Numerical analysis of 1.5D internal multiple prediction

Pan Pan and Kris Innanen

ABSTRACT

Multiples attenuation is a key process in seismic data processing, and the quality of multiples elimination will affect the final imaging directly. In this paper, we present a 1.5D MATLAB implementation of the inverse scattering series internal multiple prediction algorithm developed by Weglein and collaborators in the 1990s. This method does not require any subsurface information. However, near offset traces information will be needed for applying this method. We discuss the whole prediction operation, and illustrate the procedure with a synthetic example. Effects of various epsilon values chosen will reveal a more efficient method of choosing the epsilon value. Usefulness of our 1.5D internal multiple prediction algorithm in situations where primaries are mixed together with internal multiples, and dipping interface exists are also demonstrated.

INTRODUCTION

For the exploration of oil and gas reservoirs, multiples can be one of the main issues in applying the seismic method. The key characteristic of the inverse scattering series based method is that they do not require any a priori information from the subsurface as they are fully data-driven. Furthermore, the primary reflections remain untouched. However, source wavelet and near offset traces information will be needed for applying this method. It will compute internal multiples from all possible generators. The output of the algorithm is a data set that contains the predicted internal multiples (Hernandez and Innanen, 2012).

In this paper we review the basic principles of the inverse scattering series internal multiple prediction algorithm, which was introduced to the industry in the 1990s (Araújo et al., 1994; Weglein et al., 1997, 2003), and demonstrate its use to 1.5D data using a MATLAB implementation. This implementation has been tested with good results on band-limited synthetic data, even for situations where primaries are mixed together with internal multiples, and dipping interface exists. Our plan forward is to explore the field application of the current algorithm using a similarly staged approach as Hernandez and Innanen (2012) from synthetic, physical modeling and finally to land data environments.

THEORY

The discussions of the transformation of the data from the space and time domain to those of wavenumber and pseudo-depth, as well as derivations of 1D and 2D inverse scattering series internal multiple prediction algorithms are given in Pan and Innanen (2013). In this section, we only focus on the 1.5D internal multiple prediction algorithm.

If the data have offset but the Earth is nearly layered, a 1.5D version of the algorithm can be considered, in which

\[ k_g = k_s, \]  

then we can obtain the 1.5D algorithm.
\[
\begin{align*}
\beta_{31M}(k_g, \omega) &= \int_{-\infty}^{\infty} dz e^{ik_2z}b_1(k_g, z) \int_{-\infty}^{z-\epsilon} dz' e^{-ik_2z'}b_1(k_g, z') \\
&\times \int_{z'+\epsilon}^{\infty} dz'' e^{ik_2z''}b_1(k_g, z'')
\end{align*}
\]

(2)

where \( k_2 = 2q_g \).

Compared to the 2D algorithm, the computation cost has been dramatically reduced, with the equivalent of a single 1D prediction for every output \( k_g \) (Innanen, 2012). As fewer wavenumbers are participating in the calculation, it is much cheaper and faster than 2D.

**SYNTHETIC EXAMPLE**

We first apply the 1.5D internal multiple prediction algorithm in synthetic data, which is generated by the finite difference method, with a three-layer velocity model (see Figure 1). The depth and velocity of each layer in this model are shown in Table 1. In Figure 2, a single shot record of data is illustrated. In Figure 2a, three primaries are indicated in yellow. In Figure 2b, two internal multiples are indicated in red. Our goal is to use the primaries as subevents to predict these two internal multiples at all offsets. The CREWES acoustic finite difference function `afd_shotrec.m` is used to create the data. To avoid free surface multiples, we need to ensure that the boundary condition is absorbing on all four sides. The source and receiver interval is 10\( m \), and the record length is 3\( s \), with a sampling rate of 2\( ms \). A frequency band of \([5 \ 10 \ 30 \ 40]\) is chosen in order to get a localized wavelet. Also, we need to remove the direct wave, as direct wave is not concerned in the calculation. Deconvolution and deghosting are useful steps in preprocessing, but if the internal multiples are resolvable in the data set without these steps, they may be avoided (Innanen, 2012).

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**FIG. 1.** Three layered velocity model used to generate synthetic data and test the 1.5D internal multiple prediction algorithm.

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![Image of velocity model](image-url)
FIG. 2. Shot record calculated using the synthetic model in Figure 1. (a) Zero offset travel times of primaries are indicated in yellow; (b) Zero offset travel times of internal multiples are indicated in red.

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of ( x )</td>
<td>512</td>
</tr>
<tr>
<td>Number of ( z )</td>
<td>512</td>
</tr>
<tr>
<td>Interval sample time</td>
<td>2ms</td>
</tr>
<tr>
<td>Velocity and depth of the first interface</td>
<td>2800m/s at 640m</td>
</tr>
<tr>
<td>Velocity and depth of the second interface</td>
<td>4500m/s at 1280m</td>
</tr>
<tr>
<td>Velocity and depth of the third interface</td>
<td>5500m/s at 1800m</td>
</tr>
<tr>
<td>Wave speed of the source/receiver medium</td>
<td>1500m/s</td>
</tr>
<tr>
<td>Time step</td>
<td>1ms</td>
</tr>
<tr>
<td>Maximum time of the shot record</td>
<td>3s</td>
</tr>
<tr>
<td>Location of the source</td>
<td>(2, 256)</td>
</tr>
<tr>
<td>Frequency band (Hz)</td>
<td>[5 10 30 40]</td>
</tr>
<tr>
<td>Optimum epsilon</td>
<td>200</td>
</tr>
<tr>
<td>Source and receiver interval</td>
<td>10m</td>
</tr>
</tbody>
</table>

Table 1. Parameters of the velocity model and shot record

Secondly, we need to create the input \( b_1(k_g, z) \). We Fourier transform the data from the time domain to the frequency domain and then define a regular output grid on
\((k_g, k_z)\). Since the wavenumber \(k_z\) is conjugate to pseudo depth \(z = c_0 t/2\), we can choose an optimum grid for resampling vectors by starting with time vector:

\[
t = dt \ast \left( (1: N) - 1 \right),
\]

where \(dt\) is the sampling interval. We can thereafter map to pseudo-depth

\[
dz = c_0 \ast dt / 2,
\]

where \(c_0\) is the reference medium P-wave velocity, and then defining

\[
k_z = -N/2 : N/2 - 1;
\]

\[
k_z = k_z / (N \ast dz),
\]

as the regularly sampled output depth wavenumber.

Since the relationship between \((k_g, \omega)\) and \((k_g, k_z)\) is nonlinear, a direct change of variables from this regular grid would lead to a data set on an irregular \((k_g, k_z)\) grid (Innanen, 2012). So we compute a regular \((k_g, k_z)\) grid to get around this problem.

FIG.3. The input \(b_1(k_g, z)\) is generated using the input data and reference velocity \(c_0\).

Figure 3 is the core input to the prediction algorithm. Note it is constructed for positive \(k_g\) values only. We will fill the negative wavenumbers using conjugate symmetry, then inverse Fourier transform the input over \(k_z\), appearing in the pseudo depth domain. Three primaries are visible on the graph when \(k_g > 0\). Figure 4 is the comparison of the input zero offset trace and constructed \(b_1(k_g, z)\) stacked over \(k_g\). In Figure 4b, positions of primaries are indicated in red circles and internal multiples are indicated in blue circles.
1.5D internal multiple prediction

Finally, we put the input $b_1(k_g, z)$ into the prediction algorithm. This 1.5D prediction contains three nested loops which are lateral wavenumber, temporal frequency and pseudo depth. Optimal choices of beginning and ending integration points in all three cases will speed up computation (Innanen, 2012).

Figure 5 is the output of the 1.5D internal multiple prediction. In Figure 5a, the prediction output matches well with the travel times of internal multiples in Figure 5b. Internal multiples around 1.8s and 2.0s are correctly predicted. The zero offset travel times and moveout patterns of the internal multiples are correctly displayed in the prediction output, which means our 1.5D internal multiple prediction algorithm works well on layered synthetic data.

Now another scenario with mixed primaries and internal multiples will be tested. All the parameters are the same as the above case except for the velocity and depth of each layer, which are shown in Table 2. Figure 6 is the prediction output of this case. The 1.8s internal multiple, which is mixed with primary, has been correctly predicted. Through this figure, we can see that even though a primary and an internal multiple are mixed together, our 1.5D prediction algorithm still yields promising results.

<table>
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<td>2800m/s at 640m</td>
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<td>Velocity and depth of the second interface</td>
<td>4000m/s at 1280m</td>
</tr>
<tr>
<td>Velocity and depth of the third interface</td>
<td>5000m/s at 2000m</td>
</tr>
<tr>
<td>Wave speed of the source/ receiver medium</td>
<td>1500m/s</td>
</tr>
</tbody>
</table>

Table 2. Parameters of the velocity model

FIG. 4. Comparison of the input zero offset trace and constructed $b_1(k_g, z)$ stacked over $k_g$. (a) Input data: zero offset trace; (b) $b_1(k_g, z)$ stacked over $k_g$.
FIG. 5. The output of the 1.5D internal multiple prediction with epsilon value equals 200. (a) The prediction, in which two internal multiples are predicted. (b) The original data with both primaries and internal multiples shown in the data set.

FIG. 6. The output of the 1.5D internal multiple prediction with epsilon value equals 200. In this case, a primary and an internal multiple are mixed together. (a) The prediction, in which two internal multiples are predicted. (b) The original data with both primaries and internal multiples shown in the data set.

ANALYSIS ON EFFECTS OF EPSILON VALUES

In this section, we will perform an analysis on the effects of various epsilon values chosen. Epsilon values of 100, 200 and 300 are chosen to represent this study with the
optimal epsilon value determined to be 200. We use the first velocity model and shot record as the input to implement the test. Figure 7 is the output with epsilon value equals 100. Figure 5 is the output with epsilon value equals 200 and Figure 8 is the output with epsilon value equals 300.

**FIG. 7.** The output of the 1.5D internal multiple prediction with epsilon value equals 100. (a) The prediction, in which two internal multiples are predicted. (b) The original data with both primaries and internal multiples shown in the data set.

**FIG. 8.** The output of the 1.5D internal multiple prediction with epsilon value equals 300. (a) The prediction, in which two internal multiples are predicted. (b) The original data with both primaries and internal multiples shown in the data set.
Comparing these three figures, we can see that if the epsilon value is smaller than the optimal value, artifacts will be seen at the arrival times of primaries, while the larger value will damage the important information in the prediction output. So far, there exists no rule with which epsilon values can be directly calculated. This analysis gives us an idea on how to choose epsilon values more efficiently.

**ANALYSIS ON EFFECTS OF DIPPING ANGLES**

Our assumption that the 1.5D internal multiple algorithm can correctly predict the internal multiples is based on the Earth is nearly layered. In this part, we test the theory that even if the assumption cannot be satisfied, our 1.5D internal multiple prediction algorithm is still able to predict internal multiples and show some examples of shot records with dip to analyze the effects of dipping angles on the 1.5D algorithm. Figure 8 is the velocity model, where the generator is the dipping interface with dipping angle equals 2 degrees. Figure 9 is the shot record of the velocity model. In this Figure, the arrival times of the internal multiples are affected with the appearance of dipping angle in the generator. The prediction output of 1.5D algorithm is displayed in Figure 10.

Another test of a larger dipping angle is also performed. We choose dipping angle equals 5 degrees this time. Figure 11, 12 and 13 are velocity model, shot record and prediction output, respectively.

Comparing Figure 9 and Figure 12, we can see that as dipping angle increases in the generator, more artifacts show up in the shot record. As well, the effects on the internal multiples become greater.
1.5D internal multiple prediction

FIG. 9. Shot record calculated using the synthetic model in Figure 8. (a) Zero offset travel times of primaries are indicated in yellow; (b) Zero offset travel times of internal multiples are indicated in red.

FIG. 10. The output of the 1.5D internal multiple prediction with epsilon value equals 200. (a) The prediction, in which two internal multiples are predicted. (b) The original data with both primaries and internal multiples shown in the data set. Red lines indicate the positions of the internal multiples, which mean the zero offset travel times are correctly predicted.
FIG. 11. Three layered velocity model with the first layer’s dipping angle of 5 degrees used to test the effects of dipping angles on 1.5D internal multiple prediction algorithm.

FIG. 12. Shot record calculated using the synthetic model in Figure 11. (a) Zero offset travel times of primaries are indicated in yellow; (b) Zero offset travel times of internal multiples are indicated in red.
1.5D internal multiple prediction

Figure 13. The output of the 1.5D internal multiple prediction with epsilon value equals 180. (a) The prediction, in which two internal multiples are predicted. (b) The original data with both primaries and internal multiples shown in the data set. Red lines indicate the positions of the internal multiples, which mean the zero offset travel times are correctly predicted.

Figure 10 and Figure 13 are the output prediction of the above two cases, neither of which satisfy the assumption. Comparison of these two figures shows that the zero offset travel times in both two cases are correctly predicted. For the smaller dipping angle case, the results are more accurate as both the zero offset travel times and moveout patterns of the internal multiples are captured in the prediction. For the larger dipping angle case, when the offset becomes larger, the prediction error increases. Also, more artifacts and edge effects emerge at the arrival times of the predicted internal multiples, which are matters of ongoing study.

CONCLUSIONS

We implement a 1.5D version of the inverse scattering series internal multiple prediction algorithm developed by Weglein and collaborators in the 1990s in MATLAB. Compared to the 2D algorithm, the computation cost has been dramatically reduced. With fewer wavenumbers participating in the calculation, it is much cheaper and faster than 2D. We illustrate the procedure of predicting internal multiples with a synthetic model. This method does not need any subsurface information and is suitable for the situation of primaries and internal multiples mixed together. However, near offset traces information will be needed for applying this technique. We also perform an analysis on the effects of various epsilon values. For a smaller epsilon value, artifacts will be seen at the arrival times of primaries. For a larger epsilon value, it will damage the important information in the prediction output. By these principles, we can now choose the epsilon value more efficiently. Our 1.5D internal multiple prediction algorithm is also demonstrated to be useful for the situation where dipping interface exists. For smaller dipping angles, the
results of the algorithm are quite promising, correctly predicting both the zero offset travel times and moveout patterns. For larger dipping angles, only the zero offset travel times can be correctly predicted with errors in the larger offset. However, it can also assist in verifying the positions of internal multiples, which is helpful for later inversion and interpretation.

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REFERENCES


