Non-linear Vibroseis models for generating harmonics

Michael P. Lamoureux

ABSTRACT

Vibroseis devices are a convenient seismic source for generating energy to propagate into the earth, driving a spot on surface of the earth with a controlled force that sets up a seismic wave used in the seismic imaging experiment. For a variety of reasons, harmonics are generated when a pure frequency drives the vibroseis device – these harmonics may be considered as noise, or as extra correlated data that might be used in the imaging algorithms.

In this short project, we build several simple mathematical models for the vibroseis device, to help to understand where these harmonics come from. The simplest models are one-dimensional non-linear oscillators, where the response of the oscillator includes non-linear effects of the earth, limiting devices on the motion, and other mechanisms that may introduce harmonics. Better models include more physical details of the vibroseis device – the motion of the reaction mass, the baseplate, the control machinery, and so on. These require a system of ordinary differential equations to represent the mathematics. Not all models are able to capture all the harmonic details of the vibroseis device.

We make use of MATLAB ODE solvers for the numerical simulations, and the Gabor transform to view the time-frequency characteristics of the seismic signal generated in these vibroseis models. This provides a simple test to verify the presence and behaviour of the harmonics in these signals.

INTRODUCTION

Vibroseis sources are a popular method for generating seismic waves used in seismic imaging. There is a great deal of technical experience in industry with these devices, and a great deal has been written about the operations of these devices. For instance, the journal Geopysical Prospecting recently published an entire issue dedicated to technical articles on vibroseis devices, including these works (Sallas, 2010), (Wei, 2010), (Boucard and Ollivrin, 2010), (Ziolkowski, 2010), (Nagarajappa and Wilkinson, 2010), (Krohn et al., 2010) on the physical mechanics of hydraulic vibrators, their control mechanisms, dynamics of the baseplates, use in deconvolution, among others. Other articles, such as (Cooper, 2002) discuss some of the assumptions and rules-of-thumbs in operating these devices in the field. Many make the observations that vibroseis devices generate harmonics, which can be a problem if it detracts from the seismic imaging process.

This author was recently introduced to the technology of vibroseis through a MITACS-funded project with StatOil on harmonic separation of signals, and a PIMS/IMA Mathematical Modeling in Industry Workshop (2014). With a group of students and postdoctoral fellows, we saw a lot of vibroseis data, and a lot of potential problems with the data. This short project came out of some questions that were left open at the conclusion of the project.

Specifically, how simple (or minimally complex) a mathematical model does one need to reproduce the complex waveforms that the vibroseis produces? There is a great deal of discussion as to what may cause the complexity in the waveform. Cooper (Cooper, 2002) suggests one problem is if the hold-down weight is insufficient, the vibrator truck “tends to turn into a giant pogo stick. An 80,000 lb machine jumping up and down on the ground is NOT what we want." Wei (Wei, 2010) suggest the harmonics may be coming from a flexing of the metal baseplate itself, as well as possibly the interaction with the ground. Others (Sallas, 2010), (Boucard and Ollivrin, 2010) point out the control mechanisms, which track phase and amplitude of the generated signal, could result in complex behaviour.

There exist already several complex models of the vibroseis device, which include the control circuitry, ground interaction, and baseplate deformations: see for instance (Wei, 2010), (Boucard and Ollivrin, 2010). In this project, we are interested in simpler models, that have just enough complexity to general the harmonics we see in real data.

This paper is organized as follows: we start with some observations about the types of waveforms and harmonics we see in real vibroseis data. We then produce some non-linear models in ordinary differential equations that general harmonics when driven by a pilot signal. Finally, we look at a more complex model that has two oscillating masses, corresponding to the baseplate and reaction mass of the real vibroseis device. Along the way, we have some comments about the tools we used to create, and examine, the numerical solutions to these mathematical systems from within MATLAB.

Due to lack of time, we were not able to get far enough along in the project to get a simple model that produces all harmonics.

REPRODUCING OBSERVED WAVEFORMS

Over the course of the PIMS/IMA summer workshop on mathematical modelling*, our group saw a great deal of vibroseis data, generously provided by StatOil. While we do not present the raw data here, we can make some observations.

Vibroseis trucks record several pieces of data while in operation. First is a recording of the pilot signal, typically a sinusoid waveform with slowly increasing frequency from about 5 Hz to 200 Hz, generated precisely by a computer. This pilot signal drives the motion of a reaction mass, and the motion of the baseplate which is in direct contact with the ground. The motion of the reaction mass and baseplate is recorded by two accelerometers, mounted at or near the centre of these masses. These last two recordings provide an accurate measure of the real signal that is being produced by the device.

Observing the baseplate recordings, we would often see very asymmetrical waveforms such as the one shown in Figure 1. It was difficult to believe that such a skewed waveform could be obtained as a simple sum of a few harmonics. In fact one concern at the workshop was that this asymmetric shape must represent a very complex behaviour in the vibrations of the device. However, it turns out the situation is relatively simple. A little experimenting with summing sinusoids shows that this form

$$x''(t) = \sin(2\pi t) + \sin(4\pi t + \pi/2) \tag{1}$$

*http://www.pims.math.ca/industrial-event/140806-ipmmiw reproduces the acceleration waveform in Figure 1. That is, a simple sinusoid and one harmonic, phase shifted by $\pi/2$, is sufficient to create this asymmetric waveform.

Significantly, by integrating acceleration $x''(t)$ twice, we obtain (up to a scale factor) the periodic position waveform

$$x(t) = -\sin(2\pi t) - \frac{1}{4}\sin(4\pi t + \pi/2) \tag{2}$$

which is displayed in Figure 2. It's not hard to imagine that such a flattened waveform might be obtained when the baseplate is being forced into the hard earth – on the downward motion, it hits resistance and flattens out the motion.

Ideally, we would like to have our ODE models produce waveforms that look like those in Figures 1 and 2.

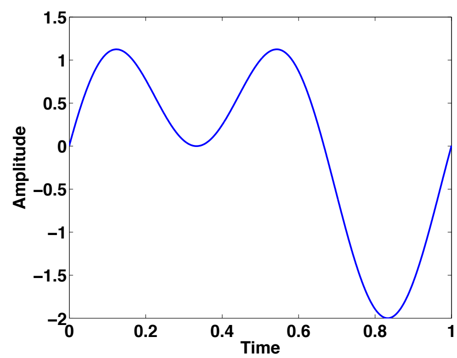


FIG. 1. Typical baseplate recording (acceleration), one cycle of the waveform.

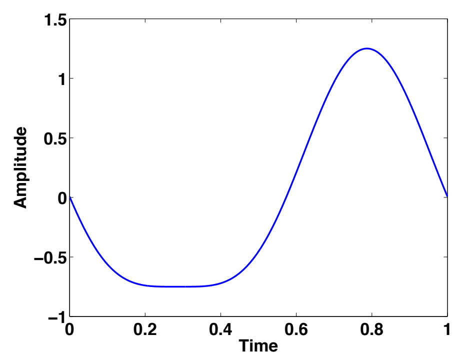


FIG. 2. Derived baseplate position, one cycle of the waveform.

BASEPLATE ACCELERATION WITH HARMONICS

The actual recording of the baseplate shows the strong presence of harmonics. The pilot signal, which drives the motion of the baseplate and reaction mass, is a simple sinusoid with an increasing frequency. It can be expressing mathematically in the form

$$p(t) = \sin(2\pi t f_{max}(\frac{t}{t_{max}})^q), \tag{3}$$

where f_{max} is the maximum frequency, t_{max} the maximum time of the pilot sweep, and q is a parameter to shape the contour of the frequency sweep.† The baseplate recording sees this sinusoid and its harmonics, creating a sum of the form

$$bp(t) = \sum_{n=1}^{N} a_n \sin(2\pi n t f_{max} (\frac{t}{t_{max}})^q), \tag{4}$$

with different weights a_n, and potentially different phase delays (not indicated here). Figure 3 shows a synthetic representation of a typical baseplate recording, in the time-frequency domain. We see for time increasing from 0 to 20 seconds, the fundamental rises from 0 Hz to about 200Hz, mirrored by harmonics with frequencies 2, 3, and 4 times the frequency of the fundamental. The pilot signal, if plotted, would only show the bottom curve, which is the driving, fundamental sinusoid.

The plot in Figure 3 is the Gabor transform of the signal from the accelerometer, a standard-time frequency display equivalent to the localized, short-time Fourier transform.

Our goal is to find a simple differential model for the vibroseis device that recreates this suite of harmonics.

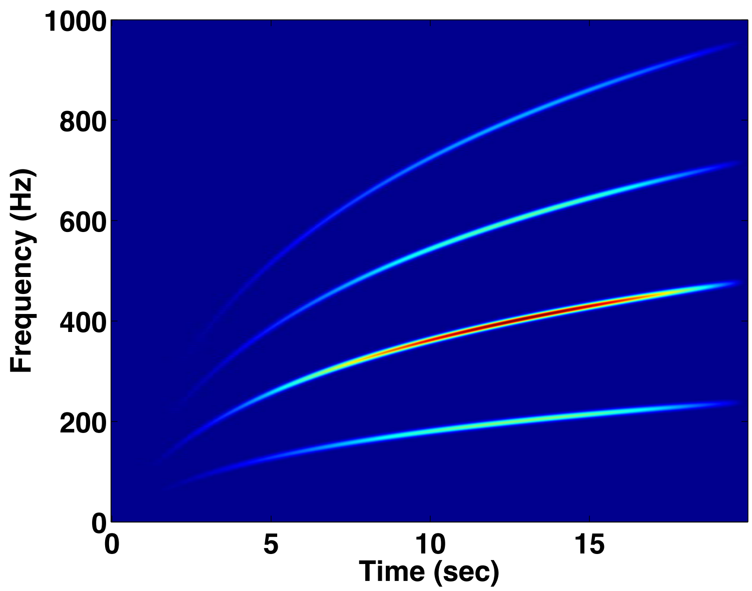


FIG. 3. Time-frequency recording of a synthetic baseplate accelerometer.

A SIMPLE ODE MODEL OF A FORCED OSCILLATOR

A simple harmonic oscillator representing a mass suspended from a spring is represented by a second order, linear differential equation in the form

$$mx''(t) + bx'(t) + kx(t) = f(t), \tag{5}$$

† $q = 1$ corresponds to a linear sweep. Note in principle the maximum frequency is actually $(1 + q)f_{max}$, since the instantaneous frequency depends on the derivative of the argument of the sinusoid.

where $x(t)$ is the displacement of an object, $x'(t)$ its velocity, and $x''(t)$ its acceleration. The constants m, k, b are the mass of the object, Hooke's constant for the spring, and a damping factor, respectively. The function $f(t)$ is a forcing term that drives the system.[‡] There is not much to say about this model – drive it with a sinusoid, and it responds with a sinusoid of the same frequency, but a different phase and amplitude. The phase and amplitude differences depend on the constants m, b, k and the frequency. Most notably, no harmonics are generated, so this cannot be a model for our vibroseis, which does produce harmonics.

A NON-LINEAR ODE MODEL GENERATING HARMONICS

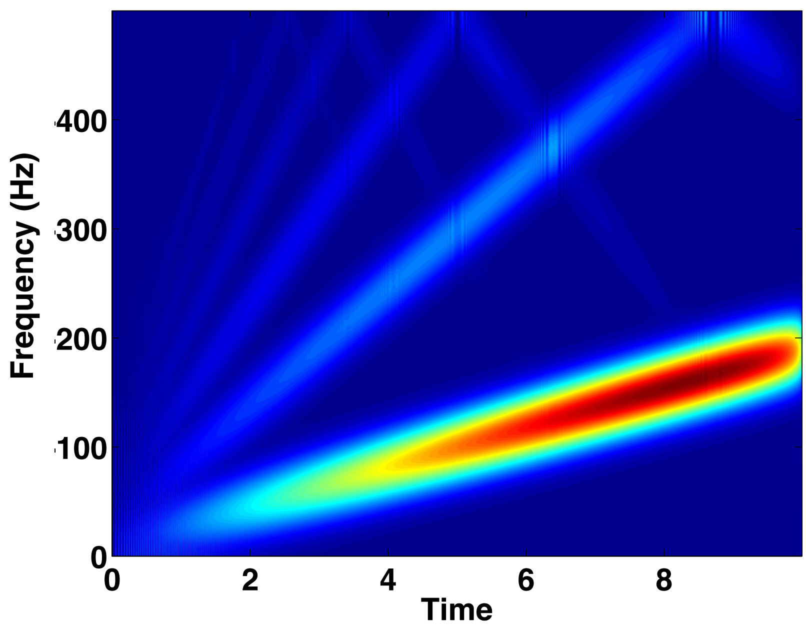


FIG. 4. Time-frequency transform for the non-linear oscillator.

In Figure 4 we see the acceleration $x''(t)$ of the forced, nonlinear oscillator shows the presence of harmonics. A careful look reveals that only the odd harmonics are present. This is different that our real Vibroseis, where even and odd harmonics are present. The equation governing this is

$$x'' + (x')^3 = A \sin(2\pi f(t)t), \tag{6}$$

where $f(t)$ is chosen as a linear sweep of frequencies. Observe that the non-linear behaviour producing harmonics is due to the cubic velocity term, $(x')^3$.

It is worth noting that the higher harmonic sweeps, once they hit the Nyquist frequency of 500 Hz, get aliased back below Nyquist and appear to be decreasing in frequency. This is a numerical artifact in MATLAB's approach to solving the ODEs – namely, it solves

[‡]Any physics book will have this equation, it combines Newton's second law of motion $F = ma$ with Hooke's law for linear springs.

them at a fine time resolution, and down-samples to the resolution requested by the user. Down-sampling without filtering introduces this aliasing.

We had hoped that a non-linear oscillator with the non-linearity in x (not x') would have a similar harmonic behaviour. A few tests for an equation of the form

$$x'' + Kx^3 = A\sin(2\pi f(t)t), \tag{7}$$

gives the time-frequency decomposition shown in Figure 5. Only one odd harmonic appears, and the oscillator gets driven to a natural oscillating frequency (about 100 Hz) and just stays trapped there. It does not follow the increasing sinusoidal sweep. In any case, it is not a good model for the vibroseis.

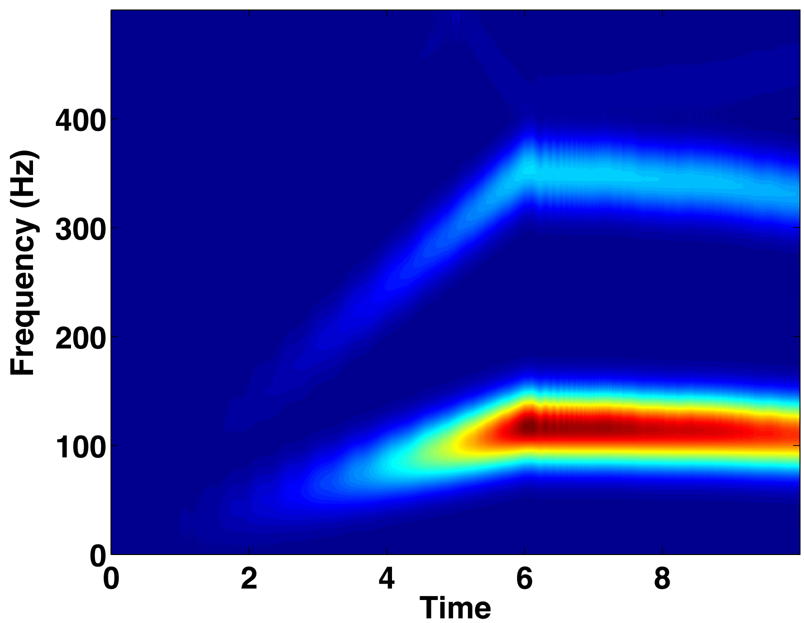


FIG. 5. Time-frequency transform for another non-linear oscillator.

We also tried a few other models that would correspond to an asymmetric restoring force, some that would suggest a waveform such as we saw in Figure 2. Unfortunately, we had no luck on this, and must leave it to future work.

COUPLED SYSTEMS OF OSCILLATORS

A more accurate numerical simulation of the vibroseis device is obtained if we model the physical components of the device with a system of ordinary differential equations. Following the description in Easley (1995), we sketch in Figure 6 the basic components of the device: a (metal) baseplate that is in contact with the ground, a reaction mass (or "hold-down" mass) that is supported above the baseplate, a spring that maintains a distance between the two, and a dashpot to damp the motion. The masses are put into motion by a force acting between them, and the ground reacts to their motion with an opposing net force.

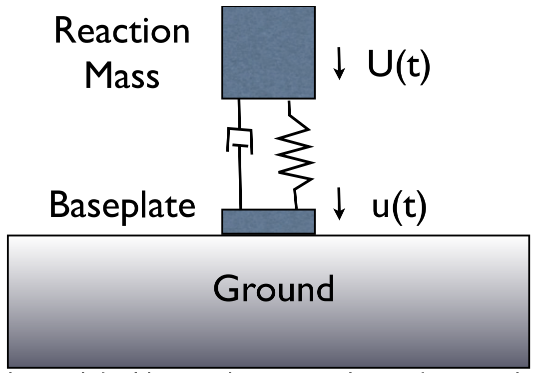


FIG. 6. Vibroseis model with reaction mass, baseplate, spring and dashpot.

By representing only the motion relative to the equilibrium (rest) position of the masses, we can set up a simplified ODE system that describes the motion, based on Newton's second law (force equals mass times acceleration). They are

$$mu'' = -b(u' - U') - k(u - U) + f + F \tag{8}$$
$$MU'' = +b(u' - U') + k(u - U) - f \tag{9}$$

where $u(t), U(t)$ are the positions of the baseplate and reaction mass, respectively, u', U' their velocities, u'', U'' their accelerations, m, M their masses, f the drive force between the two, and F the force from the earth. Constants b, k are the damping constant of the dashpot, and the spring's elasticity constant, respectively. Since we are modelling about the equilibrium points, $u = 0, U = 0$, we can ignore the force of gravity.

While Easley does not explore a numerical simulation for this model in his thesis (Easley, 1995), we can obtain numerical results in a straightforward manner using MATLAB's built-in ODE solvers. A 4-component vector y is used to represent the system, with the identification $y_1 = u, y_2 = u', y_3 = U, y_4 = U'$. The system of ODEs is expressed in matrix form as a first order system with

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}' = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -k/m & -b/m & k/M & b/M \\ 0 & 0 & 0 & 1 \\ k/M & b/M & -k/M & -b/M \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/m \\ 0 \\ -1/M \end{bmatrix} f(t) + \begin{bmatrix} 0 \\ 1/m \\ 0 \\ 0 \end{bmatrix} F(t), \tag{10}$$

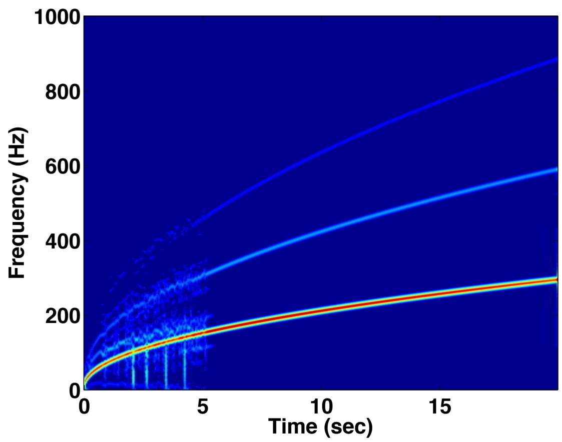
or more simply as

$$\mathbf{y}'(t) = \mathbf{A}\mathbf{y}(t) + \mathbf{v}f(t) + \mathbf{w}F(t), \tag{11}$$

with $\mathbf{A}, \mathbf{v}, \mathbf{w}$ the constant matrix and column vectors from the system above.

To drive the system, we use a non-linear sweep of sinusoids, running from 10Hz to 200Hz, which is input as the forcing term $f(t)$.

MATLAB provides an assortment of numerical solvers for such systems. Initially, we made use of the code ODE45, which is based on an explicit Runge-Kutta $(4, 5)$ formula, a one-step solver. On several runs of the solver, we obtained results as shown in Figure 7.



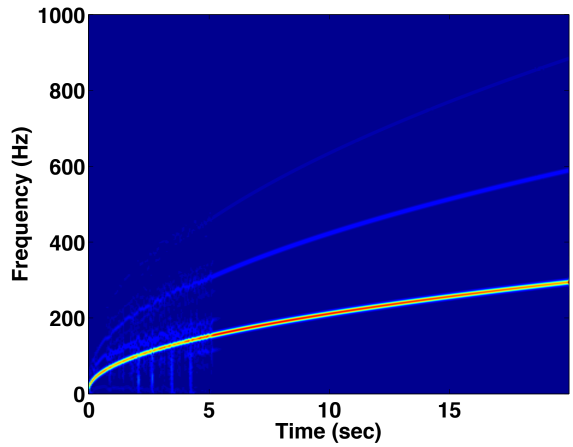
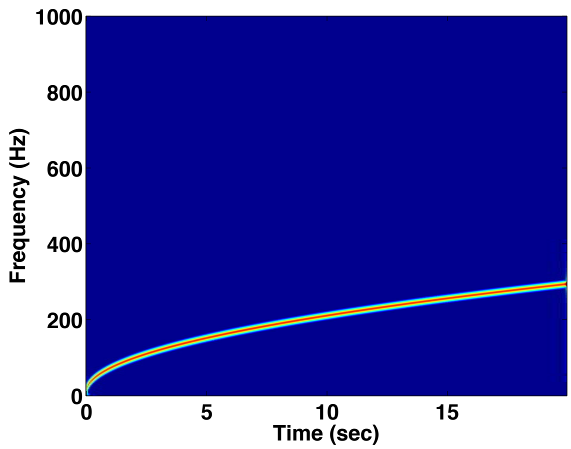


FIG. 7. Baseplate and reaction mass acceleration for linear coupled oscillators.

This clearly shows the fundamental sweep, as well as second and third harmonics, both on the baseplate and the reaction mass.

On the one hand, this is exciting – we have harmonics! What's more, we are getting both even and odd harmonics, which is exactly what we want for our baseplate model. However, this disagrees with the mathematics – for linear systems, one frequency in gives only the same frequency out. In theory, no harmonics. Something is clearly wrong here, as this is a linear system. There should be no harmonics!

This is a good lesson, though – there can be problems with any numerical code, even the well-tested routines in MATLAB. Unfortunately, this was a time-wasting distraction for us but eventually we got back on track. Switching to a more robust numerical code gives better results. We switched to MATLAB's ODE113, a variable order Adams-Bashforth-Moulton PECE solver, which is a multistep method. The results are shown in Figure 8, which show only the fundamental sweep on the accelerometer data, as expected.



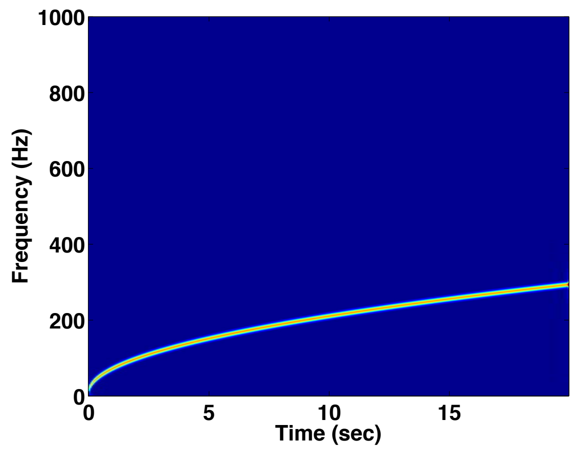


FIG. 8. Baseplate and reaction mass acceleration, with better ODE solver.

Now we introduce non-linearities. As with the 1 ODE case, it is a simple matter to introduce a cubic term in the system of differential equations. In this case, we assume the baseplate has some simple non-linear interaction with the earth, and we introduce a cubic force of the form $c(u')^3$, where the dependence is on velocity. Adjusting the constant c until some effect is seen, we in Figure 9 that the baseplate now shows odd harmonics (and aliasing, as mentioned earlier). The reaction mass only sees the fundamental.

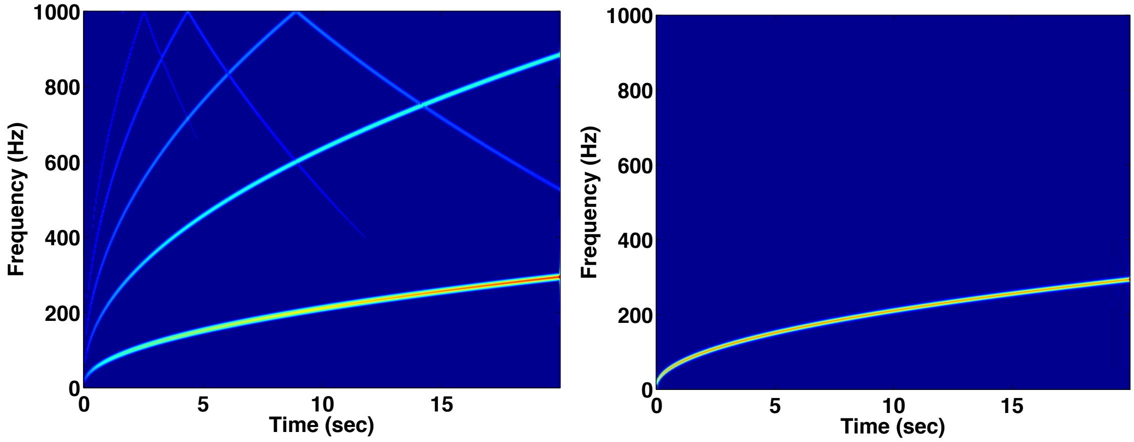


FIG. 9. Baseplate and reaction mass acceleration with non-linear earth.

This is an interesting observation, since there is some question as to whether the baseplate, reaction mass, or a weighted combination of the two is the best input for seismic data processing (e.g. deconvolution of the sweep). We see here in this simple model that the harmonics appear strongly in the baseplate data, not the reaction mass. A quick plot of the baseplate acceleration, Figure 10, in amplitude versus time (around the 4 sec mark) shows that the harmonics are not appearing the way we want – the data is very spiky, but from real experiments we expect a flattening of the waveform.

We tried a few other non-linearies, where the non-linear forcing term depended on displacement, or asymmetrically on the displacement or velocity, or even depending on the sign of the displacement or velocity. These are all reasonable models for an asymmetric response of the ground to loading, or the possibility of the vibroseis truck "pogo-sticking" up and down on the ground. Unfortunately, with our numerical experiments so far, we have not seen a good result with harmonics appearing.

FUTURE WORK

There's lots to be done here. The two-oscillator model looks promising, but we need to find a way to insert non-linearities that will give the full range of harmonics that we see in real vibroseis data. One approach will be to model a non-linear earth response, and the possibility of the "pogo stick" effect of the truck losing contact with the ground. Removing the welded-contact assumption is another approach. There are also physical devices on the truck which constrain the motion of the reaction mass and baseplate – including these would represent another mechanism that could create harmonics. Some models in the literature include a third oscillator, as representing the effective ground mass, as described in Wei (2010), which we would like to explore numerically.

There is a suggestion, also in Wei (2010), that we need to model the flexing of the metal baseplate in order to accurately account for the harmonics. This would be an interesting model, but also fraught with difficulties since we do not have easy access to full data on the

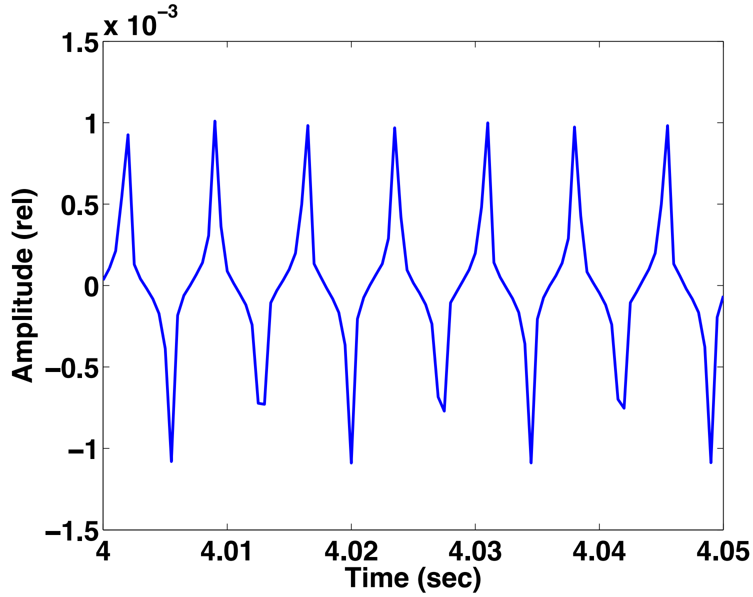


FIG. 10. Baseplate acceleration, amplitude versus time.

motion of the baseplate – usually we only have accelerometer data from the centre, or near the centre, of the baseplate. Flexing motion would need to recording at several points on the baseplate in order to verify our models.

Finally, there is the control machinery in the vibroseis device that we have not attempted to model yet. It would be an interesting component to include.

CONCLUSION

Vibroseis devices produce harmonics. Our simple numerical models of the oscillating systems in a vibroseis devices also produce some harmonics, but are significantly different than what we see in real data. More work is needed to get a simple model that has just enough complexity to capture this harmonic behaviour. Our numerical tools of MATLAB and Gabor transforms are effective methods for exploring numerical simulations of these models, especially when the proper high-order accuracy methods are use.

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REFERENCES

Boucard, D., and Ollivrin, G., 2010, Developments in vibrator control: Geophysical Prospecting, **58**, 33–40.

Cooper, N. M., 2002, Vibroseis - a Counter-Intuitive Tool: CSEG Geophysics 2002, 1–3.

Easley, D. T., 1995, Multicomponent seismology: Elastic-wave sources, composite media, anisotropy, and modeling: Ph.D. thesis, Univ. of Calgary.

Krohn, C., Johnson, M., Ho, R., and Norris, M., 2010, Vibroseis productivity: shake and go: Geophysical Prospecting, **58**, 101–122.

Nagarajappa, N., and Wilkinson, D., 2010, Source measurement effect on high fidelity vibratory seismic separation: Geophysical Prospecting, **58**, 55–67.

Sallas, J., 2010, How do hydraulic vibrators work? A look inside the black box: Geophysical Prospecting, **58**, 3–17.

Wei, Z., 2010, Modelling and modal analysis of seismic vibrator baseplate: Geophysical Prospecting, **58**, 19–31.

Ziolkowski, A., 2010, Review of vibroseis data acquisition and processing for better amplitudes: adjusting the sweep and deconvolving for the time-derivative of the true groundforce: Geophysical Prospecting, **58**, 41–53.