A hybrid method for AVO inversion

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ABSTRACT

Global optimization algorithms are generally computationally intensive processes, where a significant amount of time is required to generate a solution. Therefore, methods to improve the efficiency are desired for problems that require their use. In this study we present a hybrid method to improve the convergence rate of an AVO inversion through implementation of a trace integration method followed by a simulated annealing optimization. The method eliminates the need for an additional model parameter to represent the layering solution and reduces the compute time of the global optimization by generation of an initial model that is close to the final solution.

INTRODUCTION

Inverse problems that contain local minima in their objective function require the use of global optimization algorithms, which may need a significant amount of computational effort. Therefore, conditioning of the initial model to a state that is close to the final solution can greatly reduce the compute time. In this study, we attempt to minimize the compute time associated with an AVO inversion through implementation of a trace integration method followed by a simulated annealing optimization to refine the model. We first present the problem formulation and subsequently demonstrate the methodology to obtain the inverse solution.

PROBLEM FORMULATION

In an AVO inversion, the objective is to estimate the P- and S-wave velocities and density from the angle-dependent reflectivity. This is typically achieved through iteratively updating an initial model until the data residuals are minimized, where the initial model is often obtained from a low-pass filtered version of measured well logs. The role of the seismic data is then to provide the high frequency variations in the elastic properties. Given this description, the elastic properties can be written as

$$\alpha = \alpha_0 + \Delta\alpha \, , \tag{1}$$

$$\beta = \beta_0 + \Delta\beta \, , \tag{2}$$

and

$$\rho = \rho_0 + \Delta\rho \, , \tag{3}$$

where α is the P-wave velocity, β is the S-wave velocity and ρ is the density. The quantities with subscript 0 and prefix Δ then represent the low and high frequency contributions respectively. The quantities α_0, β_0 and ρ_0 are assumed to be known functions and are referred to as the low frequency model, which are typically used as the initial model for the inverse problem. Since the low frequency model is a smooth function, a layering solution is required and therefore adds an additional parameter to the inverse problem (e.g. Coulon et al., 2006). Alternatively, a layering solution can be defined prior to the estimation of the elastic properties. In the following, we discuss a methodology to obtain the layering solution in addition to an initial model that is closer to the final solution relative to the low frequency model.

Consider the normal incidence reflectivity function given by

$$r_{j+1} = \frac{I_{j+i} - I_j}{I_{j+i} + I_j}, \tag{4}$$

where $I_j = \alpha_j \rho_j$ is the acoustic impedance of layer j. The reflectivity is therefore obtained through a scaled difference operation on the impedance. Conversely, if the reflectivity is known, the impedance can be recovered using

$$I_{j+1} = I_1 \exp\left(2\sum_{k=1}^{j} r_k\right), \tag{5}$$

which involves integrating the reflectivity function (in practice this is assumed to be processed seismic data where the wavelet has been deconvolved) followed by exponentiation and scaling by I_1 (e.g. Oldenburg et al., 1983). An inherent problem with this method is that all frequencies are required for a proper solution. In seismic data, the low frequencies are not present due to the bandlimited response and integration generally leads to a deviation from the correct low frequency trend. To avoid this problem, low frequency information from well logs can be used to calibrate the solution. Therefore, trace integration followed by a series of filtering operations can be performed to obtain an impedance estimate.

Now, given an estimate of the impedance and the known functions of α_0, β_0 and ρ_0, an initial model can be obtained and is given by

$$\alpha^{(initial)} = \frac{I}{\rho_0} = \alpha\left(1 + \frac{\Delta\rho}{\rho_0}\right), \tag{6}$$

$$\beta^{(initial)} = \alpha^{(initial)}\left(\frac{\beta_0}{\alpha_0}\right), \tag{7}$$

and

$$\rho^{(initial)} = \frac{I}{\alpha_0} = \rho\left(1 + \frac{\Delta\alpha}{\alpha_0}\right). \tag{8}$$

Equations 6 to 8 demonstrate that the initial model for the elastic properties has an error on the order of the property reflectivities (e.g. $\Delta x/x_0$), which is a small quantity. In addition, since the impedance estimate from the seismic data provides the fluctuations about the low frequency model, we have defined layer boundaries given by the zero crossings upon the subtraction of the low frequency trend. The layer amplitudes can subsequently be perturbed using a global optimization algorithm to obtain the final solution.

INITIAL MODEL GENERATION

To demonstrate the proposed methodology, we generate synthetic data using measured well logs. Figure 1 shows the P- and S-wave velocity and density logs and the associated angle gather. The reflectivity was calculated using the Aki-Richards approximate AVO equation (Aki and Richards, 1980) and subsequently convolved with a [0 10 50 60] Ormsby wavelet to generate the seismic response.

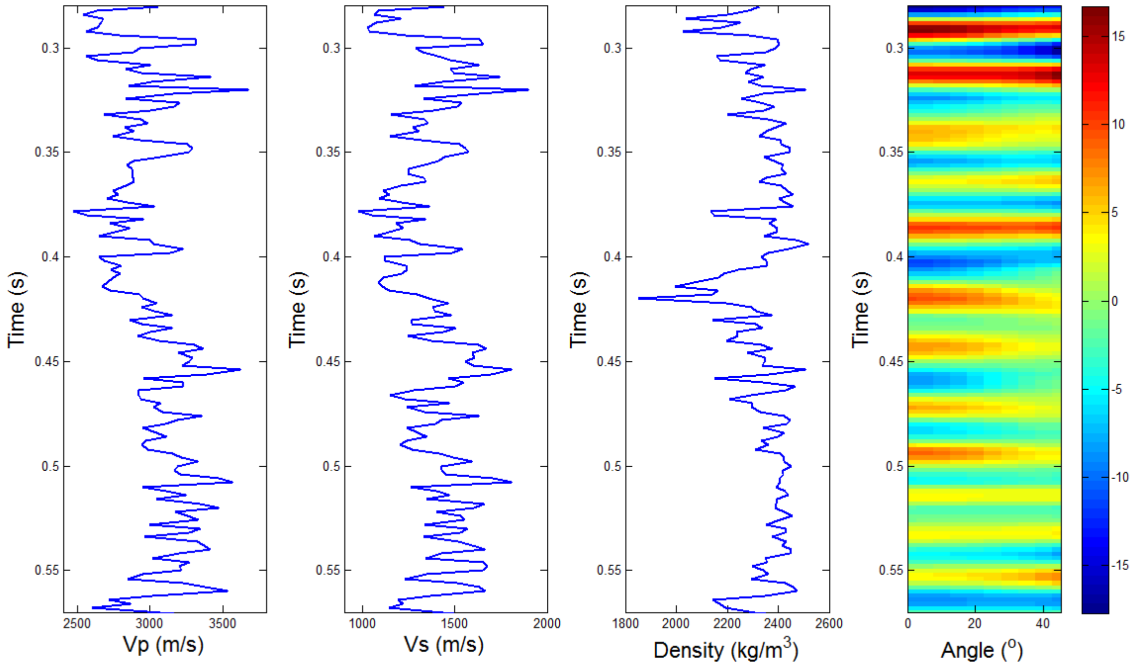

FIG. 1. P- and S-wave velocity and density well logs and the associated angle gather.

To prepare for the inversion, we first estimate the acoustic impedance from the deconvolved near angle traces in the seismic gather using equation 5. Subsequently, equations 6 to 8 are implemented to generate the initial model where the low frequencies provided by α_0, β_0 and ρ_0 were added through a series of filtering operations. Figure 2 shows the low frequency (red), true (blue) and initial (green) model, where the true model was filtered back to match the seismic frequencies.

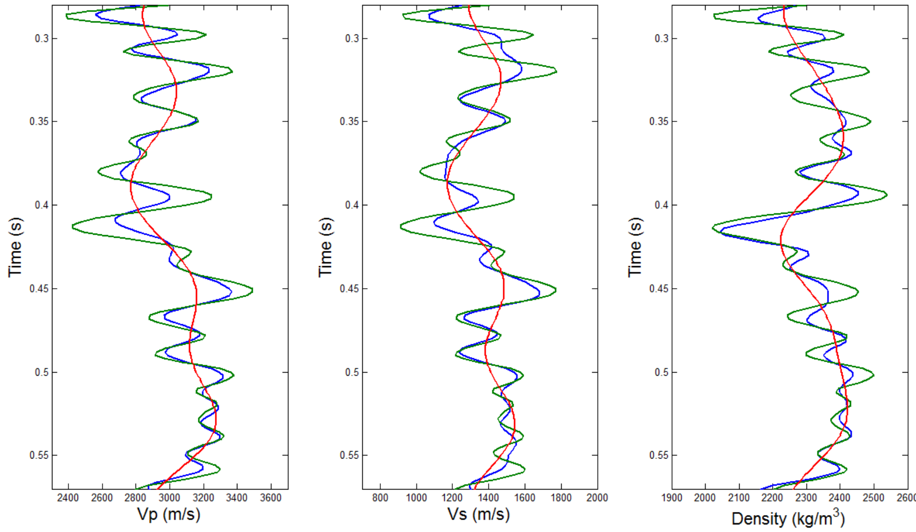

FIG. 2. Low frequency (red), true (blue) and initial (green) model for the P- and S-wave velocity and density.

INVERSION

To perform the inversion, we implement a simulated annealing algorithm as discussed in Cho and Margrave (2012) to adjust the amplitudes layer by layer. Figure 3 shows a discrete set of states in the solution space between the upper and lower bounds in color and the true model in black for the elastic properties. The optimization algorithm then perturbs the model amplitudes and selects a solution that minimizes the objective function. Since the initial model is close to the final solution, a lower initial temperature and fewer iterations are required in the simulated annealing process for convergence, therefore reducing the compute time. Figure 4 illustrates the results of the inversion where the low frequency (red), true (blue) and estimated (green) models are shown. The inversion demonstrates a reasonable fit to the well logs and produces a minimal amount of residual energy in the difference between the input seismic and synthetic as shown in Figure 5. The finite errors are then attributed to the imperfect deconvolution process and the corresponding layering solution imposed by the impedance estimate from trace integration.

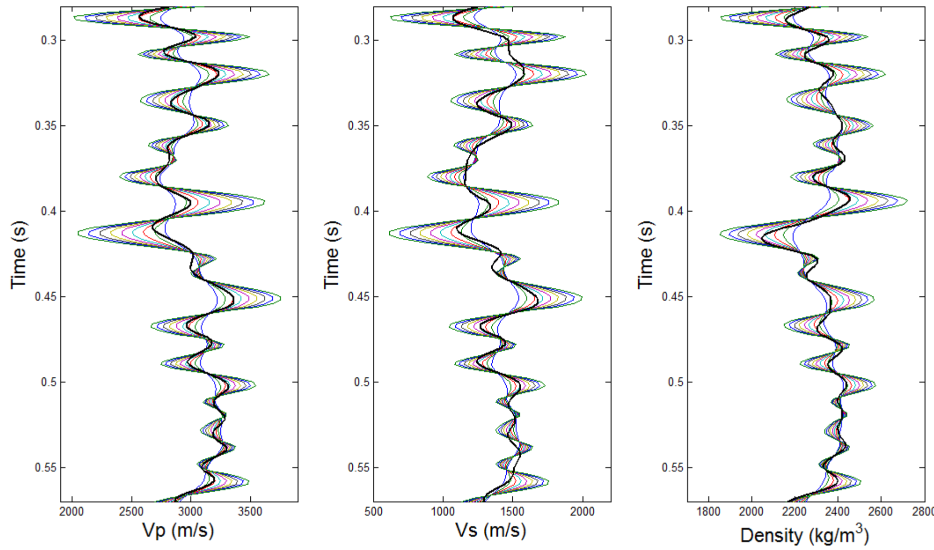

FIG. 3. Discrete set of states in the solution space displayed in color with the true model in black.

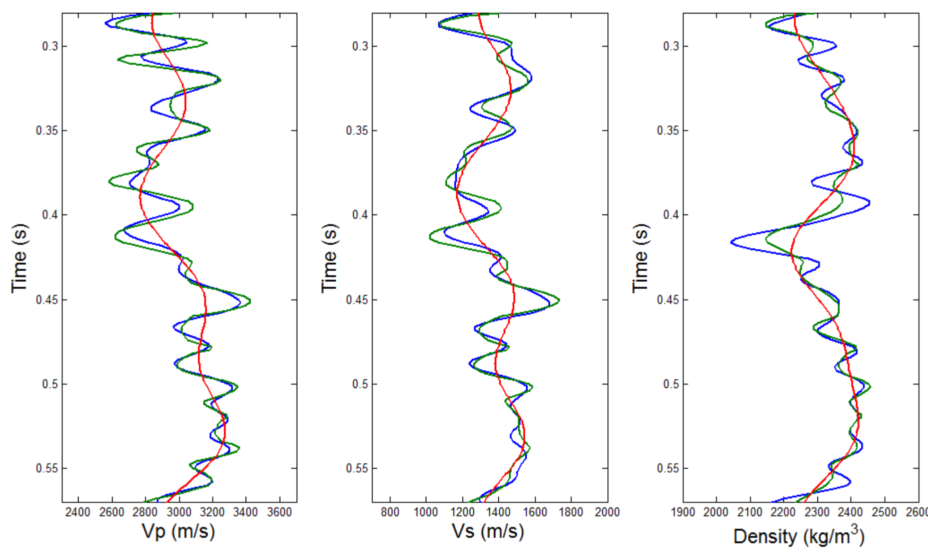

FIG. 4. Low frequency (red), true (blue) and estimated (green) model for the P- and S-wave velocities and density.

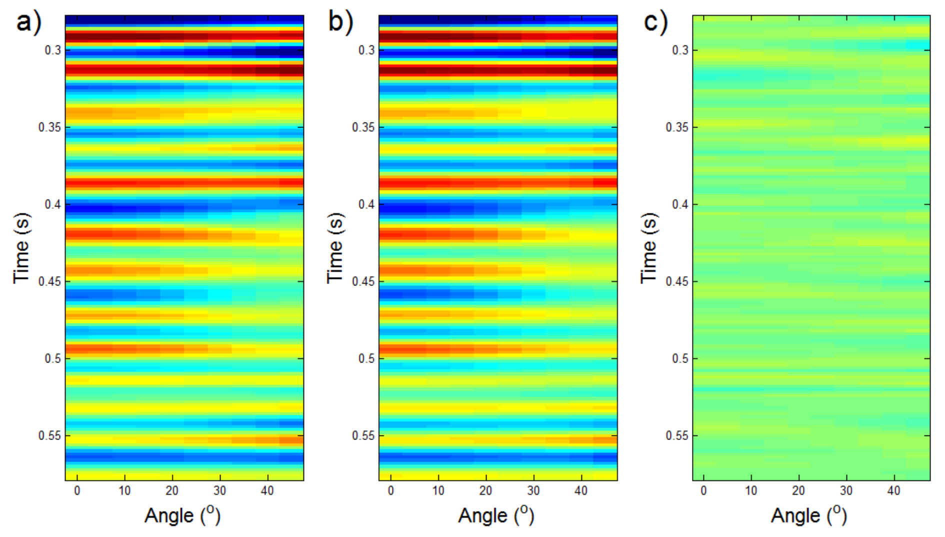

FIG. 5. Seismic angle gathers for the a) input data, b) synthetic data and c) residuals (color scale is the same for all panels).

CONCLUSIONS

A hybrid method to improve the convergence rate of an AVO inversion was presented. A trace integration method was performed to obtain a layering solution and an initial model that consist of small errors on the order of the property reflectivities. Subsequently, a global optimization algorithm was applied to refine the solution to achieve the final result. Using this approach, a separate model parameter representing the layering solution is not required. In addition, the time devoted to the computationally intensive global optimization is reduced due to a pre-defined layering solution and an initial model that is close to the final solution.

ACKNOWLEDGEMENTS

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