

Estimation of Q: a comparison of different computational methods

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ABSTRACT

In this article, four methods of Q estimation are investigated: the spectral-ratio method, a match-technique method, a spectrum modeling method and a time-domain match-filter method. Their accuracy and the reliability of Q estimation is evaluated using synthetic data. Testing results demonstrate that the time-domain match-filter method is more robust to noise and more suitable for application to reflection data than the other three methods.

INTRODUCTION

The attenuation of seismic waves is an important property of the earth, which is of great interest to geoscientist. Seismic attenuation can be quantified by the quality factor Q. The knowledge of Q is very desirable for improving seismic resolution, facilitating AVO amplitude analysis, understanding the lithology of subsurface better and providing useful information about the porosity and fluid or gas saturation of reservoir.

Conventionally, Q is estimated from transmission data, such as VSP data (Hague, 1981; Tonn, 1991), crosswell (Quan and Harris, 1997; Neep et al., 1996) and sonic logging (Sun et al., 2000). There are various methods for Q estimation such as analytical signal method (Engelhard, 1996), spectral-ratio method (Bath, 1974), the centroid frequency-shift method (Quan and Harris, 1997), the match-technique method (Raikes and White, 1984; Tonn, 1991), and the spectrum-modeling method (Janssen et al., 1985; Tonn, 1991; Blias, 2011), and each method has its strengths and limitations. An extensive comparison between various methods for Q estimation was made by Tonn (1991) using VSP data, and a conclusion was made that the spectral-ratio method is optimal in the noise-free case. However, the estimation given by spectral-ratio method may deteriorate drastically with increasing noise (Patton, 1988; Tonn, 1991). The question of reliable Q estimation remains. In addition, it is more useful to estimate Q from the surface reflection data. For Q estimation from reflection data, the tuning effect (Sheriff and Geldart, 1995) of local thin-beds should be addressed properly. Dasgupta and Clark (1998) proposed a Q versus offset (QVO) method for estimating Q from surface data, which essentially applied the classic spectral-ratio method on a trace by trace basis to the designatured and NMO corrected CMP gather. Hackert and Parra (2004) proposed an approach to remove this tuning effect from the QVO method using reference well log data. Generally, estimating Q from noisy data or surface reflection data needs further investigation.

A time-domain match-filter method for Q estimation was proposed by Cheng and Margrave (2012) and was shown to be robust to noise and suitable for application to surface reflection data. Theoretically, the match-filter method is a sophisticated wavelet-modeling method, which is a time-domain alternative to spectrum-modeling method (Janssen et al., 1985; Tonn, 1991; Blias, 2011). The spectrum-modeling method is a

modified approach to the spectral-ratio method without taking division of spectra. In addition, the match-filter method and the match-technique method (Raikes and White, 1984; Tonn, 1991) employ the idea of matching at different stages of their Q-estimation procedures. Therefore, the above four methods all have theoretical connections but are distinctly different. It is worthwhile to make a comparison between these methods in terms of their underlying theory, accuracy and reliability of estimation results.

The purpose of our work is to investigate the four different methods for Q estimation mentioned above. This paper is organized as follows: the first part introduces theory of Q-estimation methods. Then, some numerical examples will be used to evaluate their performance. Finally, some conclusions are drawn from results of the examples.

THEORY OF Q-ESTIMATION METHODS

The theory of the constant Q model for seismic attenuation is well established (Futterman, 1962; Aki and Richards, 1980). Suppose that a seismic wavelet with amplitude spectrum $|S_1(f)|$ has a amplitude spectrum $|S_2(f)|$ after traveling in the attenuating media for an interval time t . Then, we have

$$|S_2(f)| = G|S_1(f)| \exp\left(\frac{-\pi f t}{Q}\right), \quad (1)$$

where f is the frequency, G is a geometric spreading factor. More generally, G can represent all the frequency independent amplitude loss in total, including spherical divergence, reflection and transmission loss.

For Q estimation, VSP data can be approximately regarded as reflection data with isolated reflectors. So, we use the reflection data to form the Q-estimation problem. Assume that a source wavelet $s(t)$ with a spectrum $S(f)$ travel through layered earth with a corresponding reflectivity $r(t)$ in two way time, and $g(t)$ denotes the geometric spreading loss of amplitudes. Then, for an acoustic/elastic medium, the reflected signal $a(t)$ can be given by

$$a(t) = g(t) \int_{-\infty}^{\infty} s(\tau) r(t - \tau) d\tau. \quad (2)$$

Consider a locally reflected wave $a_1(t)$, i.e. a windowed part of $a(t)$ has the contribution from a corresponding subset of reflectivity, $r_1(t)$, which is around two way time t_1 . From (2), we have

$$a_1(t) \approx g(t) \int_{-\infty}^{\infty} s(\tau) r_1(t - \tau) d\tau. \quad (3)$$

Then the spectrum of the localized signal $a_1(t)$ near time t_1 can be approximated by

$$A_1(f) \approx g(t_1)S(f)R_1(f), \quad (4)$$

where $R_1(f)$ is the Fourier transform of $r_1(t)$ and we assume $g(t)$ changes slowly with respect to $s(t)$. If the attenuation of the layered medium is taken into account and the attenuation mechanism can be described by the constant Q model, equation (4) should be modified as

$$|A_1(f)| \approx g(t_1)|S(f)||R_1(f)| \exp\left(\frac{-\pi f t_1}{Q}\right). \quad (5)$$

Similarly, for a localized reflected signal $a_2(t)$ near time t_2 with a corresponding local reflectivity series $r_2(t)$, we have

$$a_2(t) \approx g(t) \int_{-\infty}^{\infty} s(\tau) r_2(t - \tau) d\tau. \quad (6)$$

when attenuation is taken into account, its amplitude spectrum of $a_2(t)$ can be formulated as

$$|A_2(f)| \approx g(t_2)|S(f)||R_2(f)| \exp\left(\frac{-\pi f t_2}{Q}\right), \quad (7)$$

where $R_2(f)$ is the Fourier transform of $r_2(t)$.

Actually, for absorptive media, the $s(\tau)$ term in equation (3) and (6) should be replaced by their corresponding evolving version $s_1(\tau)$ and $s_2(\tau)$. There are various methods for Q estimation, in which Q is usually derived from the local waves $a_1(\tau)$, $a_2(\tau)$ or their spectra. We will discuss different methods for Q estimation based on the model of local waves given in equation (3), (5), (6) and (7).

Spectral-ratio method

From equation (5) and (7), we have

$$\ln\left(\left|\frac{A_2(f)}{A_1(f)}\right|\right) = \ln\left(\frac{g(t_2)}{g(t_1)}\right) + \ln\left(\left|\frac{R_2(f)}{R_1(f)}\right|\right) - \frac{\pi f(t_2 - t_1)}{Q}. \quad (8)$$

Then, the Q factor can be estimated from fitting a straight line to the logarithmic spectral ratio over a finite frequency range. Assuming the reflectivities are essentially white and there are no significant notches in either spectrum, then the term $\ln\left(\left|\frac{R_2(f)}{R_1(f)}\right|\right)$ can be regarded as nearly constant and the estimated Q has a direct relation with the slope k of the best-fit straight line as

$$Q_{\text{est}} = -\frac{\pi f(t_2 - t_1)}{k}. \quad (9)$$

The above is the basic theory of the classic spectral-ratio method, which is originally derived for application to VSP data. From the viewpoint of Q estimation, the VSP data can be taken as a special case of the reflection data when $r_1(t)$ and $r_2(t)$ represent single isolated reflectors. So, the $\ln\left(\left|\frac{R_2(f)}{R_1(f)}\right|\right)$ term in equation (8) can be approximately constant or, more generally, frequency independent. The computed spectra are smooth when SNR is sufficiently high. In this circumstance, reliable Q estimation can be obtained.

For reflection data, the spectrum of local wavelets can be significantly affected by the corresponding local reflectors, which makes estimating Q from surface data difficult. In this case, $\left|\frac{R_2(f)}{R_1(f)}\right|$ varies with frequency, and Q is not strictly proportional to the slope of the logarithmic spectral ratio given by equation (8). Even when the data is free of noise,

the estimated Q can significantly deviate from the true value. The accuracy of the estimated result depends on both the SNR level and the degree to which $\left| \frac{R_2(f)}{R_1(f)} \right|$ can be taken as frequency independent, i.e. the extent to which $R_2(f)$ resembles $R_1(f)$. A correction method to the tuning effect of local reflectors was discussed by several publications (Raikes and White, 1984; White, 1992; Hackert and Parra, 2004). If well-log data is available, $r(t)$ can be calculated from the impedance, then correction can be made to equation (9) as (Hackert and Parra, 2004)

$$\ln \left(\left| \frac{A_2(f)/R_2(f)}{A_1(f)/R_1(f)} \right| \right) = \ln \left(\frac{g(t_2)}{g(t_1)} \right) - \frac{\pi f(t_2 - t_1)}{Q}. \quad (10)$$

Therefore, more accurate estimation can be expected by the spectral-ratio method based on equation (10). In addition, the estimation result might be more stable when appropriate smoothed versions of $R_2(f)$ and $R_1(f)$ are used.

Spectrum-modeling method

The spectrum modeling method compares just the amplitude spectra of the local wavelets. $|A_1(f)|$ is modified by varying Q until an optimum approximation to $|A_2(f)|$ is obtained. If the L2-norm criterion is used for optimization, Q can be estimated as (Blais, 2011)

$$Q_{\text{est}} = \min_Q \left\| |A_2(f)| - \alpha(Q) |A_1(f)| \exp \left(\frac{-\pi f(t_2 - t_1)}{Q} \right) \right\|^2, \quad (11)$$

where scaling factor $\alpha(Q)$ addresses the frequency-independent energy loss and can be formulated as

$$\alpha(Q) = \frac{\int_{-\infty}^{\infty} |A_2(f)| |A_1(f)| \exp \left(\frac{-\pi f(t_2 - t_1)}{Q} \right) df}{\int_{-\infty}^{\infty} |A_1(f)|^2 \exp \left(\frac{-2\pi f(t_2 - t_1)}{Q} \right) df}. \quad (12)$$

The spectrum-modeling method differs from the spectral-ratio method in the following aspects. Firstly, the criterion used to minimize the objective function for the spectral ratio-method is least-squares error, which is not necessary for spectrum-modeling. The objective function for minimization in equation (11) can be of other criteria, for instance, L1 norm. Secondly, the spectral-ratio method assumes that reflection coefficients and phase velocity of traveling waves are frequency independent (Janssen et al., 1985). Spectrum modeling does not necessarily need this assumption.

Spectrum-modeling method avoids taking spectral division, which can stabilize the estimation in case of noise. In addition, if the L2-norm criterion is used for minimization for spectrum-modeling method, the result can be significantly be affected by the matching for the frequency components with large amplitudes.

Match-technique method

A match technique for Q estimation was proposed by Raikes and White (1984). By matching the two local waves as

$$a_2(t) \approx a_1(t) * h_{12}(t), \quad (13)$$

where * denotes convolution, $h_{12}(t)$ is the forward filter predicting $a_2(t)$ from $a_1(t)$. Similarly, a backward filter $h_{21}(t)$, can be obtained by predicting $a_1(t)$ from $a_2(t)$. Then, the transfer functions $H_{12}(f)$ and $H_{21}(f)$ can be computed from $h_{12}(t)$ and $h_{21}(t)$ by taking Fourier transform. Therefore, the spectral power ratio of the two local waves is given by

$$\frac{P_2(f)}{P_1(f)} = \frac{|H_{12}(f)|}{|H_{21}(f)|}, \quad (14)$$

where $P_1(f)$ and $P_2(f)$ are the power spectra of $a_1(t)$ and $a_2(t)$ respectively. Then, Q can be estimated from the spectral power ratio by the classic spectral-ratio method.

Actually, $h_{12}(t)$ gives an approximate estimation of the attenuation operator combined with a constant scaling factor. The amplitude spectrum of the operator can be distorted in presence of noise. The spectral coherence of $H_{12}(f)$ and $H_{21}(f)$ is used to calculate confidence limit on which the spectral ratio is computed (Raikes and White, 1984). The discrepancy between $|H_{12}(f)|^2$ and $|H_{21}(f)|^{-2}$ indicates the SNR level and interference due to local reflectors. The convergence of the two curves and their confidence limits can be used to define the frequency range within which the spectral ratios are considered reliable. To sum up, the match technique for Q estimation is conducted in four stages. First, power transfer functions $|H_{12}(f)|^2$ and $|H_{21}(f)|^{-2}$ are estimated by matching the two local waves. Then, a frequency range is defined by examining the behavior of power transfer functions. Following that, the power spectral ratios over a specific frequency band are estimated from the geometric mean value of $|H_{12}(f)|^2$ and $|H_{21}(f)|^{-2}$. Finally, Q is estimated from the logarithmic spectral ratios. Generally, the match-technique method described here can be regarded as a spectral-ratio method with spectrum estimation using matching techniques.

Match-filter method

Cheng and Margrave (2012) proposed a match-filter method for Q estimation. The procedure of this method consists of three stages. First the smoothed amplitude spectra of the local waves are computed. Thomson (1982) proposed a multitaper method for smooth, high resolution spectral estimation, which has been shown to provide low variance estimation with less spectral leakage when applied to seismic data (Park et al., 1987; Neep et al., 1996, Cheng and Margrave, 2009). From equation (5) and (7), the smoothed amplitude spectra can be formulated as

$$\overline{|A_1(f)|} \approx g(t_1) \overline{|S(f)|} \overline{|R_1(f)|} \exp\left(\frac{-\pi f t_1}{Q}\right), \quad (15)$$

where the overbar indicates smoothing, and

$$\overline{|A_2(f)|} \approx g(t_2) \overline{|S(f)|} \overline{|R_2(f)|} \exp\left(\frac{-\pi f t_2}{Q}\right). \quad (16)$$

Then, the minimum-phase wavelets with amplitude spectra $\overline{|A_1(f)|}$ and $\overline{|A_2(f)|}$ can be formulated as

$$w_1(t) = F^{-1}(\overline{|A_1(f)|} e^{iH(\ln(\overline{|A_1(f)|}))}) \quad (17)$$

and

$$w_2(t) = F^{-1}(\overline{|A_2(f)|} e^{iH(\ln(\overline{|A_2(f)|}))}), \quad (18)$$

where F^{-1} denotes inverse Fourier transform; H denotes Hilbert transform. Finally, Q can be estimated by

$$Q_{\text{est}} = \min_Q \|w_1(t) * I(Q, t) - \mu w_2(t)\|^2, \quad (19)$$

where $*$ denotes convolution, and $I(Q, t)$ is the impulse-response of the constant Q theory with a quality factor value Q and travel time $(t_2 - t_1)$, which can be formulated as

$$I(Q, t) = F^{-1}\left(\exp\left(\frac{-\pi f(t_2 - t_1)}{Q} - iH\left(\frac{\pi f(t_2 - t_1)}{Q}\right)\right)\right), \quad (20)$$

μ is a constant scaling factor which accounts for frequency independent loss and can be estimated as

$$\mu = \frac{\int_{-\infty}^{\infty} (w_1(t) * I(Q, t)) w_2(t) dt}{\int_{-\infty}^{\infty} w_2^2(t) dt}. \quad (21)$$

Various methods for Q estimation need to calculate the spectrum of short-time signals. Often there are spikes or notches in the spectrum caused by noise or the tuning effect of local reflectors, which causes problem for the Q estimation. Appropriate smoothing of amplitude spectra can improve the estimation results. The multitaper method mentioned above can be used to estimate a smooth amplitude spectrum for the spectral-ratio method, spectrum-modeling method and match-technique method as well.

For the match-filter method described by equation (20), the optimal Q is found by a direct search over an assumed range of Q values with a particular increment since it is a nonlinear minimization. $w_1(t)$ and $w_2(t)$ in equation (17) and (18) can be regarded as the embedded wavelets at time t_1 and t_2 respectively. For attenuating media, the embedded wavelet evolves with time. Then, Q can be estimated by fitting the evolution of embedded wavelet to the attenuation law. Although we estimate the embedded wavelet as minimum phase, in practice, this assumption does not limit our match-filter method to minimum phase sources. The match-filter method just provides a way to match the spectra in time-domain, compared to the frequency-domain match for the spectrum-modeling method. So, the match-filter method is valid as long as the attenuation law given by equation (1) stands.

In addition, the match-filter method can be regarded as a sophisticated wavelet-modeling method. For the wavelet-modeling method (Jannsen et al., 1985), $a_1(t)$ is modified synthetically by attenuation operators corresponding to varying Q values until an optimal approximation to $a_2(t)$ is obtained. The wavelet-modeling method needs that the difference of the phase spectra of the two local waves can be approximated by the phase spectrum of a minimum-phase signal, which may be troublesome in practice. Theoretically, the wavelet-modeling method does not work well for reflection data. By estimating the embedded wavelets of minimum-phase first, the match-filter method ensures that matching of them can be conducted successfully.

The spectral-ratio method, spectrum modeling method and match-technique method are frequency-domain methods. All of them need to define a frequency range where signal dominates for better estimation. For the implementation of these three methods in this paper, the frequency band is given manually as an input parameter. Compared to spectral-ratio method and match-technique method, the match-filter method avoids taking spectral division. Compared to spectrum modeling method, the match-filter method matches the spectra in time domain. In this paper, the performance of these four methods will be evaluated by synthetic data and real VSP data.

NUMERICAL TEST

Synthetic 1D VSP data or reflection data with isolated reflectors

First, we use synthetic noise free VSP data to validate the Q estimation methods theoretically. A synthetic attenuated seismic trace was created by a nonstationary convolution model proposed by Margrave (1998), using two isolated reflectors, a minimum phase wavelet with dominant frequency of 40 Hz and a constant Q value of 80, as shown in figure 1. Using the two local events in figure 1, Q estimations from the four methods are shown in figure 2 - 7. The spectral-ratio method gives the exact estimation as shown in figure 2. From figure 3 and 4, spectrum-modeling method obtained minimum error when $Q=80.11$. The match-technique gives an estimation of 81.76 as shown in figure 5. It is close to the exact Q value but not ignorable for the ideal case, which may be caused by the approximation to estimate the forward and backward filters for the matching of the two local waves. The match-filter method gives an estimation of 80.06 when fitting error is minimized, as shown in figure 6 and 7.

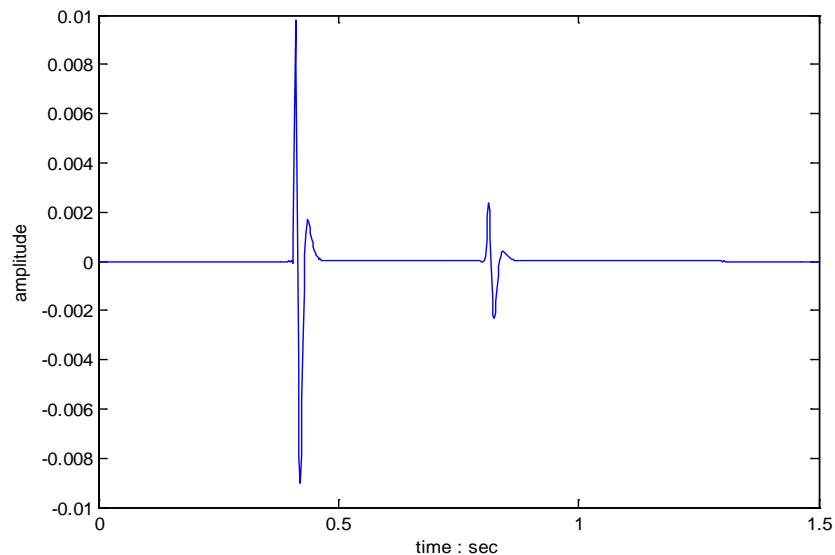


Figure 1. Synthetic seismic trace created with two events, created using two isolated reflectors, a minimum phase source wavelet with dominant frequency of 40 Hz, and a constant Q value of 80.

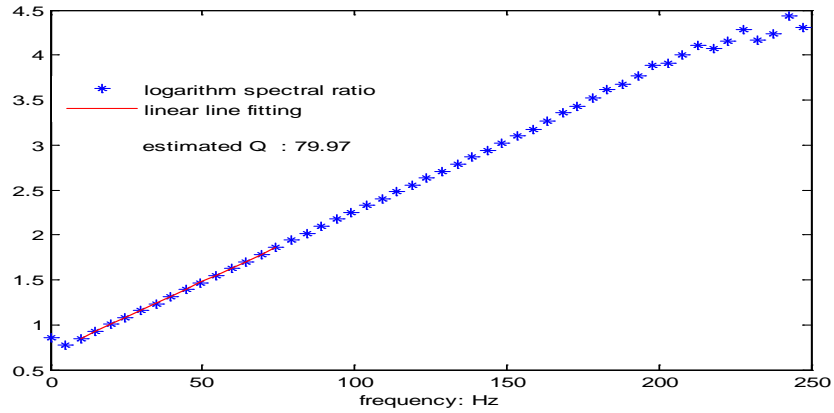


Figure 2. Q estimation by the spectral-ratio method using the two local events shown in figure 1.

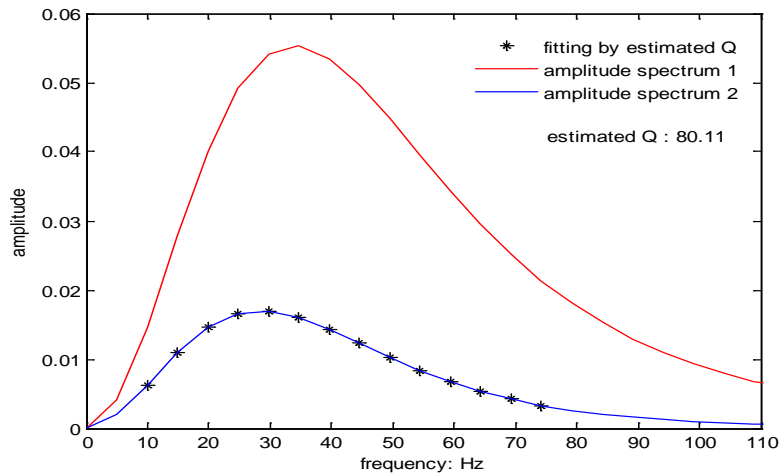


Figure 3. Q estimation by spectrum-modeling method using the two local events shown in figure 1.

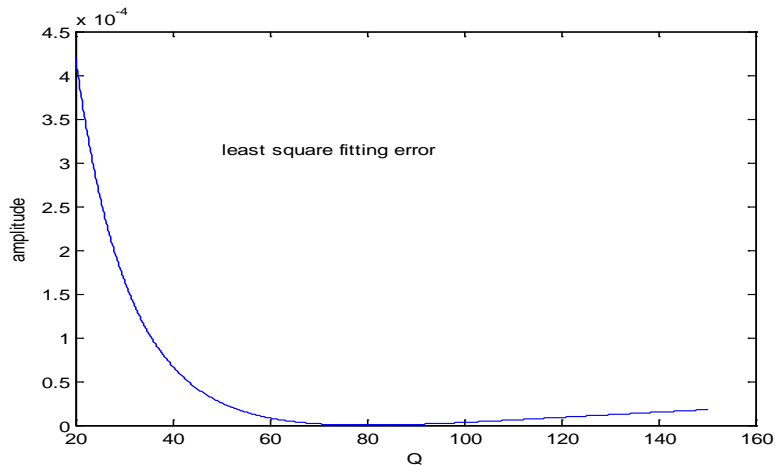


Figure 4. The fitting error curve for Q estimation by spectrum-modeling method corresponding to figure 3.

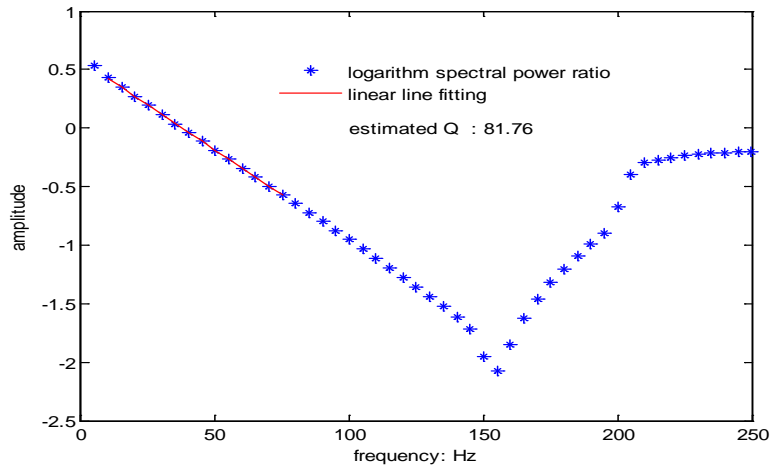


Figure 5. Q estimation by match-technique method using the two local events shown in figure 1.

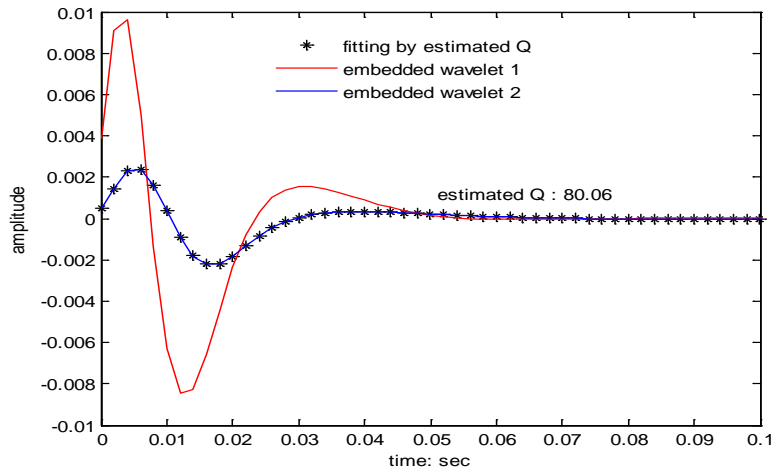


Figure 6. Q estimation by the match-filter method using the two local events shown in figure 1.

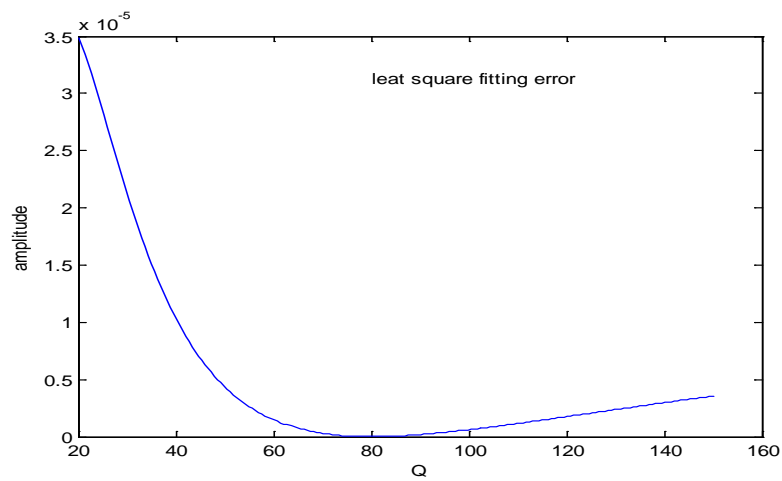


Figure 7. The fitting error curve for Q estimation by match-filter method corresponding to figure 6.

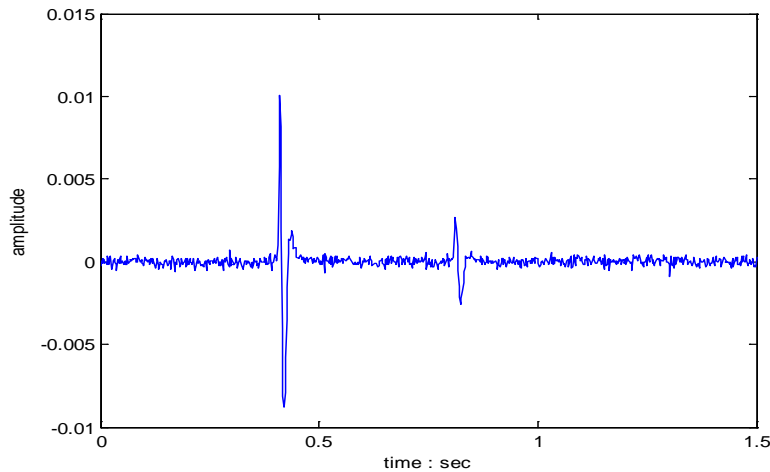


Figure 8. Synthetic seismic trace with noise, created by adding random noise to the seismic trace in figure 1 with SNR=4.

Then, random noise is added to the synthetic data to evaluate the performance of the Q estimation methods in less ideal circumstances. Figure 8 shows a synthetic seismic trace with a signal-to-noise ratio of SNR=4 (we define this in the time domain as the ratio of the RMS values of signal and noise). The amplitude spectra of the two events are shown in figure 9 and figure 10, of which the noise levels are -25 dB and -20 dB respectively. Then Q estimations are conducted using the four methods. For the three frequency-domain methods, a frequency band from 15 Hz to 75 Hz is used for Q estimation. For the match-filter method, a band-pass filter is applied to suppress the noise before estimating the embedded wavelets, and passing bands for the two local waves are 10 Hz – 140 Hz and 10 Hz – 90 Hz respectively. The smoothing of amplitude spectra using multitaper method is not conducted at this time. The results of Q estimation are shown in figure 11 – 14. We can see that the estimation results are deviated from the exact Q value because of the noise. To make a more general comparison of performance for the four estimation methods in presence of noise, 200 seismic traces are created by adding 200 different random noise series of the same level (SNR=4) to the trace shown in figure 1. Then Q estimation is conducted using these noisy data. The histograms of the estimated Q values are shown in figure 15 - 18. We can see that results of match-filter method have the mean value most close to true Q value, and the standard deviation of estimation results are comparable while spectral-ratio method has slightly larger one than other methods. So, the match-filter method gives a slightly better result than other methods. Then, the multitaper method is employed to smooth the spectrum in all four methods, and Q estimation is conducted using 200 seismic traces with a noise level of SNR = 4. The results are shown in figure 19-22. For the estimation results of the three frequency domain methods, the mean values are obviously distorted while their standard deviation values remain the same level as the case when the spectrum estimation is not employed. For match-filter method, the estimation results, as shown in figure 22, are significantly improved when the smoothing of amplitude spectra is employed, which have accurate mean value of 80.79 and a small standard deviation of 7.07. The above results indicate that the three frequency-domain methods can be sensitive to spectrum smoothing. Match-filter method, as a time-domain method, needs the embedded wavelets for matching to be smooth,

which, in turn, make proper spectrum estimation favorable. Therefore, incorporation of spectrum smoothing can help stabilize the estimation result for match-filter method.

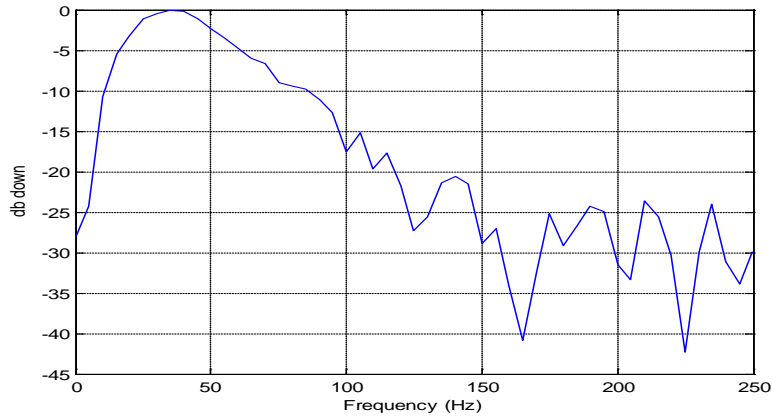


Figure 9. Amplitude spectrum of the local events (0.34s-0.54s) in figure 8.

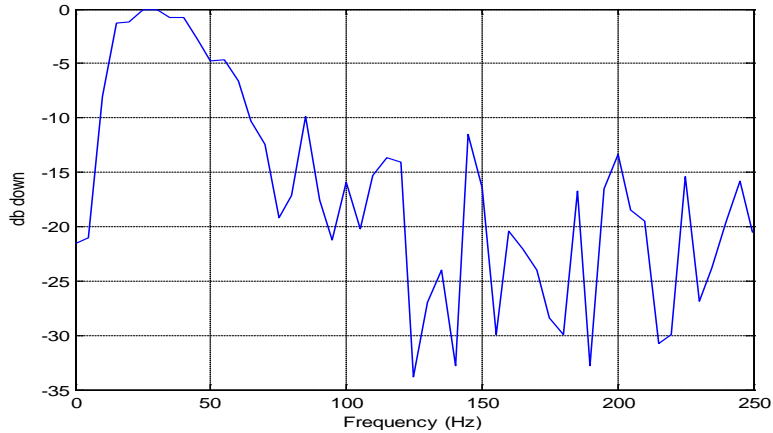


Figure 10. Amplitude spectrum of the events (0.74s-0.94s) second in figure 8.

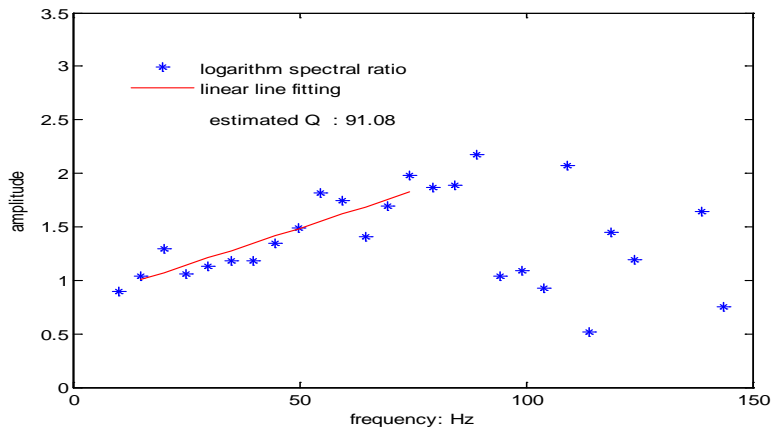


Figure 11. Q estimation by spectral-ratio method using the two local events shown in figure 8.

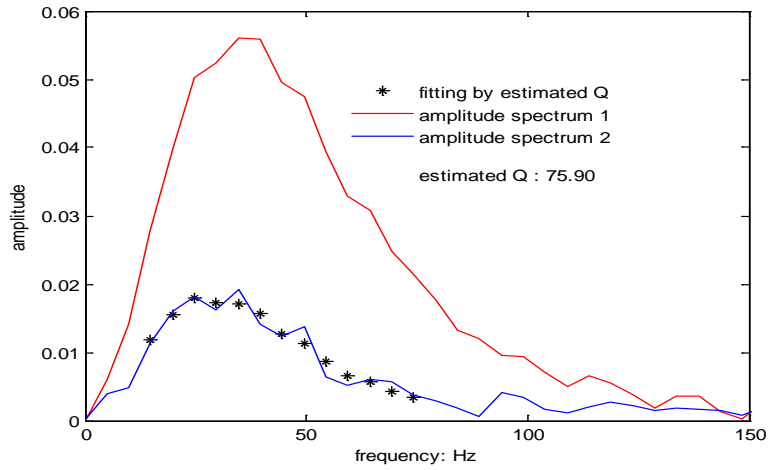


Figure 12. Q estimation by spectrum-modeling method using the two local events shown in figure 8.

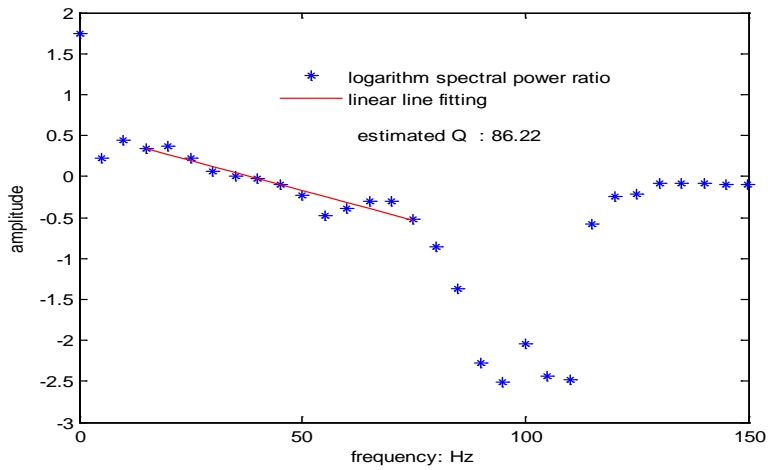


Figure 13. Q estimation by match-technique method using the two local events shown in figure 8.

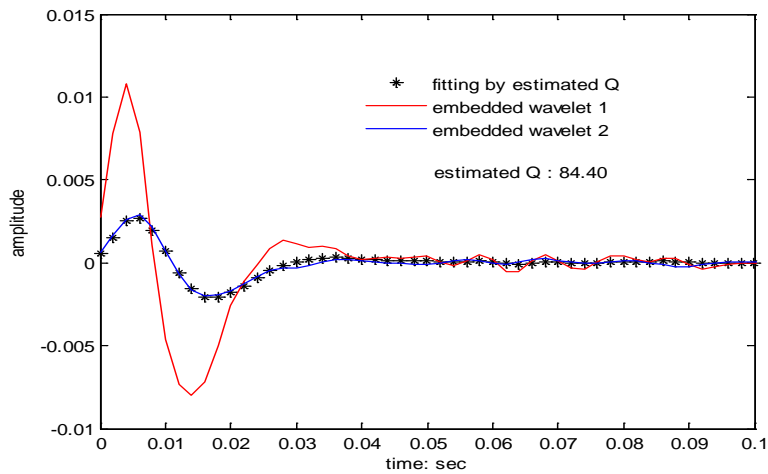


Figure 14. Q estimation by match-filter method using the two local events shown in figure 8.

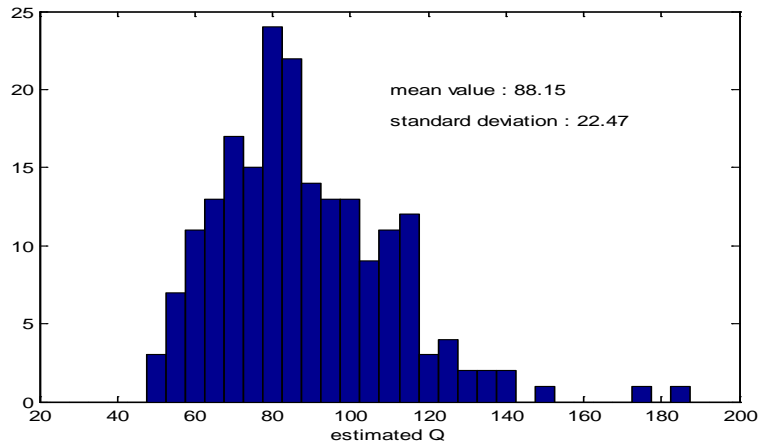


Figure 15. Histogram of the Q values estimated by spectral-ratio method using 200 seismic trace (similar to the one shown in figure 8) with noise level of SNR=4.

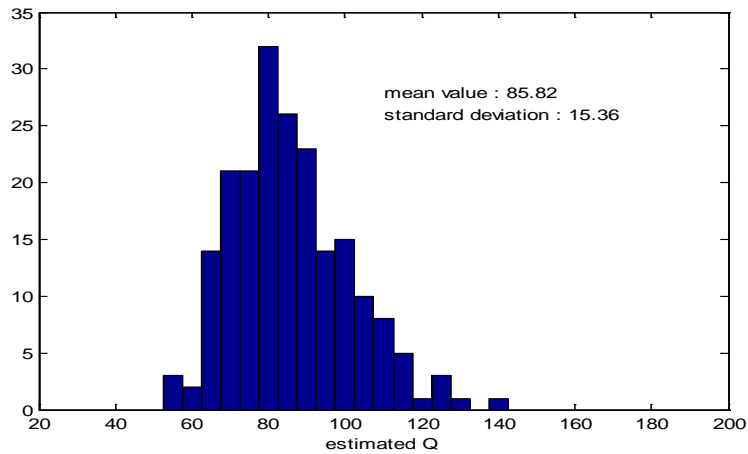


Figure 16. Histogram of the Q values estimated by spectrum-modeling method using 200 seismic trace (similar to the one shown in figure 8) with noise level of SNR=4.

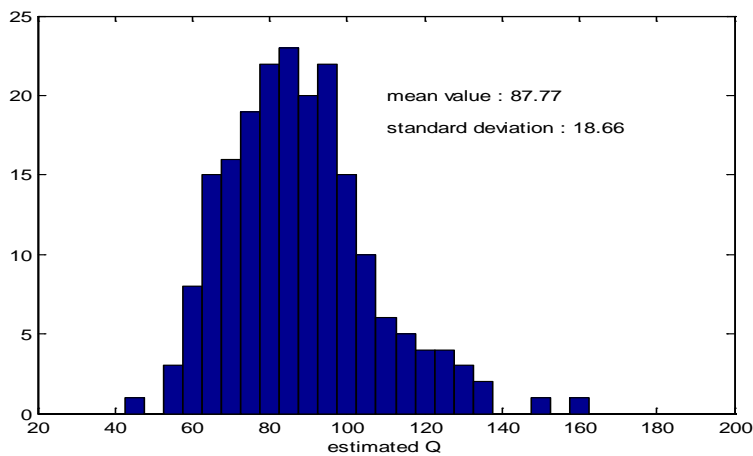


Figure 17. Histogram of the Q values estimated by spectral-ratio method using 200 seismic trace (similar to the one shown in figure 8) with noise level of SNR=4.

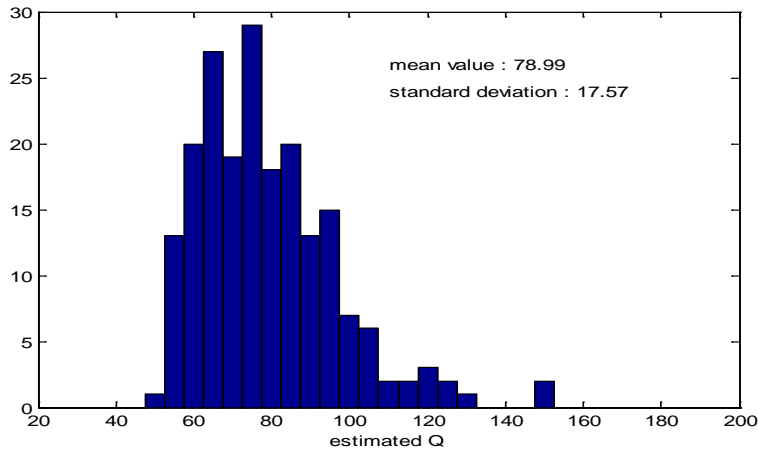


Figure 18. Histogram of the Q values estimated by the match-filter method using 200 seismic trace (similar to the one shown in figure 8) with noise level of SNR=4.

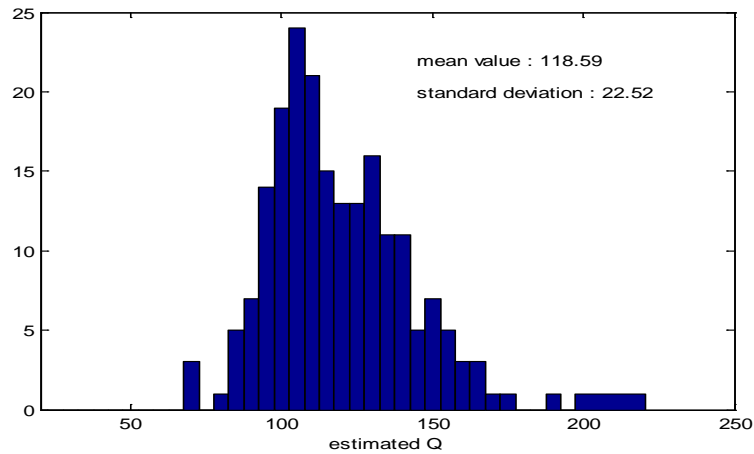


Figure 19. Histogram of the Q values estimated by spectral-ratio method using 200 seismic trace with noise level of SNR=4 (multitaper method for spectrum estimation is employed)

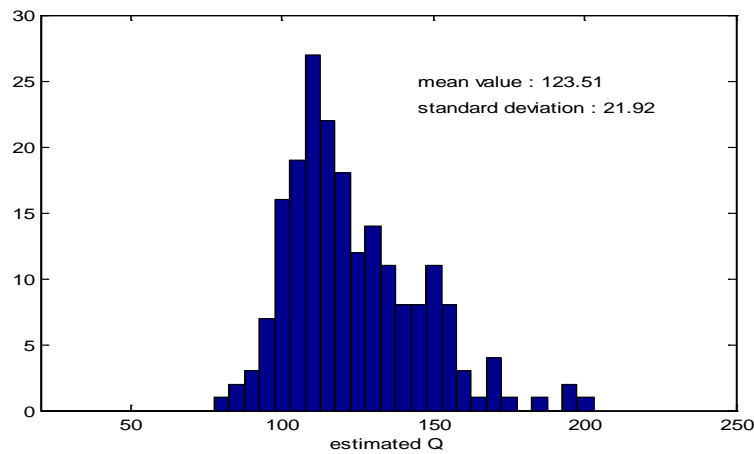


Figure 20. Histogram of the Q values estimated by spectrum-modeling method using 200 seismic trace with noise level of SNR=4 (multitaper method for spectrum estimation is employed)

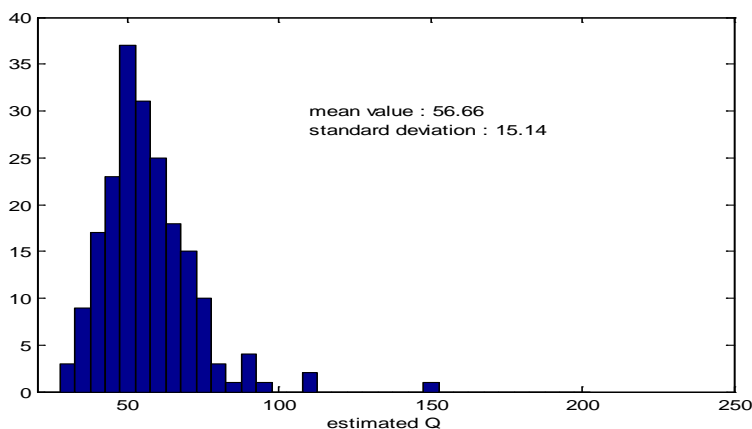


Figure 21. Histogram of the Q values estimated by match-technique method using 200 seismic trace with noise level of SNR=4 (multitaper method for spectrum estimation is employed).

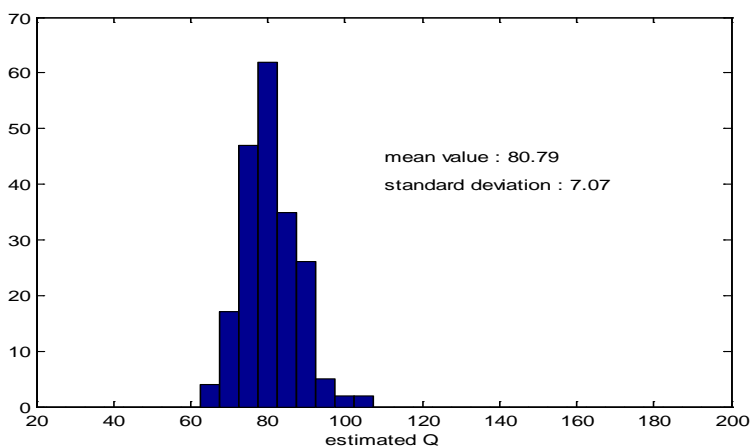


Figure 22. Histogram of the Q values estimated by the match-filter method using 200 seismic trace (similar to the one shown in figure 5) with noise level of SNR=4 (Multitaper method for spectrum estimation is employed).

To evaluate the effect of spectrum smoothing to Q estimation further for the four methods, we use the noise free VSP data to conduct the Q estimation with spectrum estimation, even though the spectrum estimation is not necessary. For the spectral-ratio method, spectrum-modeling method and match-technique method, the band-limited amplitude spectra of the two local events in figure 1 are shown in figure 23, which are estimated by multitaper method with frequency bands of 10Hz- 140Hz and 10Hz- 90Hz respectively. We can see that the original amplitude spectra are modified by the spectrum estimation. Following Q estimation are based on these estimated spectra. For match-technique method, the two local waves are band-limited to 10Hz- 140Hz and 10Hz- 90Hz respectively, and the spectrum estimation by multitaper method is applied to the prediction filter for the two local waves, which are used to compute the spectral power ratio for Q estimation. The results for these four methods are shown in figure 24 – 27. We can see that estimated Q values for the three frequency-domain methods are significantly deviated from the true value. It indicates that the attenuation law between the original amplitude spectra can be distorted by the modification imposed by spectrum estimation. For match-filter method, it still gives a quite accurate estimation of 76.55,

compared to the exact value 80. It indicates that match-filter method is less sensitive to the modification of amplitude spectra caused by spectrum estimation. Theoretically, the frequency band used to filter the local waves can affect the result of match-filter method. If a band-pass filter with lower high-pass frequency is applied to the local wave in the deep zone, the loss of high-frequency energy will be attributed to attenuation, which will lead to estimated value greater than the true value. Therefore, in order to give accurate estimation, match-filter method needs the match of frequency band for the local waves as well. From figure 9 and 10, we can see that 90Hz and 140Hz correspond to the frequency components of the amplitude spectra that have magnitude about -20dB respectively. Frequency band 10Hz – 140Hz for local wave in shallow zone roughly matches the frequency band 10Hz – 90Hz for the one in deep zone. Then, the estimated Q value is close to the true value. When the frequency band is poorly chosen for match-filter method, the result can be distorted. If we use a frequency band of 10Hz-70Hz for the wave in deep zone, the band-limited amplitude spectra estimated by the multitaper method are shown in figure 28, which lead to a distorted Q estimation shown in figure 29. For this case, the high-frequency energy loss of the local wave in deep zone caused by band-pass filtering is attributed to Q attenuation, which in turn leads to a significantly smaller Q value than the true one.

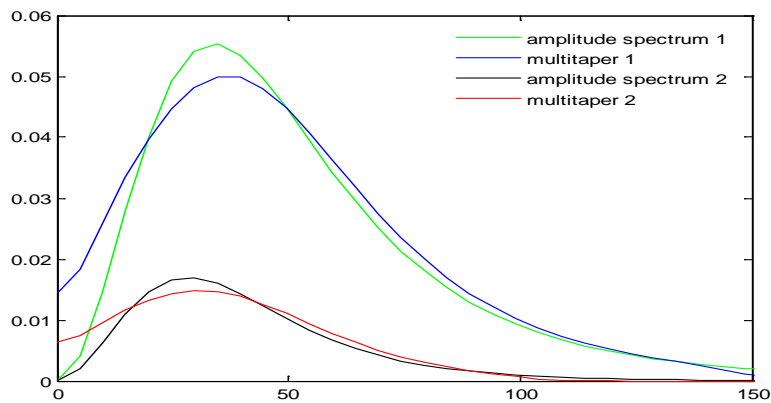


Figure 23. Spectrum estimation of for the two events (0.34-0.54s, 0.74s-0.94s) in figure 1 by multitaper method with frequency-band limit of 10Hz – 140Hz and 10Hz – 90Hz respectively.

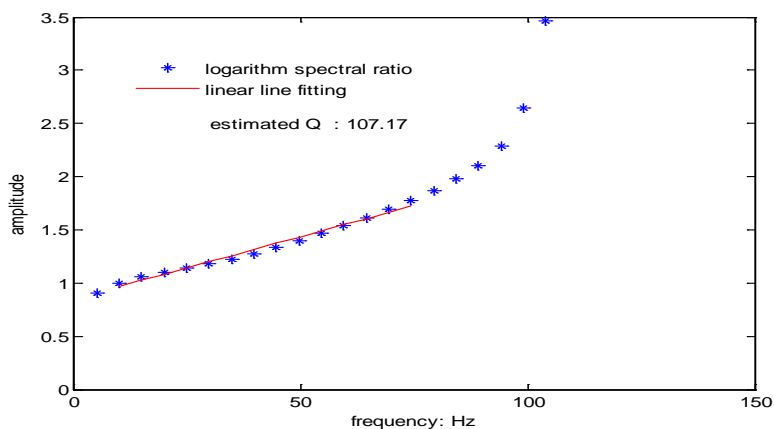


Figure 24. Q estimation by spectral-ratio method using the amplitude spectra estimated by multitaper method shown in figure 23.

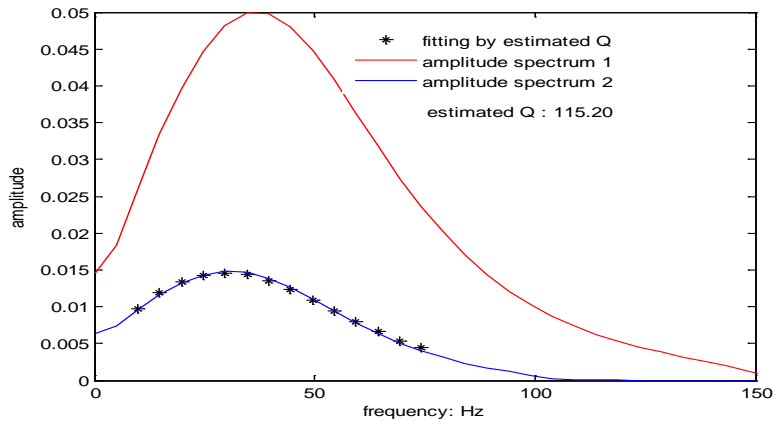


Figure 25. Q estimation by spectrum-modeling method using the local events in figure1; Spectrum estimation for the two events by multitaper method is employed with frequency band 10Hz-140Hz and 10Hz – 90Hz respectively.

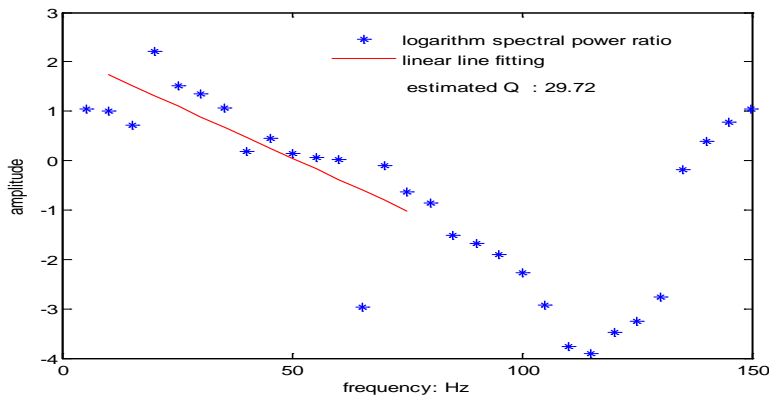


Figure 26. Q estimation by match-technique method using the band-pass filtered local events shown in figure 1 with frequency band 10Hz-140Hz and 10-140Hz respectively (multitaper method for spectrum estimation of prediction filter is employed) .

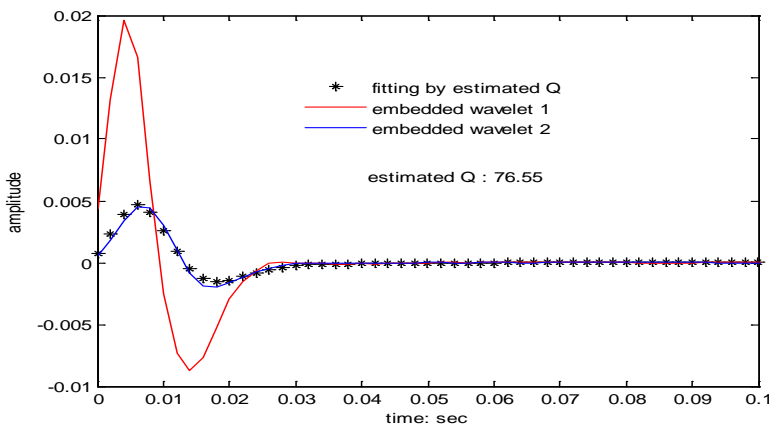


Figure 27. Q estimation by match-filter method using the local events in figure1; Spectrum estimation for the two events by multitaper method is employed with frequency band 10Hz-140Hz and 10Hz – 90Hz respectively.

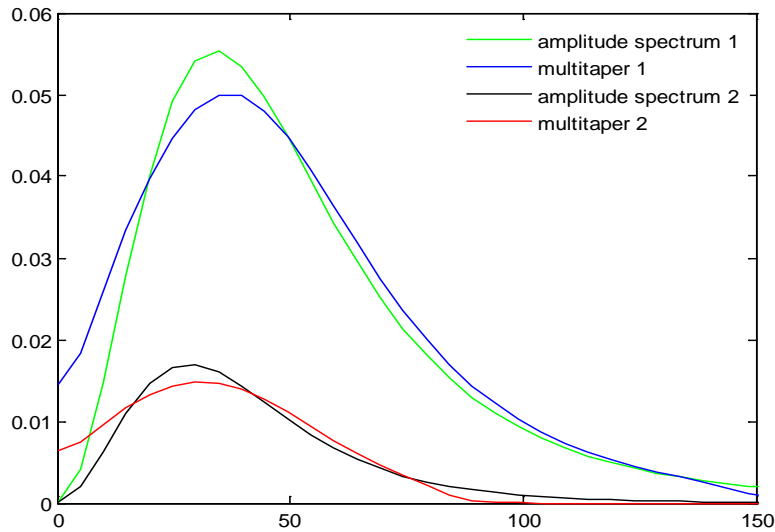


Figure 28. Spectrum estimation of for the two events (0.34-0.54s, 0.74s-0.94s) in figure 1 by multitaper method with frequency-band limit of 10Hz – 140Hz and 10Hz – 70Hz respectively.

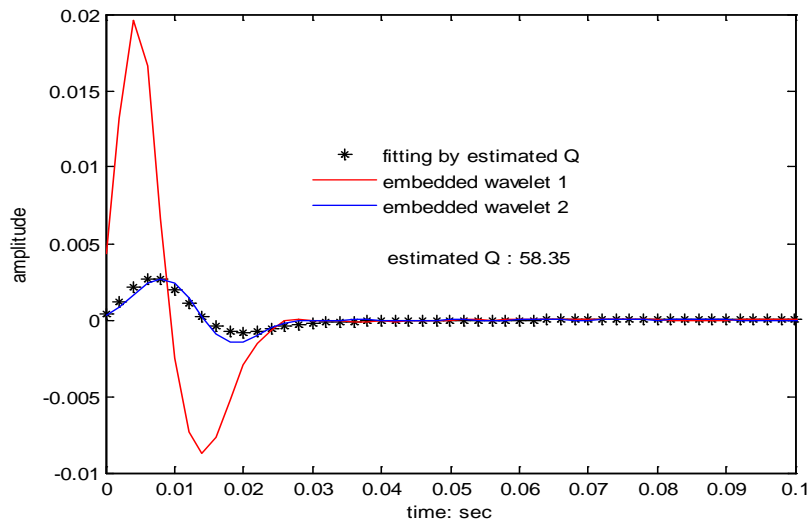


Figure 29. Q estimation by match-filter method using the local events in figure1; Spectrum estimation for the two events by multitaper method is employed with frequency band 10Hz-140Hz and 10Hz – 70Hz respectively.

In addition, the case of extensive noise is used to evaluate the four methods. The Q estimation is conducted using 200 seismic traces with a noise level of SNR = 2. Spectrum estimation by multitaper method is employed for match-filter method, which is not applied to other three frequency-domain methods. As shown in figure 30 - 32, the three frequency domain methods have become inaccurate with results that have significantly deviated mean value and large standard deviation. However, the match-filter method, as shown in figure 33, still gives good estimation with a mean value of 80.02 and standard deviation of 11.82. Based on the above results, match-filter method is more robust to noise.

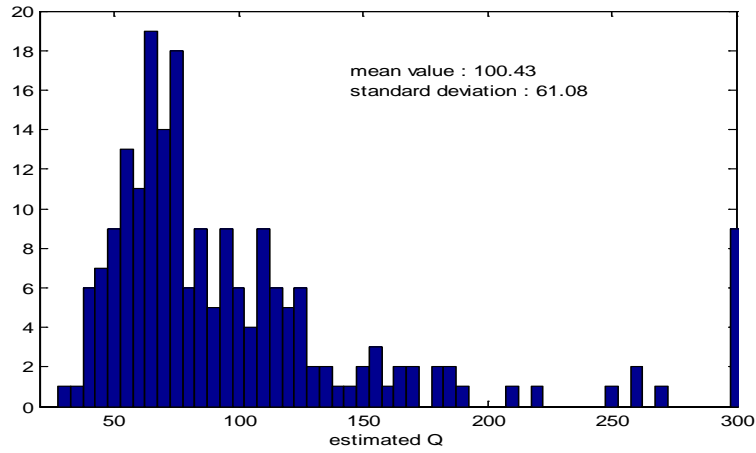


Figure 30. Histogram of the Q values estimated by spectral-ratio method using 200 seismic trace (similar to the one shown in figure 8) with noise level of SNR=2.

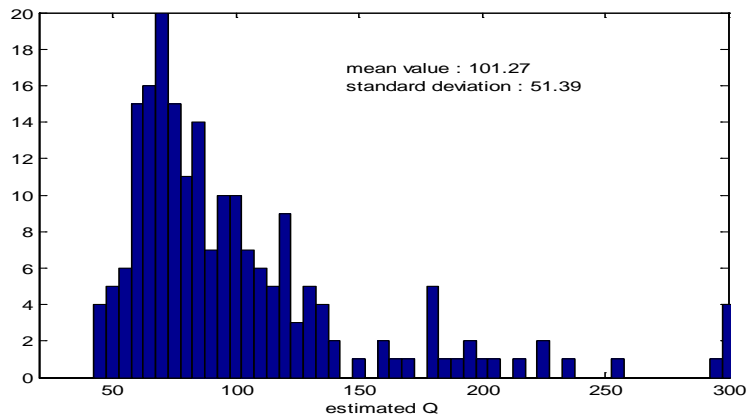


Figure 31. Histogram of the Q values estimated by spectrum-modeling method using 200 seismic trace (similar to the one shown in figure 8) with noise level of SNR=2.

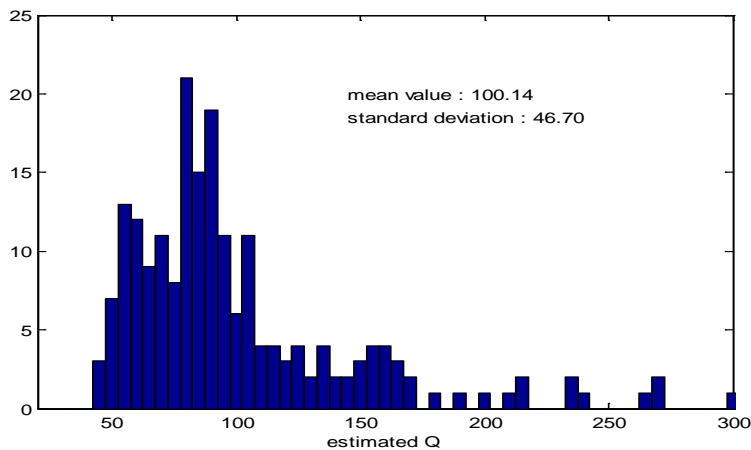


Figure 32. Histogram of the Q values estimated by match-technique method using 200 seismic trace (similar to the one shown in figure 8) with noise level of SNR=2.

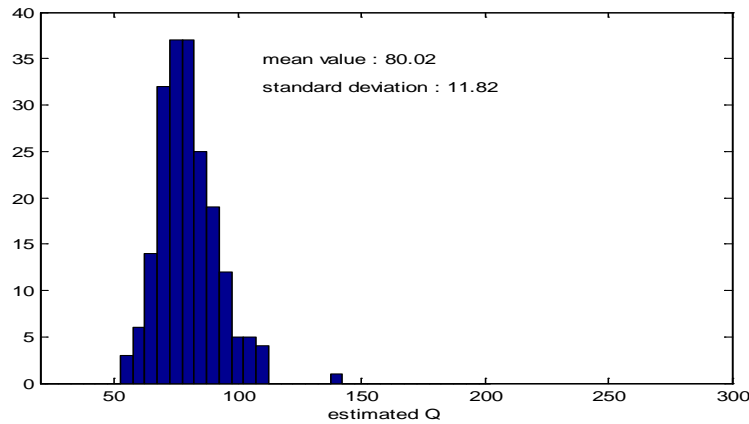


Figure 33. Histogram of the Q values estimated by the match-filter method using 200 seismic trace (similar to the one shown in figure 8) with noise level of SNR=2 (Multitaper method for spectrum estimation is employed).

Synthetic 1D reflection data

Surface reflection data is the most common seismic data. Whether or not these Q estimation methods are suitable for application to reflection is worthy of investigation. A synthetic seismic trace is created using a random reflectivity series, a minimum phase source wavelet with dominant frequency of 40Hz and a constant Q of 80, as shown in figure 34. Two local reflected waves are obtained by applying time gates of 100ms-500ms and 900ms-1300ms to the attenuated seismic trace. For the two windowed local waves, their spectrum estimation by multitaper method is demonstrated by figure 35. The spikes and notches in the original spectra of local reflected waves are obvious due to the tuning effect of local reflectors. Now, spectrum estimation is necessary, and multitaper method is employed for all the four methods. Q estimation is conducted using the obtained local reflected waves, and the results are shown in figure 36 - 39. We can see that, even without noise, the estimation results are deviated from the true value due to the tuning effect.

Then, attenuated seismic traces are created using 200 different random reflectivity series, from which 200 pairs of local reflected waves are obtained to conduct the Q estimation experiment using the four Q estimations. The results are shown in figure 40 - 43. We can see that the match-filter method gives best result with the closest mean value of 82.49 and the smallest standard deviation of 16.86. Next, the four Q estimation methods are further evaluated using reflection data with noise level of SNR=4 and SNR=2. The corresponding results are shown in figure 44- 51 We can see that the three frequency methods give unreliable results with significantly distorted mean value and large standard deviation value, while match-filter method is insensitive to noise level and gives good estimation results for both cases.

From above results, we can see that multitaper method can give an appropriate estimation of the amplitude spectrum of windowed reflection data. The three frequency domain methods investigated in this paper are sensitive to the spectrum modification caused by noise and the tuning effect of local reflectors. The match-filter method is more suitable to be applied to reflection data, and is robust to noise.

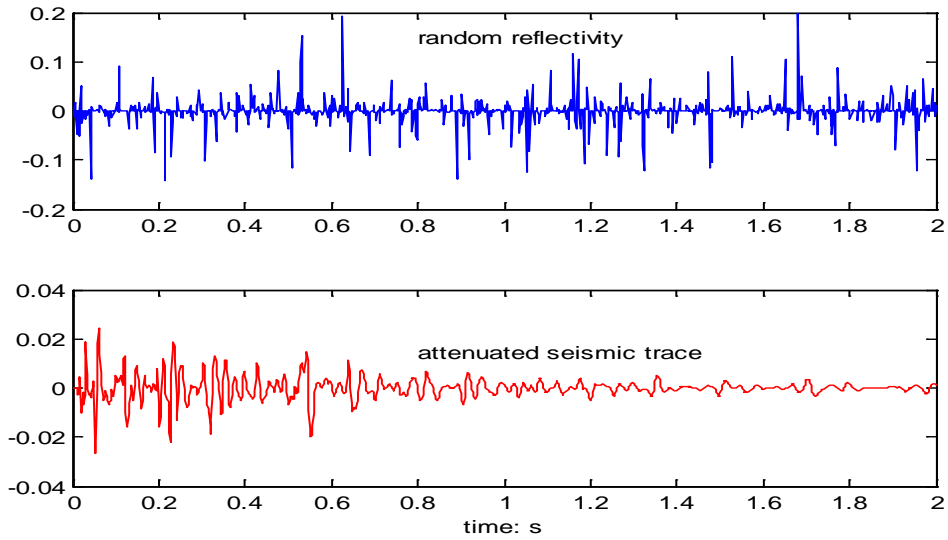


Figure 34. A random reflectivity series (upper). An attenuated seismic trace created using the reflectivity series, a minimum phase wavelet with dominant frequency of 40Hz and a constant Q of 80.

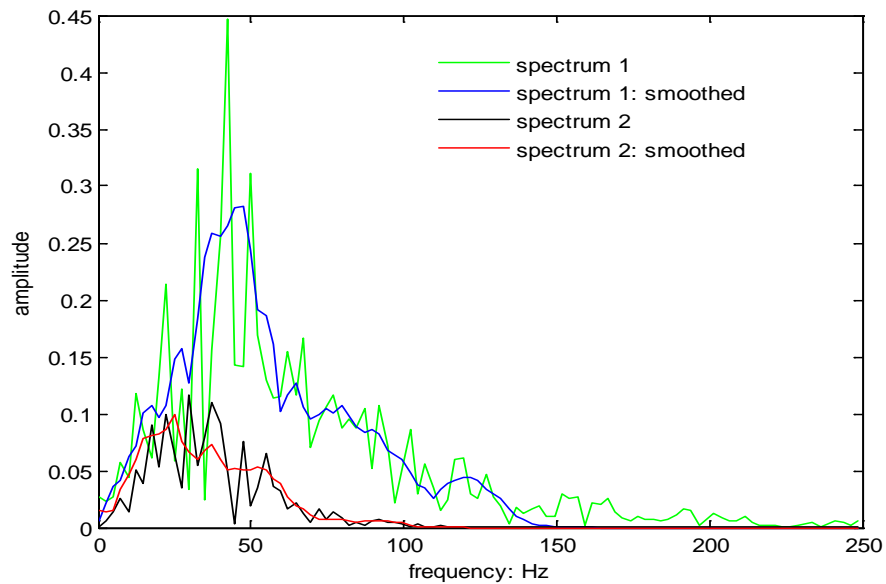


Figure 35. Amplitude spectrum of the 100ms-500ms part of the seismic trace in figure 34 (Green). Amplitude spectrum estimated by multitaper method for the 100ms-500ms part of the seismic trace in figure 34 (Blue). Amplitude spectrum of the 900ms-1300ms part of the seismic trace in figure 34 (Black). Amplitude spectrum estimated by multitaper method for the 900ms-1300ms part of the seismic trace in figure 34 (Red).

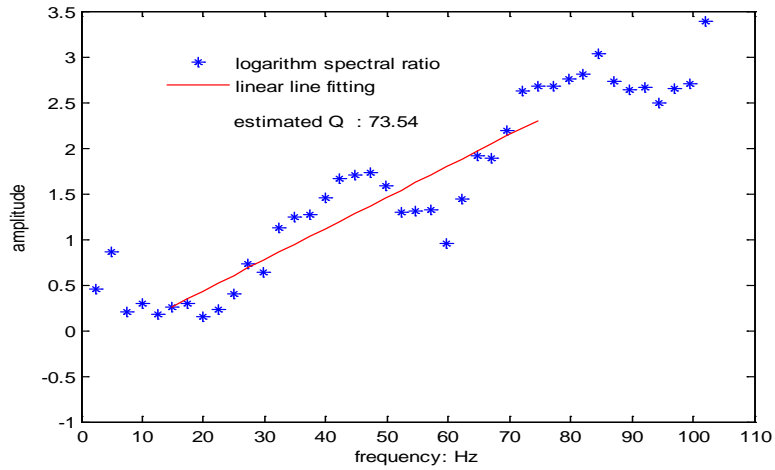


Figure 36. Q estimation by spectral-ratio method using the 100ms-500ms and 900ms-1300ms parts of the seismic trace shown in figure 34.

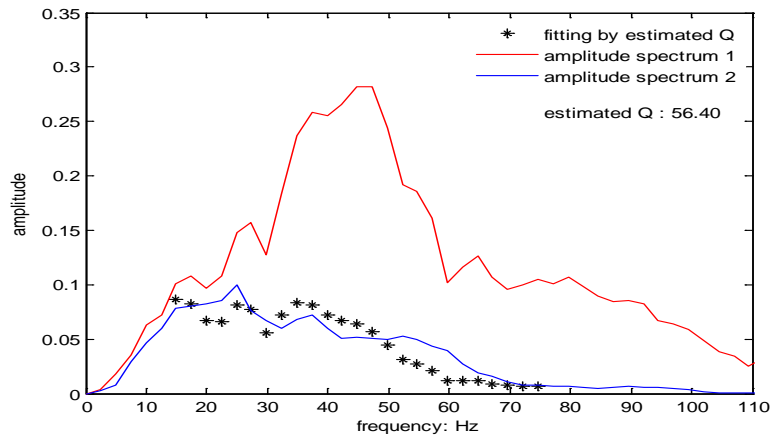


Figure 37. Q estimation by spectrum-modeling method using the 100ms-500ms and 900ms-1300ms parts of the seismic trace shown in figure 34.

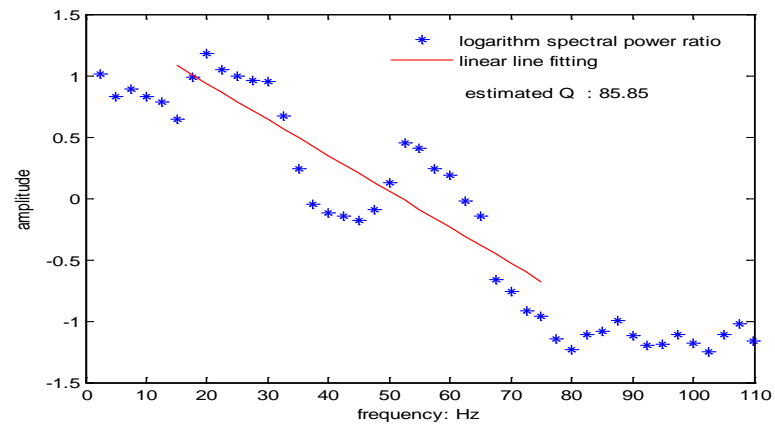


Figure 38. Q estimation by match-technique method using the 100ms-500ms and 900ms-1300ms parts of the seismic trace shown in figure 34.

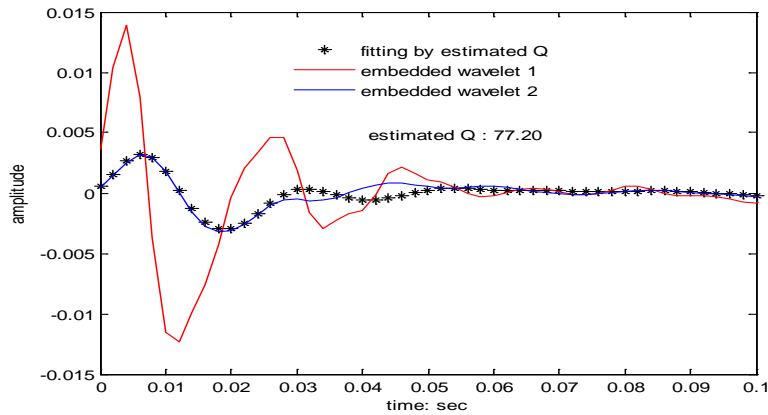


Figure 39. Q estimation by match-filter method using the 100ms-500ms and 900ms-1300ms parts of the seismic trace shown in figure 34.

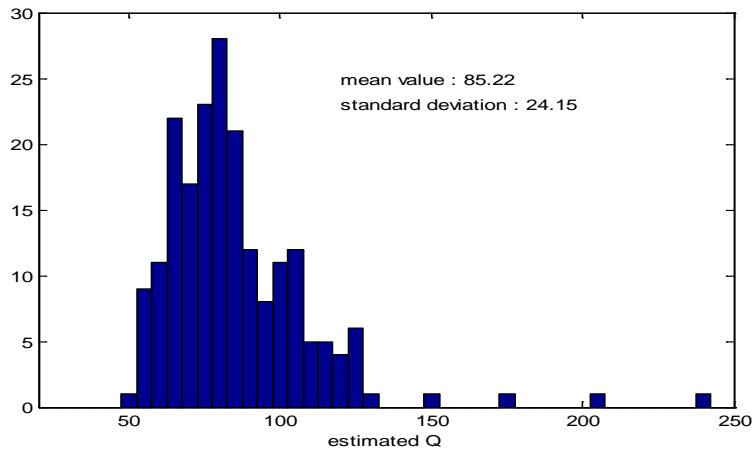


Figure 40. Histogram of the Q values estimated by spectral-ratio method using the 100ms-500ms and 900ms-1300ms parts of 200 seismic traces without noise, which are similar to the one shown in figure 32.

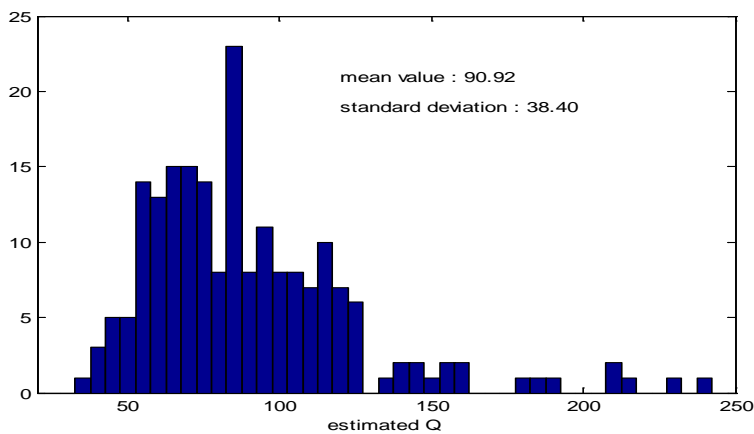


Figure 41. Histogram of the Q values estimated by spectrum-modeling method using the 100ms-500ms and 900ms-1300ms parts of 200 seismic traces without noise, which are similar to the one shown in figure 34.

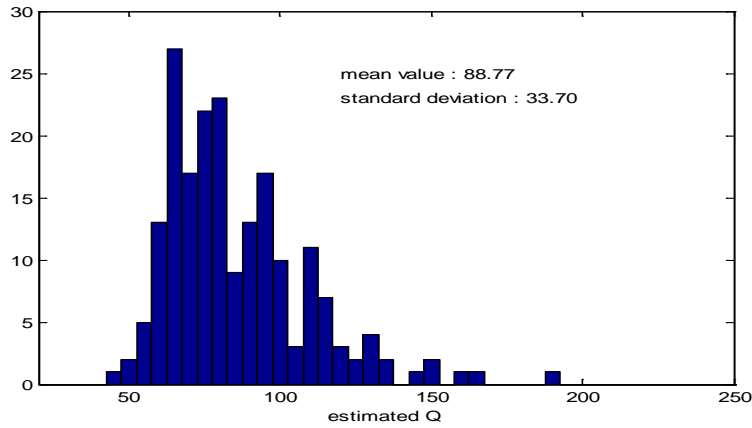


Figure 42. Histogram of the Q values estimated by match-technique method using the 100ms-500ms and 900ms-1300ms parts of 200 seismic traces without noise, which are similar to the one shown in figure 34.

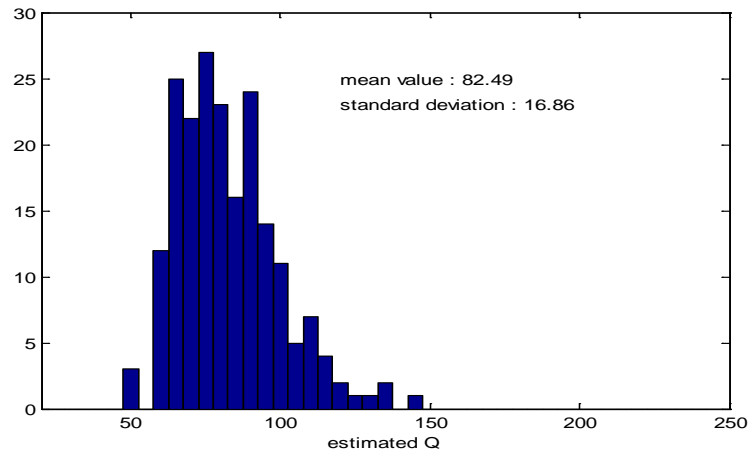


Figure 43. Histogram of the Q values estimated by match-filter method using the 100ms-500ms and 900ms-1300ms parts of 200 seismic traces without noise, which are similar to the one shown in figure 34.

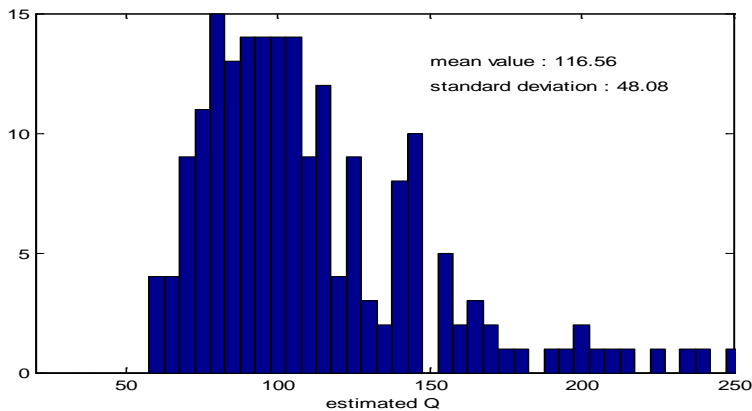


Figure 44. Histogram of the Q values estimated by spectral-ratio method using the 100ms-500ms and 900ms-1300ms parts of 200 seismic traces with noise level of SNR=4, which are similar to the one shown in figure 34.

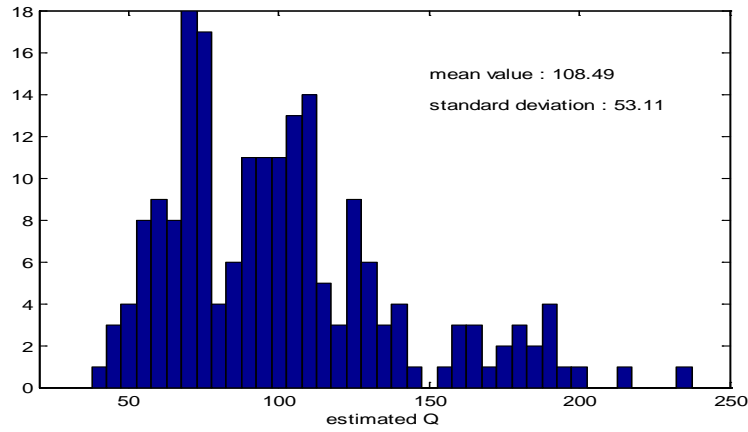


Figure 45. Histogram of the Q values estimated by spectrum-modeling method using the 100ms-500ms and 900ms-1300ms parts of 200 seismic traces with noise level of SNR=4, which are similar to the one shown in figure 34.

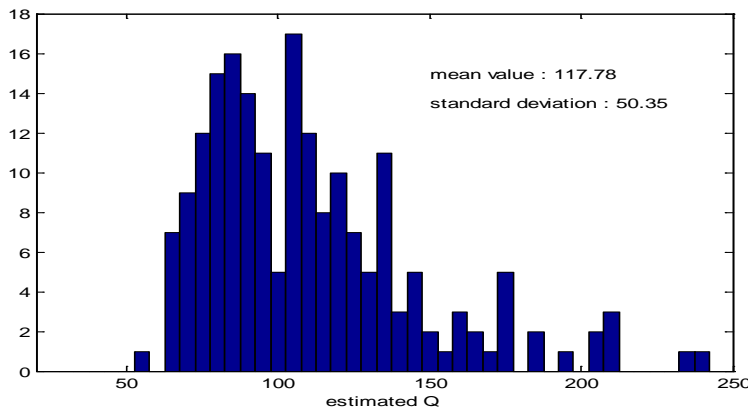


Figure 46. Histogram of the Q values estimated by match-technique method using the 100ms-500ms and 900ms-1300ms parts of 200 seismic traces with noise level of SNR=4, which are similar to the one shown in figure 34.

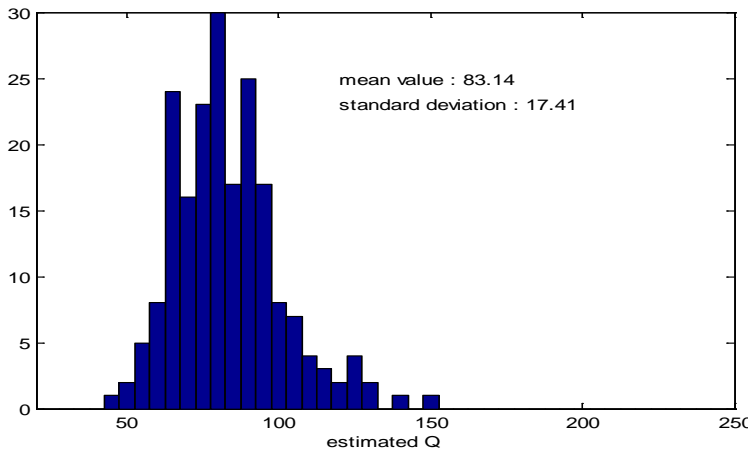


Figure 47. Histogram of the Q values estimated by match-filter method using the 100ms-500ms and 900ms-1300ms parts of 200 seismic traces with noise level of SNR=4, which are similar to the one shown in figure 34.

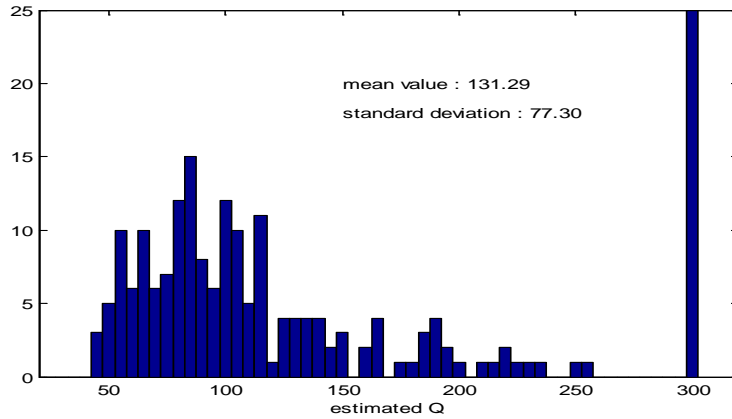


Figure 48. Histogram of the Q values estimated by spectral-ratio method using the 100ms-500ms and 900ms-1300ms parts of 200 seismic traces with noise level of SNR=2, which are similar to the one shown in figure 34.

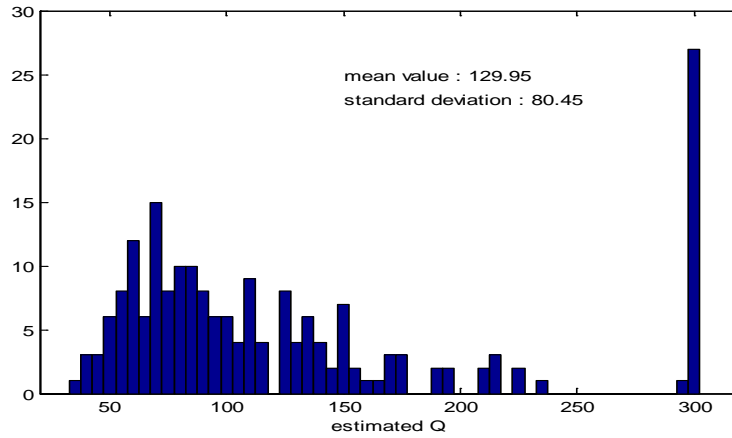


Figure 49. Histogram of the Q values estimated by spectrum-modeling method using the 100ms-500ms and 900ms-1300ms parts of 200 seismic traces with noise level of SNR=2, which are similar to the one shown in figure 34.

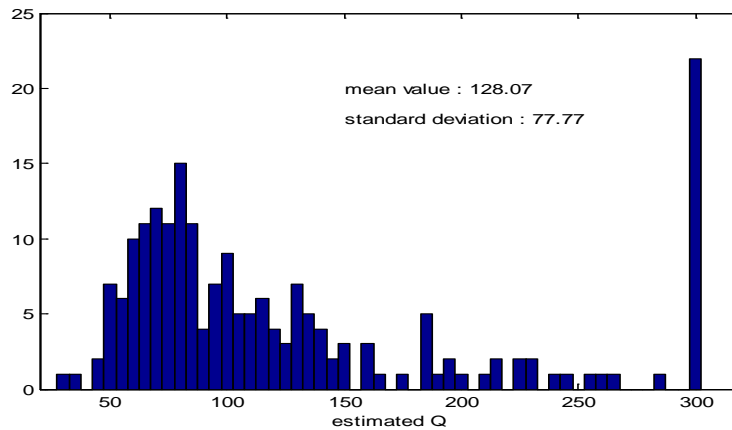


Figure 50. Histogram of the Q values estimated by match-filter method using the 100ms-500ms and 900ms-1300ms parts of 200 seismic traces with noise level of SNR=2, which are similar to the one shown in figure 34.

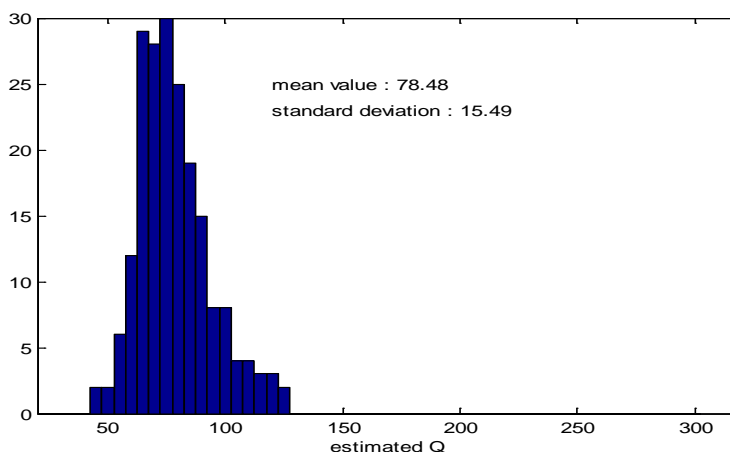


Figure 51. Histogram of the Q values estimated by match-filter method using the 100ms-500ms and 900ms-1300ms parts of 200 seismic traces with noise level of SNR=2, which are similar to the one shown in figure 34.

Real VSP data

Figure 52 shows field zero-offset P-wave VSP data. Since the VSP data consists of downgoing waves and upgoing waves, it is necessary to obtain the downgoing waves for Q estimation. First, the first breaks of VSP data are picked and their corresponding time is shown in figure 53. Linear move out is applied to align the events of VSP data. Then, median filtering is applied to the aligned VSP data for upgoing wave suppression. The downgoing wave VSP data is shown in figure 54.

With a fixed trace interval of 100, 230 pairs of windowed VSP traces shown in figure 54 are chosen for Q estimation, of which the first pair are the VSP trace 101 and trace 201 and the last pair are VSP trace 330 and trace 430. At first, the multitaper method is not used for the three frequency domain method, and the results are shown in figure 55. We can see that the estimation results are similar and have the same trend of variations at most cases, while match-filter method and spectrum-modeling method gives more stable results at some cases. Then multitaper method is used to smoothing amplitude spectra for the three frequency domain method, and the results are shown in figure 56. We can see that the spectrum smoothing stabilizes the Q estimation for the spectral-ratio method, while match-technique method is sensitive to spectrum smoothing.

Then, 80 pairs of windowed VSP traces, shown in figure 54, with fixed trace interval of 250 are used to investigated the four method, of which the first pair are the VSP trace 101 and trace 351 and the last pair are VSP trace 180 and trace 430. When spectrum estimation is not conducted for the three frequency domain method, the results for Q estimation are shown in figure 57. With a larger trace interval (travel-time difference), the attenuation between the two trace becomes more measurable. We can see that the results of spectral-ratio method and match-technique method are more stable, and the four methods give more consistent estimation. Then, multitaper method for spectrum smoothing is employed for the three frequency domain methods. The corresponding Q-estimation results are shown in figure 58. With spectrum smoothing, the results of

spectral-ratio method are stabilized. We also can see that spectral-ratio method, spectrum-modeling method and match-filter method give quite close estimation results.

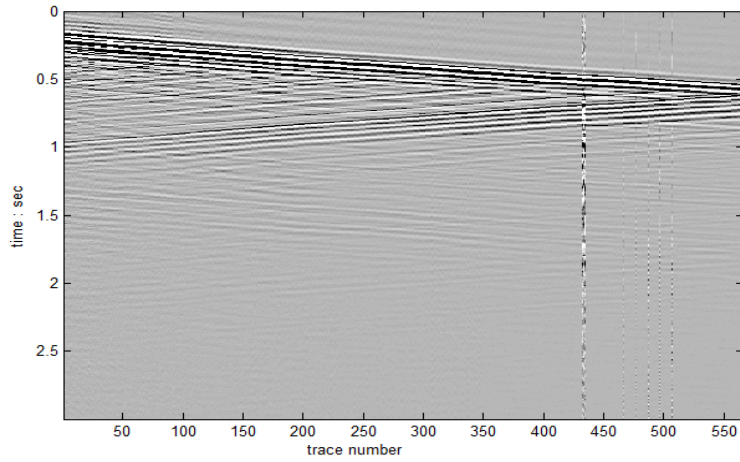


Figure 52 Ross Lake VSP data (vertical component P-wave).

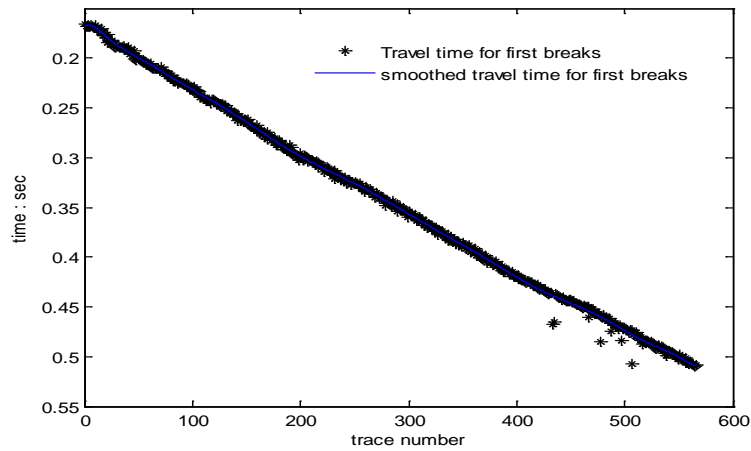


Figure 53. First breaks of VSP data shown in figure 52.

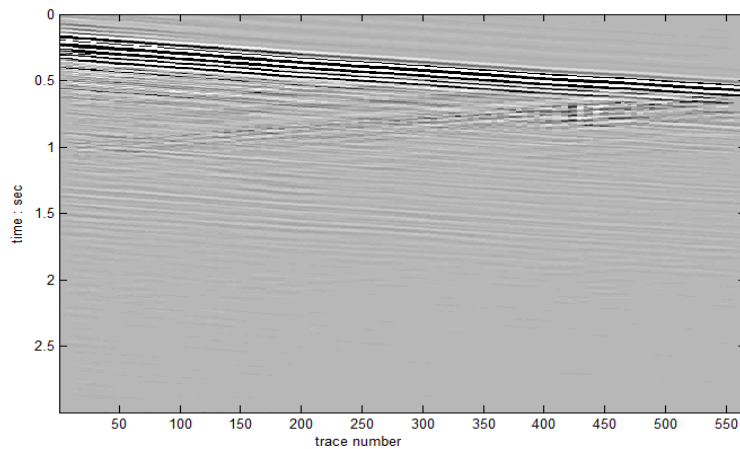


Figure 54. VSP data with upgoing wave suppression.

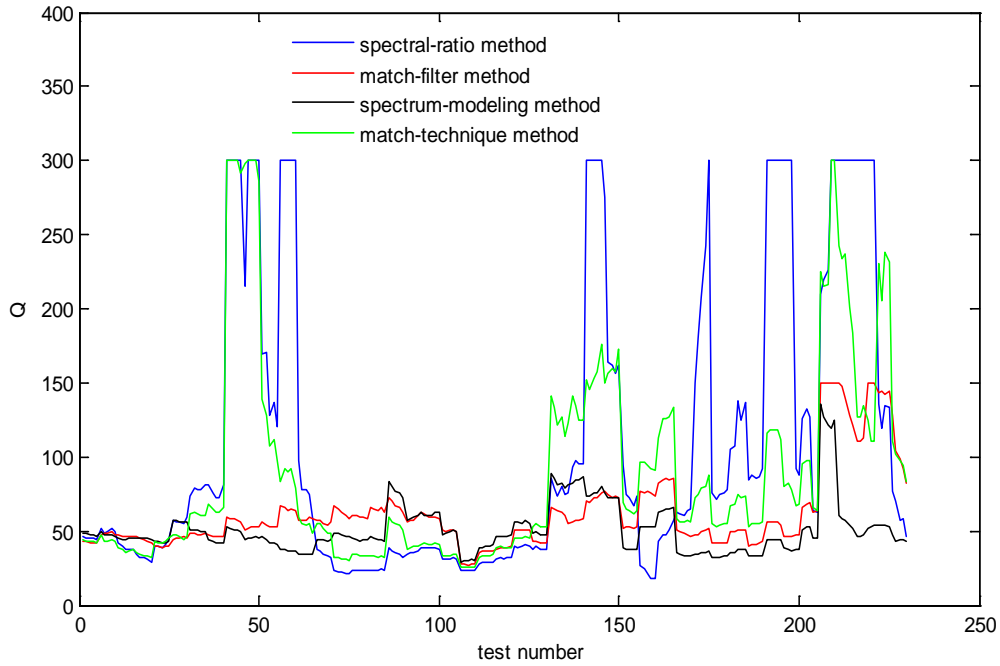


Figure 55. Q estimation using 230 pairs of VSP traces shown in figure 54 (Each pair has a fixed trace interval of 100; the first pair are the VSP trace 101 and trace 201 and the last pair are VSP trace 330 and trace 430); Multitaper method for spectrum estimation is not employed for the three frequency domain methods.

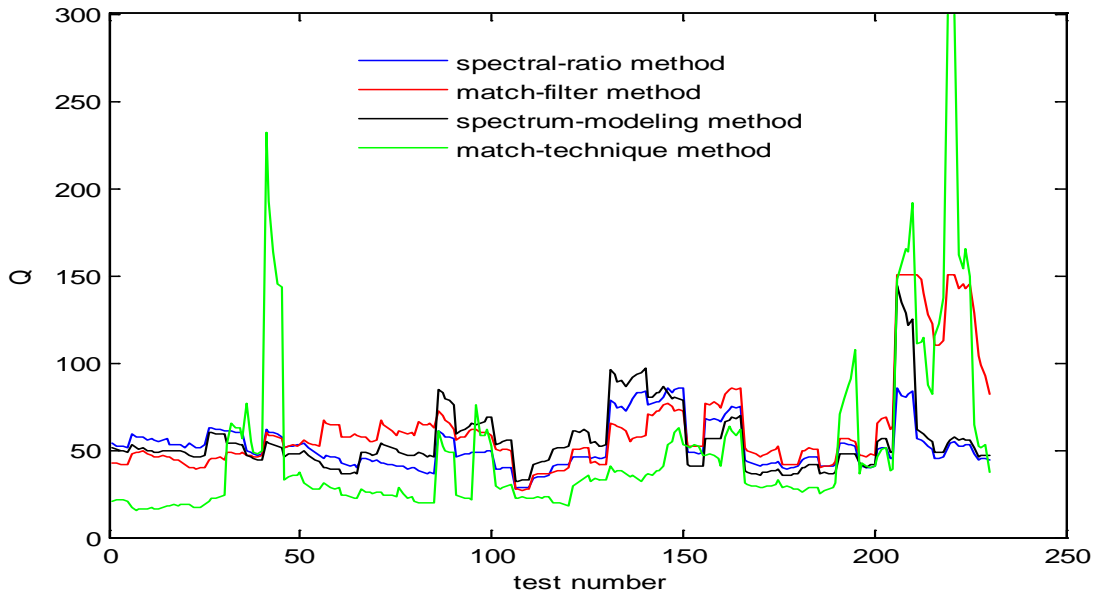


Figure 56. Q estimation using 230 pairs of VSP traces shown in figure 54 (Each pair has a fixed trace interval of 100; the first pair are the VSP trace 101 and trace 201 and the last pair are VSP trace 330 and trace 430); Multitaper method for spectrum estimation is employed for the three frequency domain methods.

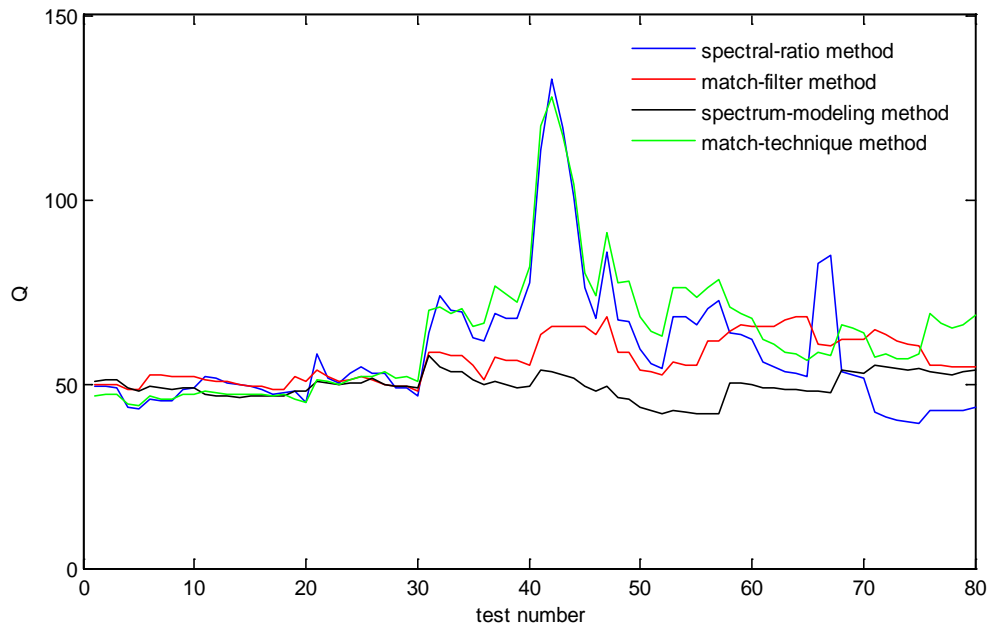


Figure 57. Q estimation using 80 pairs of VSP traces shown in figure 54 (Each pair has a fixed trace interval of 250; the first pair are the VSP trace 101 and trace 351 and the last pair are VSP trace 180 and trace 430); Multitaper method for spectrum estimation is not employed for the three frequency domain methods.

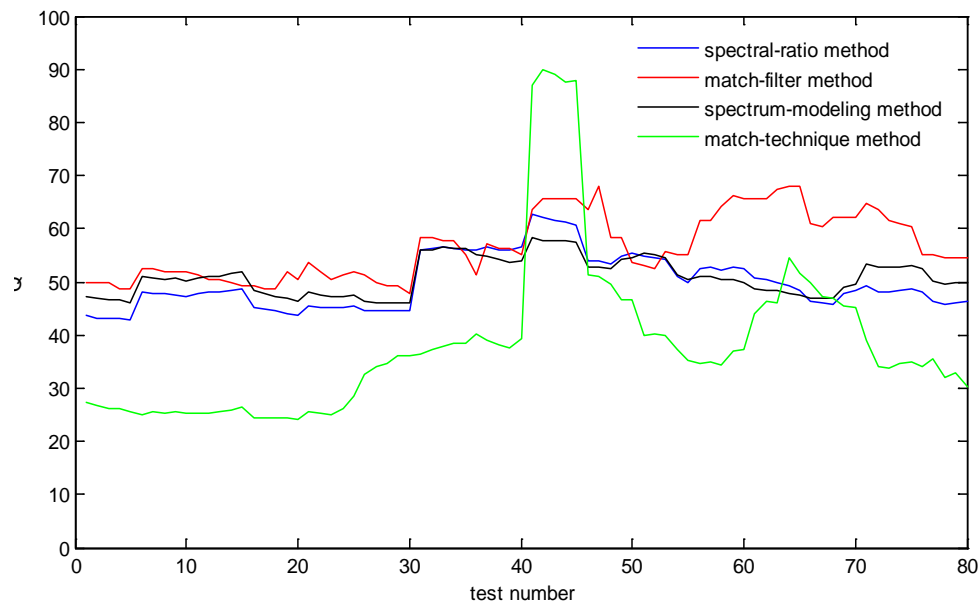


Figure 58. Q estimation using 80 pairs of VSP traces shown in figure 54 (Each pair has a fixed trace interval of 100; the first pair are the VSP trace 101 and trace 201 and the last pair are VSP trace 180 and trace 430); Multitaper method for spectrum estimation is employed for the three frequency domain methods.

CONCLUSION AND DISCUSSION

The relative performances of spectral-ratio method, spectrum-modeling method, match-technique method and match-filter method are evaluated in this paper. Testing on synthetic seismic traces shows that the match-filter method, compared to the classic spectral-ratio method, is robust to noise and more suitable to be applied to reflection data. Testing on real VSP data shows that match-filter method and spectrum-modeling method are more stable compared to spectral-ratio method and match-technique method, since no spectral division is involved in their algorithm, and all the four methods can obtain similar results at most cases when VSP data with high SNR is used for Q estimation.

Spectral-ratio method, spectrum-modeling method and match-technique method, as methods in frequency domain, can be sensitive to the modification of amplitude spectrum caused by application of spectrum estimation, noise and the tuning effect of local reflectors. For match-filter method, appropriate spectrum smoothing can improve the estimation of embedded wavelets, and, in turn, make the estimation result more stable. Theoretically, the result of the match-filter method can be affected by the frequency band used to estimate the embedded wavelets. Accurate estimation results require a rough match of the frequency bands for embedded wavelets, which can be chosen based on the evaluation of their amplitude spectra of original local waves.

When applied to reflection data, match-filter method is quite insensitive to noise, which may indicate that the spectrum estimation of local waves by multitaper method is mainly affected by the tuning effect of local reflectors instead of noise.

ACKNOWLEDGEMENTS

We would like to thank the sponsors of CREWES project for their financial support. We also would like to thank Andrew James Carter and Rainer Tonn for their reminder of reference work to the match-filter method investigated in this paper.

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