria using the mask function of the difference section. In order to apply this adaptive POCS subtraction strategy to the seismic data we implement the Equation 8 for each single frequency of the data in the *f-x* domain.

EXAMPLES

Synthetic linear seismic events

We begin our analysis of the POCS time-lapse survey subtraction with a synthetic seismic example. Figure 1a resembles a synthetic baseline survey composed of 3 linear events. Figure 1b shows the seismic section for monitor survey. The monitor survey has four linear events, on which three events coincide with the events on the baseline surveys. Also two of the common events between baseline and monitor survey have different amplitude. Next, we randomly eliminate 50% of the traces in baseline and monitor surveys and the resulted sections are shown in Figures 2a and 2b, respectively. Figures 2c and 2d show the *f-k* spectra of the data in Figures 2a and 2b, respectively.

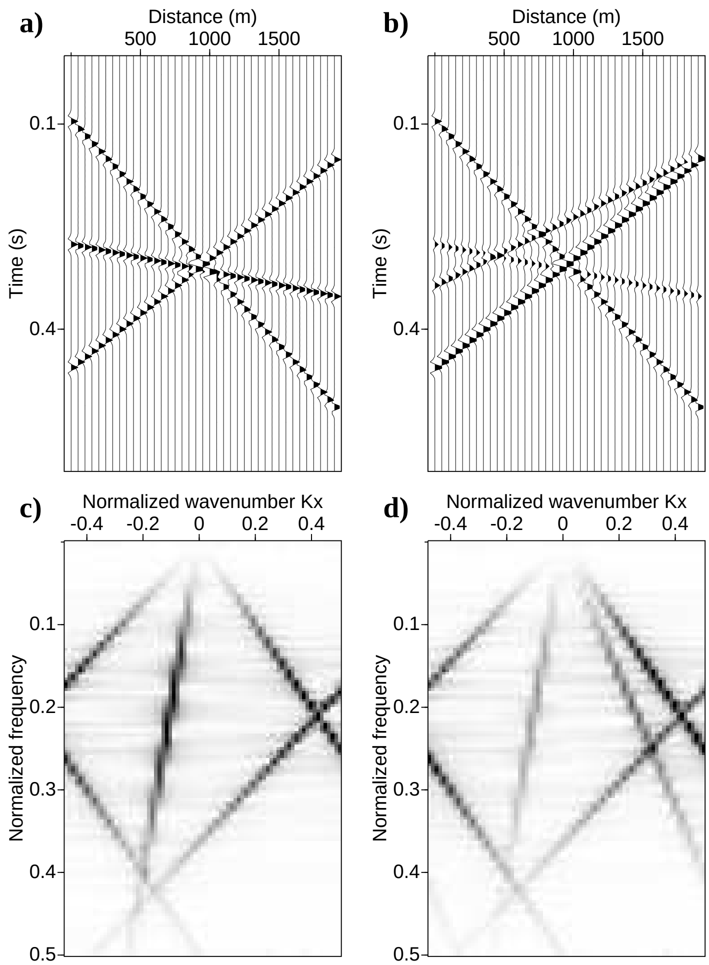

FIG. 1. a) Synthetic baseline survey section with 3 linear events. b) Synthetic monitor survey section with one new event and 2 events with different amplitudes compared to the baseline survey section. c) and d) are the *f-k* spectra of a and b, respectively.

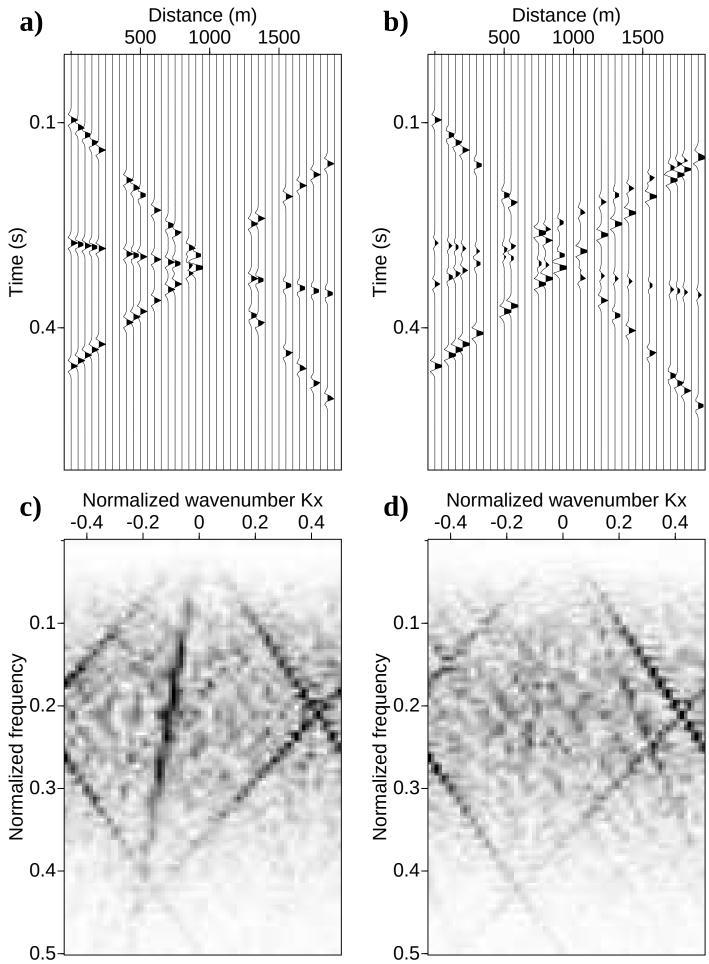

FIG. 2. a) Data in Figure 1a after eliminating 50% of the traces. a) Data in Figure 1b after eliminating 50% of the traces. c) and d) are the *f-k* spectra of a and b, respectively.

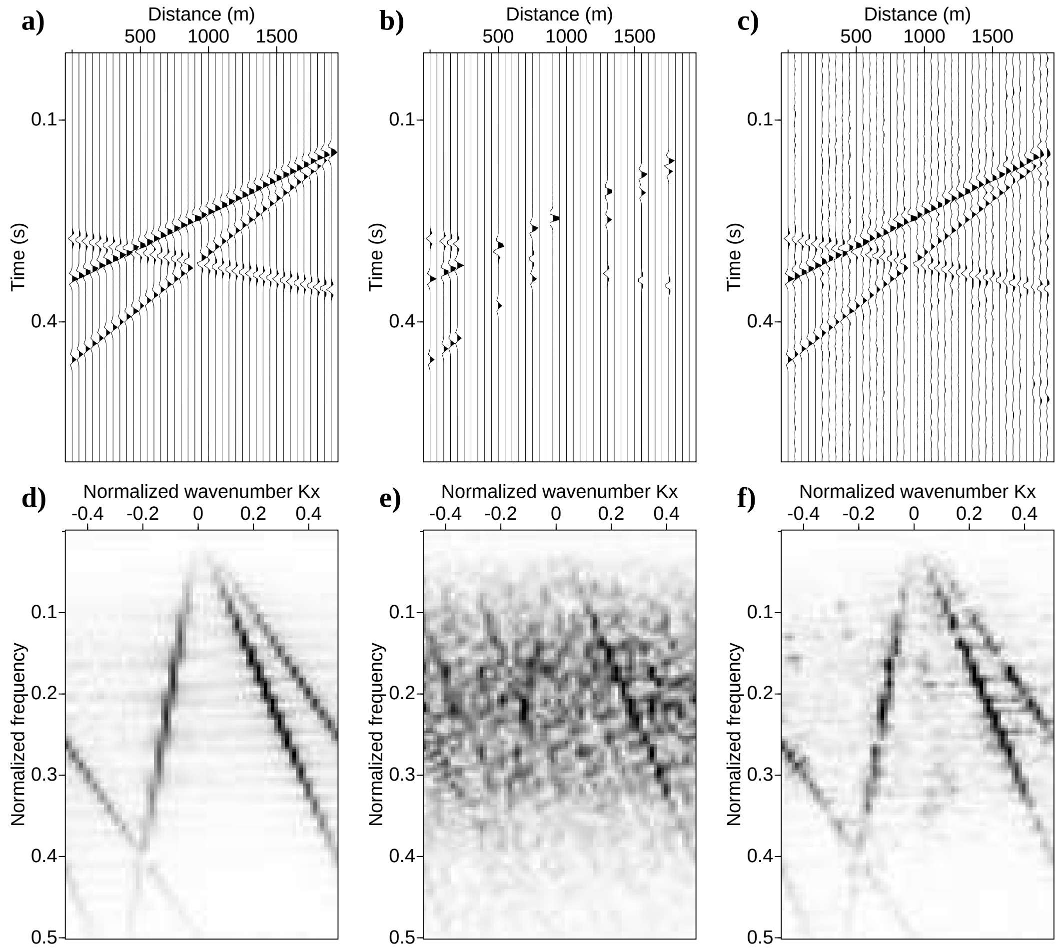

FIG. 3. a) The difference between Figures 1a and 1b. b) The difference between the common available trace in Figures 2a and 2b. c) Reconstructed difference section using proposed method. d), e), and f) are the *f-k* spectra of a, b, and c, respectively.

Figure 3a shows the subtraction of the original baseline and monitor surveys. For the randomly sampled baseline and monitor surveys, we can only subtracted the traces that are available in both baseline and monitor surveys. Figure 3b depicts difference section of baseline and monitor surveys for the common traces in both surveys. It is clear that the percentage of available traces in the difference section is lower than the randomly sampled baseline and monitor surveys. Figure 3c shows the reconstructed difference section using the proposed algorithm in this article. The reconstructed difference section shows a nice amplitude recovery of events. This clearly shows that the proposed algorithm does not sacrifice the recovery of low amplitude events in the presence of high amplitude ones. Figures 3d, 3e, and 3f show the *f-k* spectra of data in Figures 3a, 3b, and 3c, respectively. It is clear that the reconstruction of Figure 3f by just using the information in Figure 3d and sparsity measures would be very challenging. The simultaneous usage of the information in both baseline and monitor surveys results, however, in a successful reconstruction of the difference data.

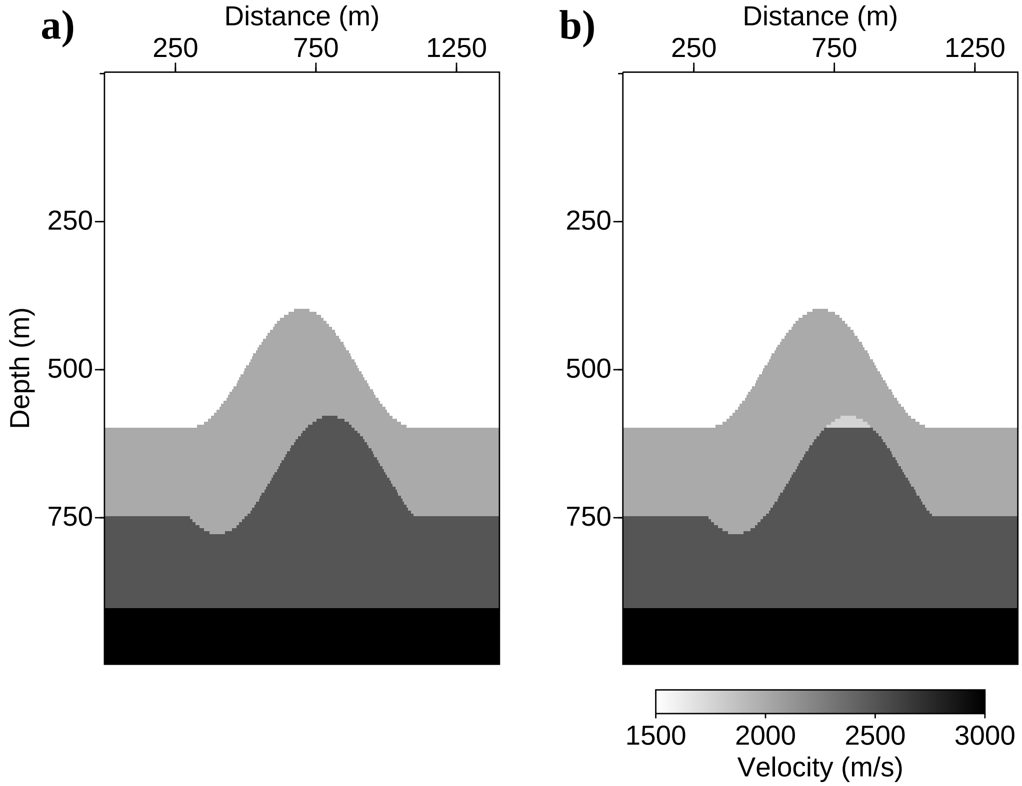

FIG. 4. Synthetic subsurface model for a) baseline survey and b) monitor survey.

Synthetic time-lapse survey

To further investigate the performance of the proposed algorithm we modeled a synthetic time-lapse survey. Figures 4a and 4b shows the earth models for baseline and monitor surveys, respectively. The subsurface baseline model contains three layers with two anticline structures. To simulate the monitor model we placed a low velocity region under one of the anticlines. Figures 5a and 5b show the modeled data using the finite-difference wave-field solution for baseline and monitor surveys, respectively. The data set contains 29 shot gathers each with 281 traces. For illustration purposes we have only plotted 1 out of 4 shot gathers in Figures 5a and 5b. Figure 5c shows the difference between Figures Figures 5a and 5b.

We picked one shot gather from simulated time-lapse survey to examine the performance of proposed reconstruction method. Figures 6a, 6b, and 6c shows the baseline, monitor, and difference section, respectively. Next we randomly eliminated 50% of the traces from both baseline and monitor shot gathers to obtain the sections in Figures 7a and 7b, respectively. Figure 7c shows the difference between randomly sampled baseline and monitor surveys. Notice that only 30% of traces are available in Figure 7c which are the common traces between randomly sampled baseline and monitor surveys. We used the adaptive POCS subtraction routine with spatial windows of 40 traces and 20 overlap traces between adjacent spatial windows to reconstruct the difference section. Figures 8a and 8b show the difference section with missing traces and the reconstructed difference sections, respectively. Figure 8c shows the reconstruction error, namely the difference between Figures 8b and 6c. The reconstructed difference section shows a very good amplitude fidelity

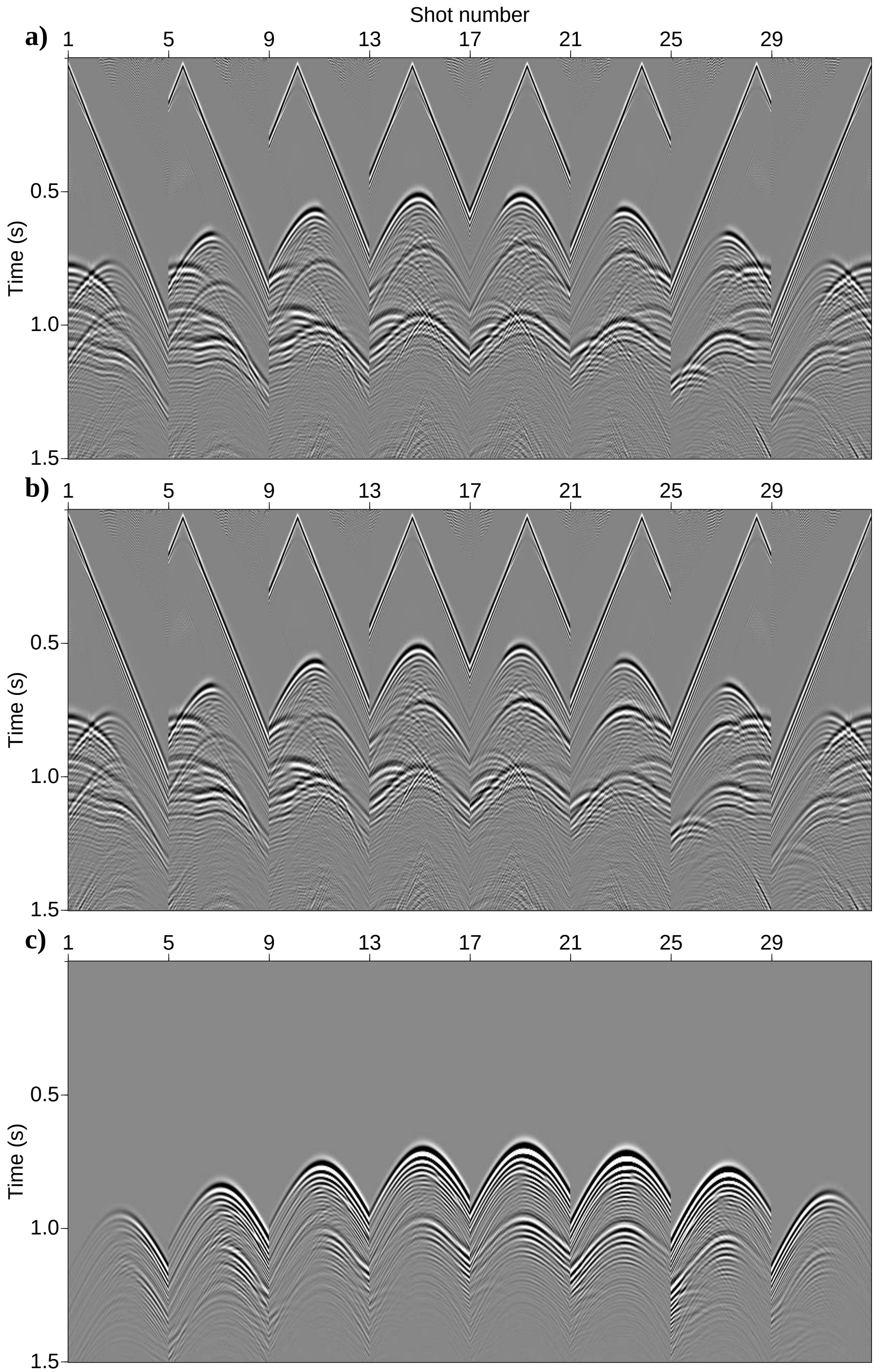

FIG. 5. Finite-difference modeled seismic shot gathers. a) Baseline survey shot gathers. b) Monitor survey shot gathers. c) The difference between a and b.

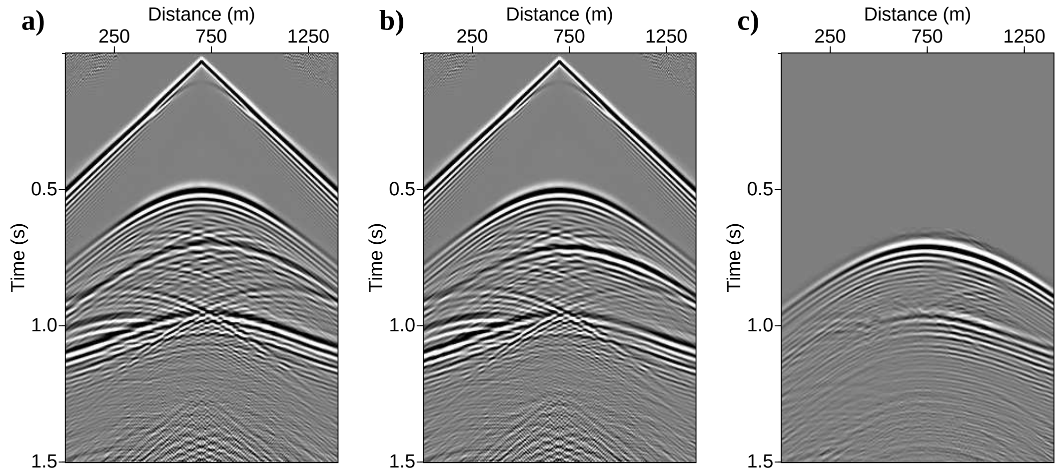

FIG. 6. a) Original baseline survey shot gather. b) Original monitor survey shot gather. c) The difference between a and b.

to the original difference section.

DISCUSSION

One of the crucial factor that affects the success and convergence of POCS algorithm is the implantation of the thresholding criteria. The most practical way of finding the optimal threshold value is the data-driven approach. To achieve this goal, first the data in Fourier domain is sorted in ascending order and the top $\alpha\%$ of the coefficients are kept and the remaining small values are eliminated. As the number of iterations increases one can make the α value larger in order to recover the events with low amplitude. Also, for the method proposed in this article one needs to use larger threshold values λ for the difference section than baseline and monitor data. This is due to the fact that the mask function for the difference section $\mathbf{M}_s$ is built from combining the mask functions of baseline $\mathbf{M}_b$ and monitor $\mathbf{M}_m$ surveys. This will guarantee the elimination of the artifacts that might fall under the mask function of the difference section. It is also important to smooth the corners of the mask functions to avoid creating Fourier artifacts by convolving it with a smoothing (Gaussian, Hanning, ...) window.

The proposed algorithm works only for the regular grids with randomly missing traces. In the case of pure irregular sampling the data should be binned into a regular grid. The binning as well as statics in the data might cause some mismatches between the baseline and monitor survey traces. If these effects are mild the sparsity measures imposed inside the POCS algorithm can remove them. However, in the presence of strong statics some pre-processing steps is necessary to remove the statics before applying the adaptive POCS subtraction of time-lapse surveys.

CONCLUSIONS

We propose an adaptive POCS subtraction strategy to obtain the difference data section from irregularly sampled time-lapse surveys. The proposed method utilize the coherent

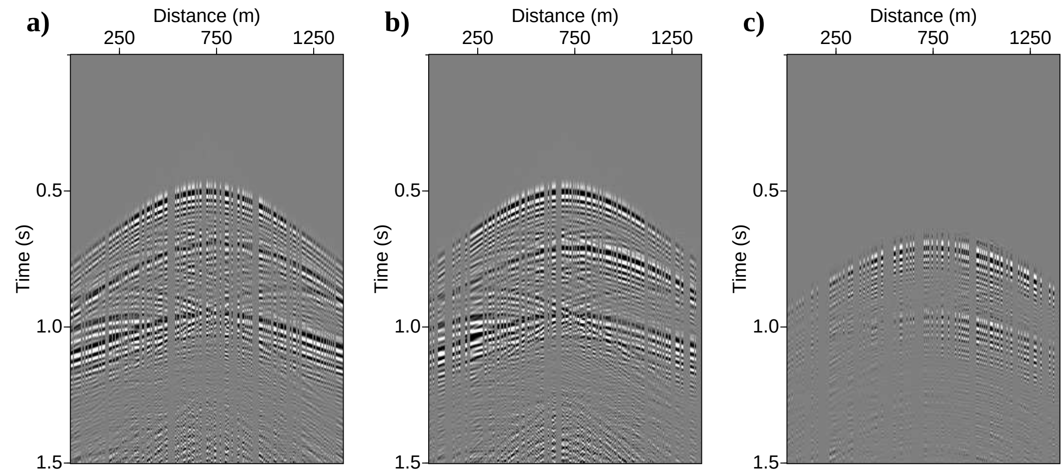

FIG. 7. a) Baseline survey shot gather with 50% randomly missing traces. b) Monitor survey shot gather with 50% randomly missing traces. c) The difference between a and b for the common available traces with only 30% available traces.

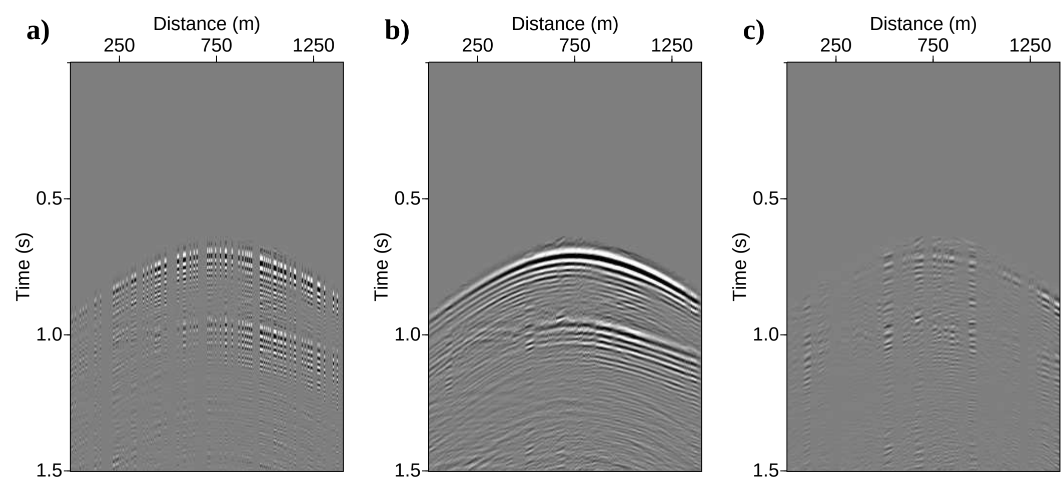

FIG. 8. a) Original difference section with 70% missing traces. b) Reconstructed difference section using the proposed adaptive POCS subtraction. c) The difference between Figure 6c and b.

events of both baseline and monitor surveys for robust recovery of difference data. The POSC reconstruction is applied independently for each frequency of seismic data in the *f-x* domain. The proposed POCS algorithm is very effective in recovering the difference section with very small number of available traces. The performance of method is examined using the synthesized time-lapse survey examples.

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