Aliasing and reverse-time migration

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ABSTRACT

Reverse-time migration (RTM) is a powerful migration method provided that an accurate velocity model can be constructed. In any discrete operation on a sampled field we need to consider aliasing before, during, and after that operation. Reverse-time migration takes a sampled seismic experiment and creates an image of the subsurface. In reverse-time migration the sampled wavefield and the forward modelled shotfield are propagated as a solution to the wave equation. This often requires re-sampling the data to a regular sampled and finer grid for wavefield propagation. Additionally aliasing can occur when cross-correlating the two wavefields to form an image.

INTRODUCTION

Aliasing occurs in prestack Kirchhoff migration when the data is sweeped from steep angles into its correct position. Steeply dipping data is aliased so that seismic data needs to be interpolated to reduce the trace spacing or by applying anti-aliasing filters (Gray, 1992; Biondi, 2001; Zhang et al., 2003) during migration which results in the inability to image steeply dipping beds and faults.

Aliasing can occur in two places for prestack wavefield continuation migrations. The downward propagated record wavefield and the forward modelled shotfield must be adequately sampled for the propagation algorithm. Aliasing can also occur during the application of the cross-correlation imaging condition. This requires that the downward propagated shot and receiver fields are sampled at half the Nyquist sampling (Zhang et al., 2003).

BORN FORWARD MODELLING AND ITS INVERSE

Let $G_0(\vec{x}, \vec{x}_{\vec{s}}, t)$ the causal Green's function for the acoustic wave equation,

$$\left[\frac{\partial^2}{\partial t^2} - c_0(\vec{x})^2 \Delta\right] G_0(\vec{x}, \vec{x}_{\vec{s}}, t) = \delta(\vec{x} - \vec{x}_{\vec{s}}), \tag{1}$$

with a background velocity c_0 and where $\Delta = \partial_{xx} + \partial_{yy} + \partial_{zz}$.

The Born scattering approximation can be used to create a synthetic seismic shotrecord or as a bases for a linearized inversion. For seismic data recorded at point $\vec{x}_{\vec{r}}$ for a shot at location at $\vec{x}_{\vec{s}}$ with a velocity $c(\vec{x})$, a background velocity c_0 and a velocity perturbation $\delta c(\vec{x}) = c(\vec{x}) - c_0(\vec{x})$, the weak scattering approximation (Schuster, 2002) in the frequency domain is

$$D(\vec{x}_{\vec{r}}, \vec{x}_{\vec{s}}, \omega) = \int G_0(\vec{x}_{\vec{r}}, \vec{x}, \omega) G_0(\vec{x}, \vec{x}_{\vec{s}}, \omega) m(\vec{x}) d\vec{x}, \tag{2}$$

where $m(\vec{x}) = 2\delta c/c_0^3$ is the scattering potential. Let G_0^* be the solution to the adjoint of equation (1). An approximate inverse of equation (2) can be formed by applying the adjoint of the forward modelling operator,

$$m(\vec{x}) \approx \omega^2 \int G_0^*(\vec{x}, \vec{x}_{\vec{s}}, \omega) G_0^*(\vec{x}_{\vec{r}}, \vec{x}, \omega) D(\vec{x}_{\vec{r}}, \omega) d\vec{x}_{\vec{r}}. \tag{3}$$

The Green's function $G_0^*(\vec{x}, \vec{x}_{\vec{s}}, \omega)$ is called the forward propagated shotfield. A band-limited version of G_0^* can be calculated by finite-differencing solution of equation (1) with the delta function multiplied with a wavelet $W(\omega)$. The greens function applied to the recored data $G_0^*(\vec{x}_{\vec{r}}, \vec{x}, \omega) D(\vec{x}_{\vec{r}}, \omega)$ is called the back propagated receiver field. In the time domain the receiver field is,

$$\begin{aligned} R_{\vec{x}_{\vec{s}}}(t, \vec{x}) &= G_0(\vec{x}_{\vec{r}}, \vec{x}, t) \otimes_t D(\vec{x}_{\vec{r}}, t) \\ &= \int_0^{T_{max}} G_0(\vec{x}_{\vec{r}}, \vec{x}, \tau) D(\vec{x}_{\vec{r}}, t + \tau), \end{aligned}$$

where T_{max} is the length of the seismic record and $\otimes_t$ is cross-correlation in the time variable. Integrating the recored wavefield $D(\vec{x}_{\vec{r}}, t+\tau)$ with Green's function $G_0(\vec{x}_{\vec{r}}, \vec{x}, \tau)$ can be accomplished by solving the wavefield equation using the trace as a time shifted source,

$$\left[\frac{\partial^2}{\partial t^2} - c_0(\vec{x})^2\Delta\right] R(\vec{x}, t) = D(\vec{x}, t_{max} - t). \tag{4}$$

For converted waves from a P-wave source the analogous forward born modelling that is kinematically correct is

$$D^{PS}(\vec{x}_{\vec{r}}, \vec{x}_{\vec{s}}, \omega) = \int G_0^P(\vec{x}_{\vec{r}}, \vec{x}, \omega) G_0^S(\vec{x}, \vec{x}_{\vec{s}}, \omega) m^{PS}(\vec{x}) d\vec{x}, \tag{5}$$

here m^{PS} is the scatter potential for PS conversion. The forward Born approximation for elastic waves (Snieder, 2002) will have much better dynamic amplitudes. Applying the adjoint to the recorded PS-wavefield gives an approximate inverse,

$$m^{PS}(\vec{x}) \approx \omega^2 \int G_0^{P*}(\vec{x}, \vec{x}_{\vec{s}}, \omega) G_0^{S*}(\vec{x}_{\vec{r}}, \vec{x}, \omega) D^{PS}(\vec{x}_{\vec{r}}, \omega) d\vec{x}_{\vec{r}}. \tag{6}$$

Migration aliasing

In the cross-correlation imaging condition the backpropgated receiver field $R(t, \vec{x})$ is cross-correlated at zero lag with forward propagated shotfield $S(t, \vec{x})$. The simplest imaging condition is

$$I(\vec{x}) = \int_0^T S(t, \vec{x}) R(t, \vec{x}) dt, \tag{7}$$

for a shotrecord of length T. This imaging condition is equivalent to by Parsavel's theorem to the imaging condition in the frequency domain used by wavefield continuation migrations,

$$I(\vec{x}) = \int_0^{\omega_{max}} S(\omega, \vec{x}) R(\omega, \vec{x}) d\omega. \tag{8}$$

In practice both receiver and shot fields are sampled with a spacial sampling of δx. The Nyquist frequency for any component of the wavenumber is $|k_{x_i}| = \pi/2\delta x$. Once the wavefields are correlated the largest wavenumber will double the maximum in either shot or the receiver field Zhang et al. (2003). Alternatively equation (7) in the wavenumber domain is

$$I(\vec{k}) = \int_0^T S(t,\vec{k}) * R(t,\vec{k}) dt, \tag{9}$$

where $*$ denotes convolution in the wavenumbers. If the data has a maximum frequency f_{max} and a minimum velocity of the model is c_{min} then the maximum wavenumber is $k_{max} = 2f_{max}/c_{min}$.

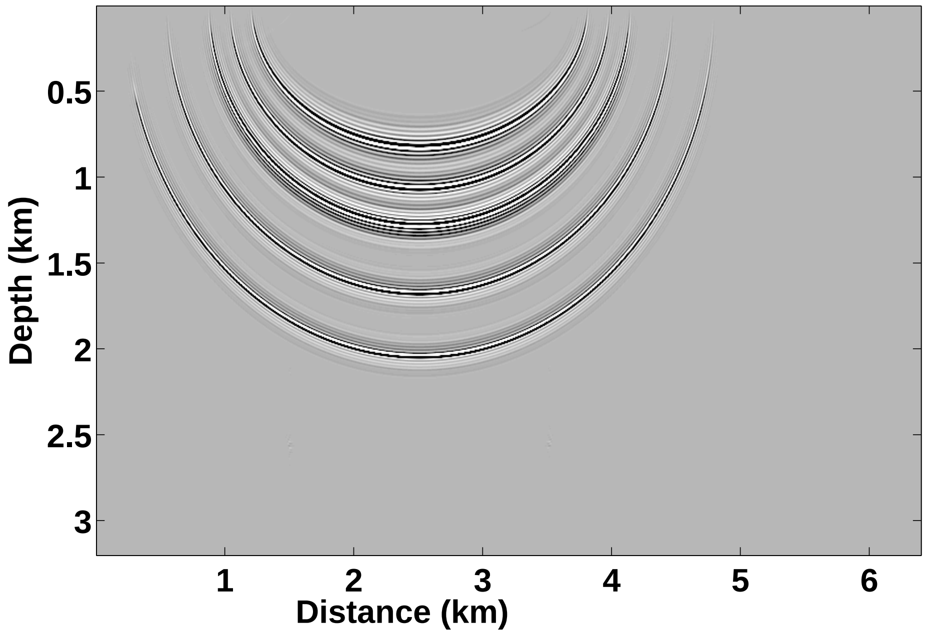
Since $R(t,\vec{k})$ and $S(t,\vec{k})$ are convolved in the wavenumber domain the maximum frequency in I will doubled that in R or S. Hence prior to the application of imaging condition $R(t,\vec{k})$ and $S(t,\vec{k})$ must be sampled at half the Nyquist frequency sampling rate.

DISCUSSION

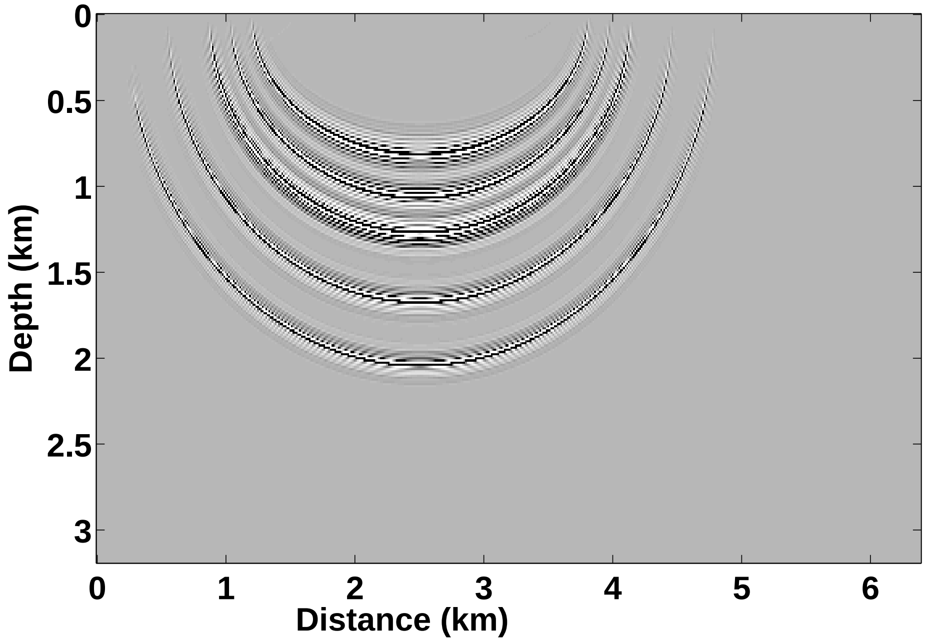
We now look at a few migration impulse responses to better understand migration aliasing. In the first example a constant $2000m/s$ velocity. The migration trace is a far offset trace with 5 Ormsby wavelets with a maximum frequency of $60Hz$ and the grid spacing is $12m$. Figure 1(a) is the impulse response of a single trace to reverse-time migration with the shot and receiver field resampled to half of the Nyquist. Figure 1(b) is the same impulse response however there was no resampling done and the migrated image contains aliasing.

CONCLUSIONS

We presented an aliasing condition for the imaging condition for reverse-time migration. If the the shotfield and backpropagated receiver field have spatial frequencies up to the Nyquist then it necessary to resample these fields to half the Nyquist prior to the application of the imaging condition.



(a)



(b)

FIG. 1. (a) Impulse migration response for a single far offset trace resampling prior to crosscorration to half Nyquist. (b) Impulse migration response without resampling.

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