Angle gathers for depth migrated *PP* and *PS* data from flat and dipping reflectors

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ABSTRACT

Angle gathers have been identified as an appropriate domain for amplitude and velocity analysis of prestack migrated data, especially in complex geological settings, where other gathers appear prone to errors. This report analysis angle gathers from depth migration of multicomponent data, PP and PS, using synthetic data and a shot-profile PSPI (wave-equation) migration method. Two methods were applied. A ray based method was implemented to obtain the angle gathers, assuming a known velocity model. The characteristics of the method and its application to P-wave and converted wave data and to horizontal and dipping reflectors are illustrated through data generated using two simple models, namely one with a flat interface and the other one with a dipping interface. Ray tracing, as well as Finite Difference (FD) modeling, were used to obtain the synthetic data for migrationfor comparison. An analysis of amplitudes was considered by means of a comparison between the theoretical results according to the Zoeppritz equations, and the actual results obtained. Another method, due to Ricker and Sava (2002), and based in the extended imaging condition, was also applied to the same data. It showed meaningful results with PP waves. However the results with PS-waves appear less evident.

INTRODUCTION:

Angle gathers have been identified as a convenient domain to analyze seismic prestack data, since they are closer to the physical reflection at the location of interest. It is especially true in complex geological settings, where other data sets like offset gathers are less meaningful. Two main applications have been found to this data gathers: velocity analysis for better imaging and amplitude variation with angle for lithological information. Migrated data in the angle domain has been recognized appropriate to extend theamplitude vs offset method to complex areas (e. g. Resnick et al., 1986, Mosher et al., 1996).

Much research has been focused on this topic, mostly applied to conventional seismic data with P waves and with many migration flavors and particular issues such as anisotropy and 3D data. Some effort has been also focused on converted waves. However the latter application is not widely used in the industry. Besides, angle gathers in depth appear as quite an appropriate domain to relate PP and PS waves, since both of them should coincide at the same depth location and with the same angle of incidence. In fact, this event corresponds to the converted wave generation. Exploration of these relationships motivated this study.

A number of methods for angle domain analysis have been developed for Kirchhoff migration, e.g. Xu et al., 2001. Other techniques have been focussed on wave equation migration methods, which can be unidirectional (such as PSPI or Phase Shift Plus Interpolation) or bidirectional (such as RTM or Reverse Time Migration). This report analyses angle gathers with a unidirectional wave equation depth migration method. Two main approaches have been used for this type of wave equation migration, namely survey-sinking (or source-receiver) and shot-profile (or shot record). The latter has been used in this study. namely shot profile migration, that migrates each shot record independently and later put these results together to generate the image. On the other hand, the survey-sinking method generates the image by simultaneously migrating source and receiver data corresponding to an earth location.

The algorithm applied is an implementation of the PSPI method, according to Ferguson and Margrave. (2005). Shot-profile prestack migration works according to the imaging principle as defined by Claerbout (1971): “reflectors exist at points in the ground where the first arrival of the downgoing wave is time coincident with an upgoing wave”. This principle allows to select the events, after the backward propagation of the source wavefield and the receiver wavefield, assuming a velocity model. Methods to obtain angle gathers from these data is the first issue to be considered.

Two methods to obtain angle gathers are investigated in this report. A ray based method is proposed, which starts with a velocity model to obtain directions of the wavefield and the geometry of the geology. Then these results are related to the migrated data. Then a method proposed by Rickett and Sava (2002) is studied. This method applies the extended imaging condition concept, which was originally developed for survey-sinking migration of P-wave data, however adapted to shot-profile migration. The characteristics and the application of these methods to PP and PS data are illustrated through simple models, one flat and the otherone dipping. Amplitude analysis is considered from a theoretical solution compared with the data results obtained.

The two methods mentioned are illustrated and analyzed, one of them takes advantage of the ray trace approach and the other one uses the extended image condition approach. Amplitude properties are discussed for the ray trace method. A discussion about the two methods and about the possible steps to follow is shown at the end.

ANGLE GATHERS WITH RAY TRACING

A method that takes advantage of the ray trace approach is presented in this section and it is also applied to some synthetic models.

From the velocity model it is possible to obtain the propagation direction using ray-tracing. Besides that, in principle the velocity model allows us to obtain information about the geometry of the reflectors, since the gradient of the velocity field corresponds to the interfaces, which are the locations where the velocity filed changes the most. Ray tracing gives the slowness for a specific velocity model. The MATLAB code *shotrayvxz.m* was used for this purpose. As for migration the code *pspi_shot_cz.m*, adapted to depth migration of PP and PS waves from topography was used.

From the ray tracing it is possible to obtain the slownesses in the x and z directions, according to

$$\vec{p} = \left(\frac{\partial x}{\partial t}, \frac{\partial y}{\partial t}\right)$$

whearease with the wave direction at the ray locations it is possible to reconstruct a field of directions across the entire space of the model. The layering of the model can be obtained from the velocity model, since it corresponds to the gradient:

$$\vec{g} = \left(\frac{\partial v}{\partial x}, \frac{\partial v}{dz}\right)$$

Then the angle of incidence can be obtained from the dot product of incidence angle and the layering gradient of the geological model, as follows:

$$\cos\ \theta = \frac{\vec{p} \cdot \vec{g}}{|\vec{p}||\vec{g}|}$$

Figure 1 is a cartoon that sketches the algorithm corresponding to this method. The final product is a matrix with the angles of incidence for each one of the locations in the geologic model and for each shot.

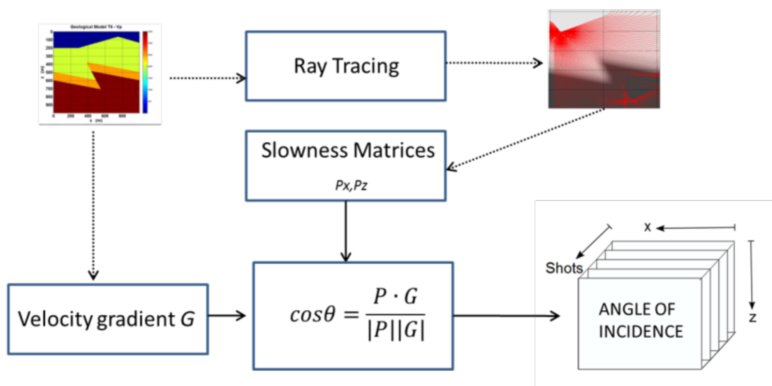

Fig. 1. Scheme of the ray tracing method to obtain the angles of incidence from the velocity field.

EXPERIMENTAL DATA

Two geological models are considered for the analysis of these methods, one with a flat reflector, and the other one with a dipping reflector, as shown in Figures 2a and 2b. It is 1000 m wide and has a depth of 1000 m. Five energy sources were located on the surface separated 100 m from each other, as illustrated in Figure 2a, including their identification number (1 to 5). The receivers were located 5 m apart along the surface, such that there are 200 in total. They are identified also with numbers from left to right.

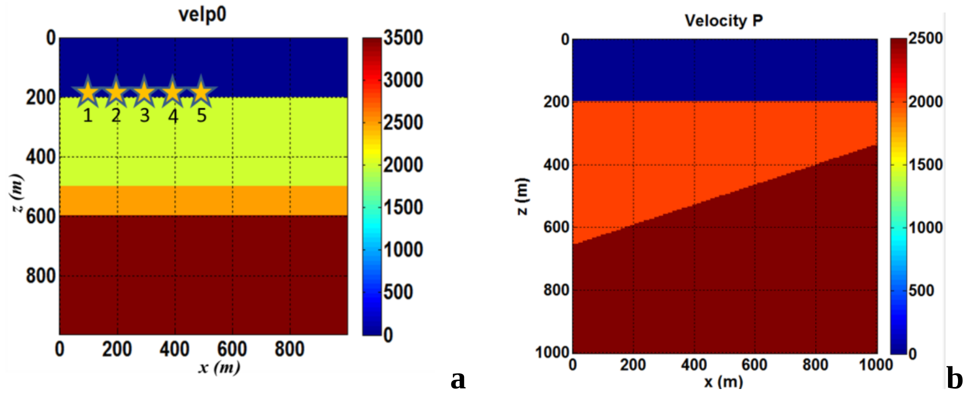

Fig. 2. (a) Model 1: Flat layers with free surface. Five shots, identified with stars, are located on the surface.(b) Model 2: dipping interface. The same shot distribution as in 2(a) was used.

A zero phase Ricker wavelet was used as the source of energy. The synthetic data for migration was obtained using Ray tracing (RT) and Finite difference (FD) methods. , since RT allows the separation of the seismic events and FD generates data closer to real. However, after comparing some results, no meaningful difference was noticed, so most of the experiments were carried out with the RT data.

Figure 3 illustrates the procedure. Figure 3a show the RT applied to shot 5 and the dipping model to obtain the slownesses, and Figure 3b illustrates an angle gather corresponding to receiver location 50, at x=250 m. Notice in this figure that there is a limited range in the angles with data, since this data results from the interpolation of the data from only five shots, each one with a limited migration aperture. Figure 4 illustrates the final step that generates the angle gathers.

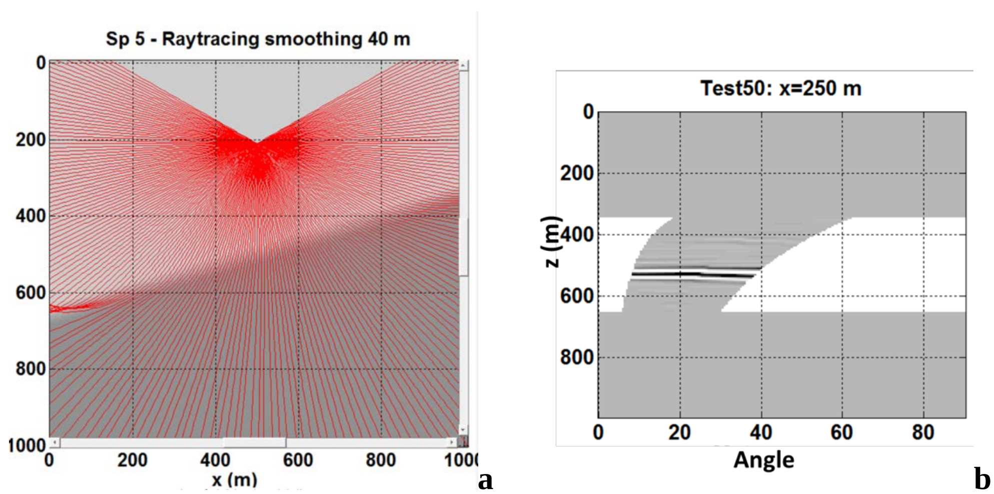

Fig. 3.Two stages of the ray tracing method: (a) Ray tracing (RT) on the dipping model, to obtain the incidence angles. (b) Example of an image angle gather for the location x=250 m of the 2D line.

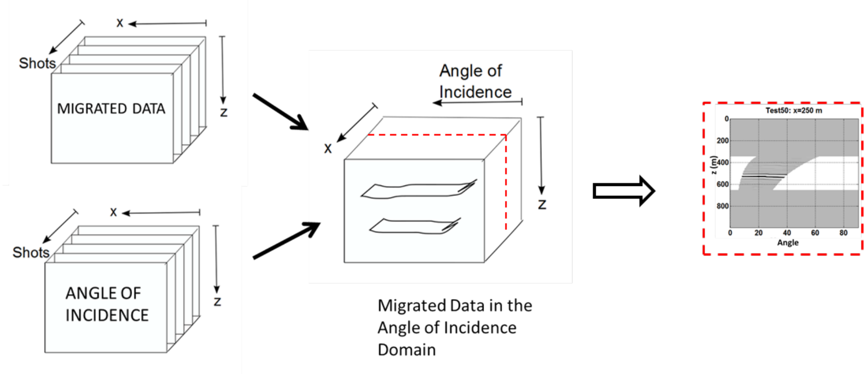

Fig. 4. Scheme of the step that generate the angle gather. Both data sets, angle of incidence and migrated data, are in a compatible volume, which allows to generate the mapping angles of incidence-migrated data, and generate angle of incidence image gathers for each location, as shown with an example..

Figure 5 shows the results of an amplitude versus offset analysis carried out on the data, using the migrated data resulting from RT and from FD, and for the PP (Figure 5a) and the PS (Figure 5b) events. The theoretical results from the calculation using the Zoeppritz equationsare also shown. The vertical scale was updated to make all these sets roughly equivalent. These amplitudes resulted of the stacking of all the amplitudes with angle for the case of the dipping reflector. A similar result was obtained with the flat reflector. It can be noticed that both data (RT and FD) agree closely, and show good correlation with the Zoeppritz prediction. However this agreement decreases for bigger angles, closer to the critical angle (53° in this case), especially for the P-wave case. It can be attributed partially to the illumination characteristics of the survey. However the possibility of being related to the migration methods deserves more detailed analysis.

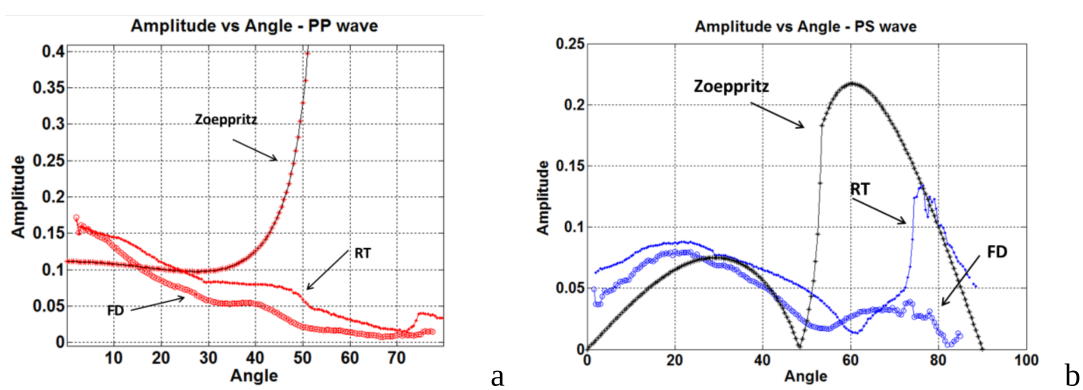

Fig.5. Amplitude vs. Incidence Angle from the ray trace method, comparing migrations of Ray Tracing (RT) and Finite Difference (FD) data, and the theoretical Zoeppritz calculation. (a) PP wave and (b) PS waves.

ANGLE GATHERS WITH THE EXTENDED IMAGING CONDITION METHOD:

The imaging condition, as defined by Claerbout (1971), can be represented as

$$Im(x,z) = \int U(x,z,\omega)D(x,z,\omega)^* \, d\omega$$

where U corresponds to the upgoing wavefield, D to the downgoing wavefield, both in the frequency domain, and the asterisk marks the complex conjugate.

Some authors have noticed that this imaging condition selects an average of the reflectivity, neglecting the amplitude variation with the angle of incidence. De Bruin et al., (1990) presented a method to obtain reflectivity in the angle domain to perform AVO analysis using the *slant-stack* or *τ-p* transform, after compensation of the propagation effects, and then selecting the zero time . This method can be applied to shot-profile migration.

Methods oriented to survey-sinking migration were proposed by Prucha et al. (1999) and by Sava and Fomel (2003). The former is applied before the imaging condition and the later after the imaging condition. A method to obtain angle gathers for shot-profile migration was proposed by Rickett and Sava (2002), based on the Sava and Fomel (2003) approach. This approach is also known as the extended imaging condition method. Not only zero offset data are correlated, but the offsets in an specific range. The extended image can be expressed as follows:

$$Im(x,z,h) = \int U(x+h,z,\omega)D(x-h,z,\omega)^* \, d\omega$$

Where h is the offset. Figure8, later, illustrates this idea with an example of the data of the flat model.

From these data migrated with the extended imaging condition, as shown by Sava and Fomel (2003), the following relation between the migrated data can be applied:

$$\tan\gamma = -\left.\frac{\partial z}{\partial h}\right|_{t,\boldsymbol{x}} = -\frac{|\boldsymbol{k}_h|}{k_z}$$

Figure 6, taken from the demonstration in the appendix of Rickett and Sava (2002), illustrates the meaning of the variables. As shown, γ is half the opening angle between the source direction and the receiver direction. It would correspond to the incidence and reflection angle in the case of PP waves, but not for converted waves, as shown by Rosales et al. (2008).

Figure 7 shows the algorithm used by Rickett and Sava (2002), which uses the Radial Transform (below), and an option using the Radon Transform (above). The Radon transform method was used in the examples presented here. It is based in the following relation:

$$A(z,\mu) = \int H(z, z+\mu.h)dh$$

Where μ is defined as

$$\mu = -\frac{\partial z}{\partial h}$$

that is to say, it is the tan γ.

The migration with the extended imaging condition is illustrated in Figure 8. Figure 8a shows the result of the standard migration, that is to say, the same result as for the extended method with offset zero. Figure 8b shows the data migrated for all the offsets selected for a specific x-location, corresponding to a surface location of 300 m in this case, and for PP events. The former result is the input to the Radon transform analysis to obtain the common image angle gathers.

A result of the application of this method is shown in Figure 9. In this example the angle gathers of the five shots for the data corresponding to the location x=300m are stacked (Fig. 9a), hence obtaining a more representative angle gather for that location. An illustration of a ray is presented in Figure 9b, which allows a theoretical calculation for comparison. The angles are into the range expected.

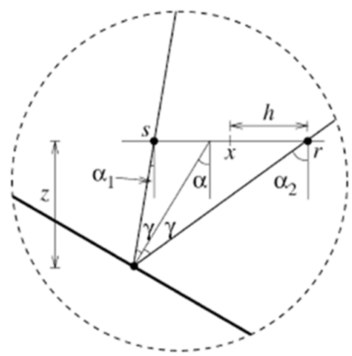

Fig 6. Relation among the variables according to the approach of Rickett and Sava (2002). This is a depth location, so the offset *h* should be understood as offset at the image point.

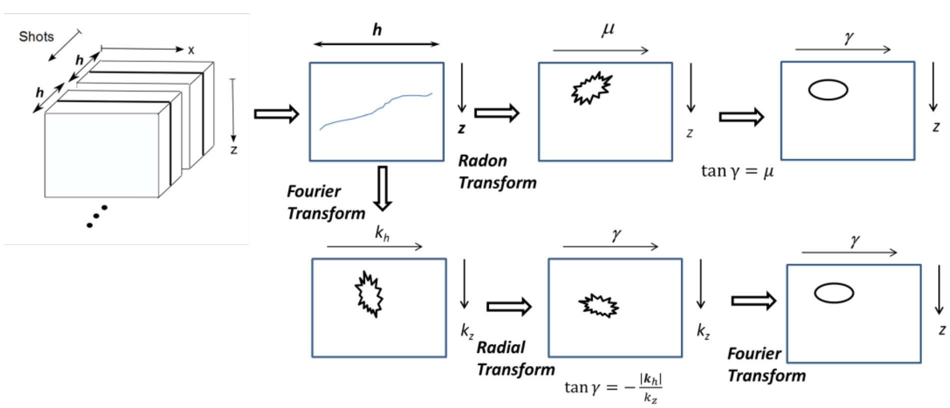

Fig. 7. Angle gathers analysis method, according to Rickett and Sava (2002). Two options with the same result are presented, one using the Radon Transform in the *h-z* (offset-depth) domain, and the other one using the Radial Transform in the k_h-k_z (wave numbers) domain. The Radon Transform approach was used in this work.

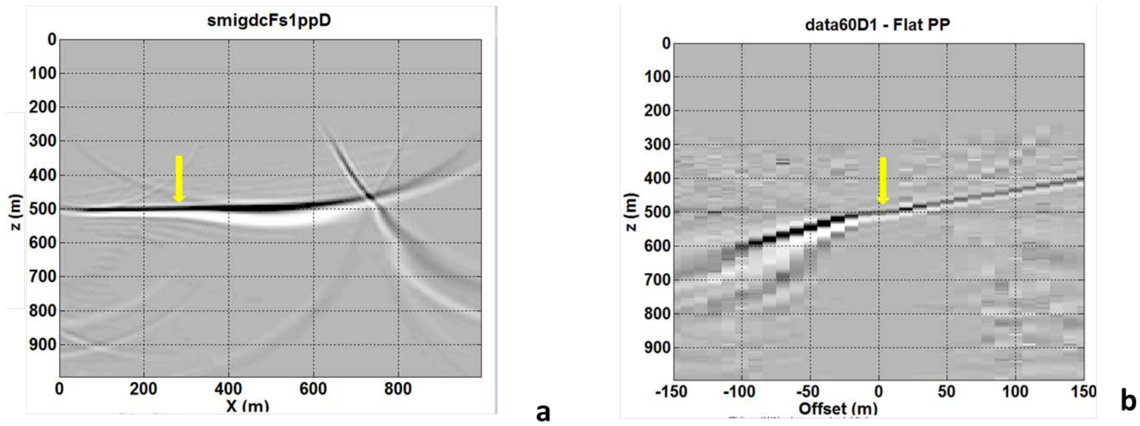

Fig.8. (a) The conventional migration section and (b) the extended image for the location x=300m The point shown by the arrows is the same one, since its offset is zero and its location is x=300m.

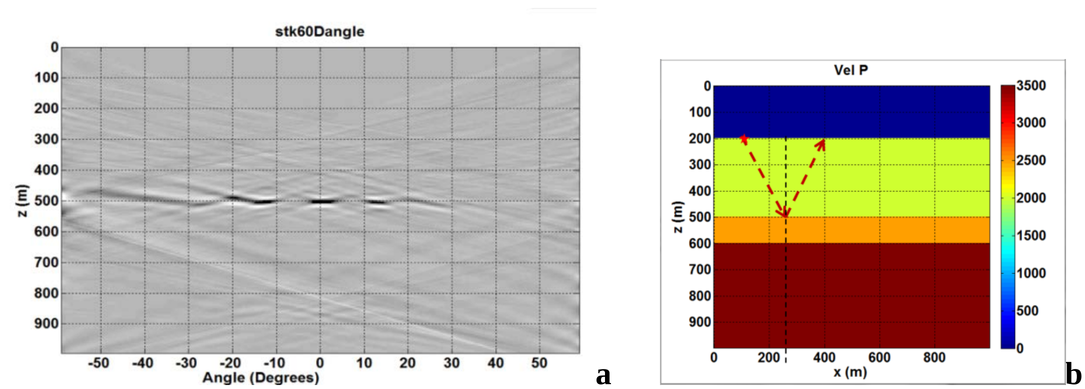

Fig. 9. (a) Angle gather for the location x=300m from five shots. Notice the energy concentrated at locations corresponding to specific angles, each one corresponding to a shot. (b) Illustration of the corresponding angle of incidence for shot 1.

Figure 10 shows some more analysis of amplitudes. Figure 10a is an average of amplitudes with angle for the x=300m location, which can be considered another way to see the results of Fig. 9a. Figure 10b is the stack of all the amplitudes vs. angle for two shots (1 and 5), which is possible in this experiment since the properties of the model are the same for all the shots and receivers. It can be compared with the result of a similar experiment carried out for the ray tracing method (Fig. 5), e.g. with the Zoeppritz results. These amplitudes appear reasonably similar, however there is a noticeable difference in energy between shot 5 and shot 1.

Figure 11a shows the amplitude as a function of the angle of the PP wave for each depth location along the seismic line from shot 1, and Figure 11b a similar result for shot 5. Notice the asymmetry of Shot 1, and the symmetry of Shot 5, as expected, since Shot 1 is close to an end and Shot 5 is in the middle of the model. The limited span on both can be related to the migration aperture. The ratio between angle and location (i.e., the distance to the source) appears strongly linear.

Figure 12 and 13 shows the results of a similar analysis carried out over the converted wave (PS). In Figure 12 the angle gathers of the five shots for the data corresponding to the location x=300m are stacked for the PS event, with the expectation of some information about the angles of incidence. The angles are clearly smaller, which can have an explanation on the assumptions of the method, which was derived for PP waves. Rosales et al. (2008) presented a derivation for angle gathers of converted waves. A test on this data with this method was not consistent with the expected result, so more extended analysis is required.

Figure 13a, similarly to Fig. 11, shows the amplitude as a function of the angle of the PS wave for each depth location along the seismic line from shot 1, and Figure 13b a similar result for shot 5. Noticeable differences can be observed, such as the non-linear ratio between distance and angle, low angles, and energy extended to more locations.

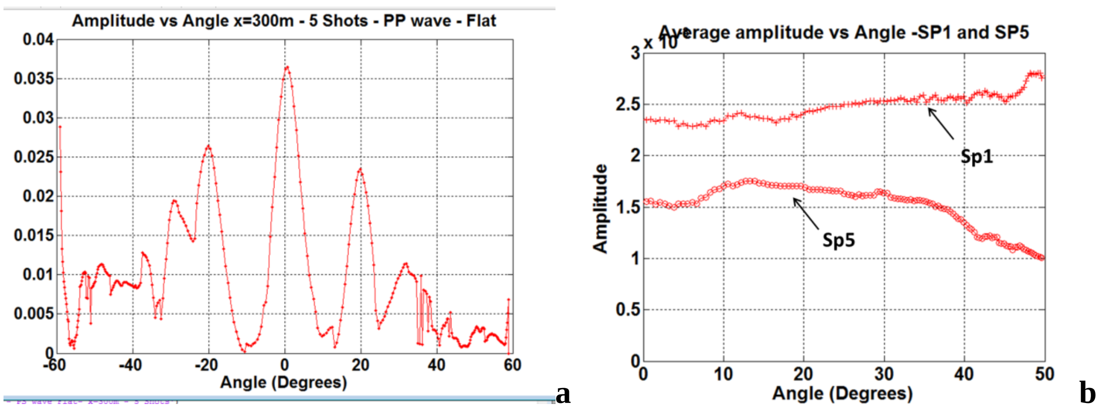

Fig. 10. Amplitude for the angle gathers, PP . (a) Average at the location x=300m for the 5 shots (b) Average for all the locations at the shots 1 and 5.

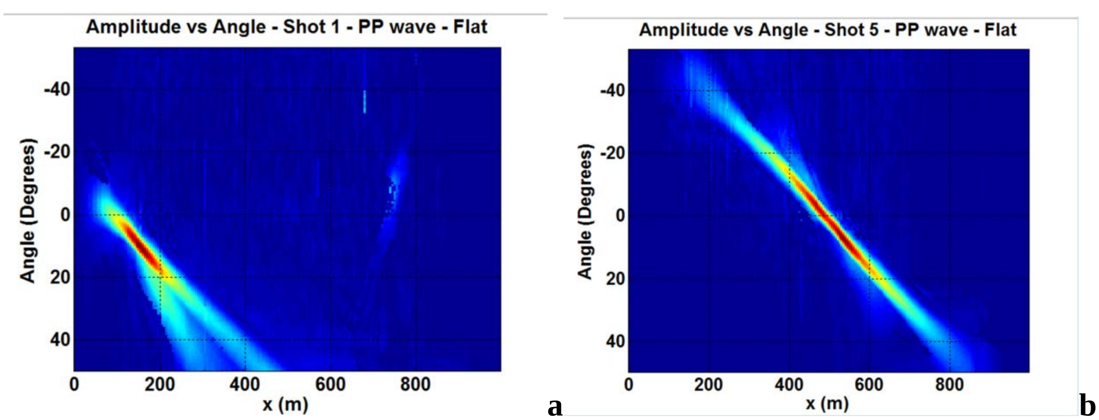

Fig. 11. Amplitude as a function of the angle of the PP wave for each depth location along the seismic line from two shots. (a) Shot 1 (100 m) and (b) Shot 5 (500 m). Notice the asymmetry of Shot 1, and the symmetry of Shot 5, as expected, and also the limited span on both, which is related to the migration aperture.

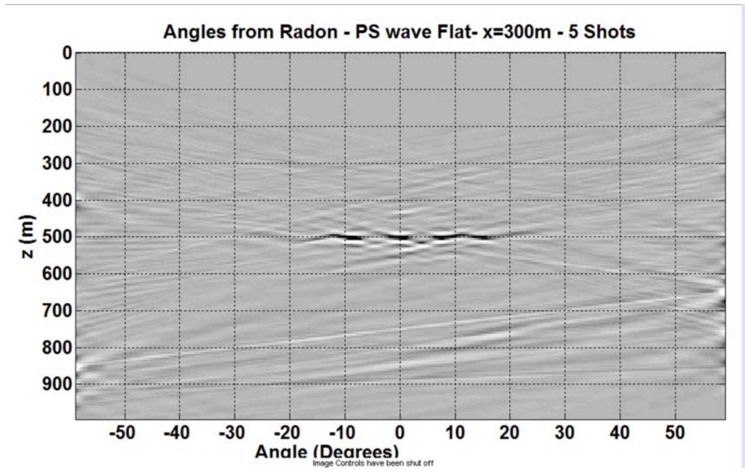

Fig. 12. Angle gather of the PS wave for the location x=300 from five shots. Notice the energy concentrated at locations corresponding to specific angles, each one corresponding to a shot.

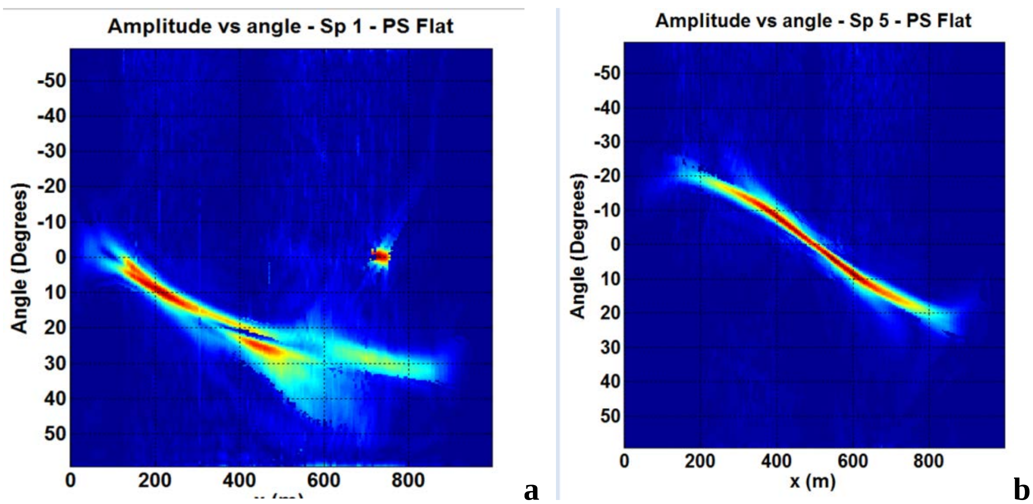

Fig. 13. Average amplitude with the angle for each location for (a) shot 1 and (b) shot 5. Notice that the relationship is not linear and the angles are smaller.

SUMMARY AND CONCLUSIONS

Two approaches for angle gathers were applied to PP and PS synthetic data obtained with simple models. Ray tracing uses the data from the velocity field and after that angle gathers are selected from the migrated data. The extended image method obtain the angles from the data. These resulting angles are reasonable for PP data. Preliminary results do not show good agreement for PS data.

From the analysis of the amplitudes vs. angle obtained after migration and the theoretical results according to Zoeppritz equations, qualitative agreement with the theory can be observed in the raytracing method. Reasonable results were also found with the PP wave on the extended imaging method, but not for the PS data.

This amplitude survey required analysis of an statistically meaningful amount of data, since there are strong differences looking at individual results, although the models were quite simple. Methods like that appear important for more complex cases.

The extended image condition using offset data for converted waves requiresfurther investigation. Angle gathers for converted waves and the velocity error sensitivity analysis for PP and PS waves are topics for additional research.

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