Effects of discrepancies between modeled and true physics in anacoustic FWI

Scott Keating and Kris Innanen

ABSTRACT

In this report, anacoustic FWI with an unknown attenuation model type is investigated. Numerical experiments are performed using a SLS true model type. Assuming a known Q model type which differs from the true Q model type led to very poor results, arguably worse than the results of an acoustic FWI. By relaxing the requirement of known model type and recovering anacoustic parameters over small frequency bands, results were dramatically improved. This recovery of model parameters of a known attenuation model type locally in frequency is a promising approach for coping with unknown attenuation model type.

INTRODUCTION

Full waveform inversion (FWI) is a technique which seeks to recover the properties of the subsurface by minimizing the mismatch between measured seismic data and synthetic data generated using an estimate of these subsurface properties. Ideally, once the data are replicated, the estimated model will match the true subsurface properties. Crucial to this idea is the assumption that the wave physics which give rise to the observed data are adequately accounted for in the modeling which generates the synthetic data. While accurately modeling all of the aspects of seismic wave propagation is an extremely demanding task, sufficient complexity in the modeling needs to be present to account for the major features of the measured data.

Where the mechanisms at play in the subsurface are well understood, generating synthetic data which display the same effects is an achievable goal. For example, seismic amplitude variation with offset (AVO) is known to be an effect of elastic wave propagation, so if the modeling used accounts for elastic wave physics, AVO effects can be correctly interpreted in FWI. The alternate case, where the physics associated with the observed data are not as well understood, presents significant obstacles in FWI. Attenuative and dispersive effects may be more appropriately grouped into this second case. No single attenuative-dispersive model is held to be correct for the general case of seismic wave propagation (Ursin and Toverud (2002), Liu et al. (1976)), and this poses a difficult problem for the use of FWI on data where these effects play a significant role. If the attenuative-dispersive model assumed in the FWI is different from that which best describes the true behaviour of the earth, then the model which best matches the data will not necessarily be similar to the true subsurface, and could introduce significant errors.

This raises important questions about a possible anacoustic or anelastic FWI. Specifically, it is important to know whether attenuation compensation still takes place, and whether deviations from a background value in the assumed attenuation-dispersion variable occur at the same spatial location as the anomalies in the attenuative-dispersive behaviour of the true subsurface. The first of these questions is important insofar as an anacoustic or anelastic FWI is being pursued with the objective of improving estimates of acoustic or elastic parameters. If the improved recovery of these non attenuative parameters is the goal, then the second question is of little importance, provided that the effective dispersion and attenuation characteristics of the medium are accurately accounted for. The second question is important if an anacoustic or anelastic FWI is used in the hopes of recovering the locations of attenuation or dispersion changes in the subsurface, likely a significantly more difficult problem. In this report, the consequences of assuming an attenuation model which differs from the true one are investigated.

ATTENUATION MODELS

This report investigates changes in both the mathematical expressions used to model attenuation and dispersion, and the spatial distribution of the variables defined in these expressions. To aid clarity, in this report changes in the mathematical description of the attenuation mechanism will be referred to as changes in anacoustic model type, whereas the distribution of variables within these model types will be referred to as anacoustic or Q models.

There are many model types that have been used to describe seismic attenuation, but in many of the most popular the assumption of constant or nearly constant quality factor Q is made. This quality factor can be defined in several different ways, here it is stated as

$$\frac{1}{Q(\omega)} = \frac{\Delta E}{2\pi E} \quad , \tag{1}$$

where E is the peak strain energy stored in the medium in a given cycle, and ΔE is the energy lost in the same cycle (Aki and Richards (2002)). If Q is constant with respect to frequency, then attenuation is linearly proportional to frequency. Much empirical evidence supports the idea that a constant or nearly constant value of Q accurately describes seismic wave propagation. A constant Q value in a non-dispersive medium will violate causality (Aki and Richards (2002)), so many models adopt a frequency dependent Q which is nearly constant over the range of seismic frequencies, and introduce dispersion. As there are many ways to create a function which is nearly constant over the range of seismic frequencies, there are many different nearly constant Q model types. Most of these are nearly indistinguishable over the range of seismic frequencies (Ursin and Toverud (2002)). These nearly constant Q model types are largely based on the empirical observation of constant Q, and are seldom based on physical mechanisms (Liu et al. (1976)). The nearly constant Q model type considered in this report is the Kolsky-Futterman (KF) model (Kolsky (1956), Futterman (1962)) in which Q is treated as constant with respect to frequency and

$$c(\omega) = c(\omega_0)\left[1 + \frac{1}{\pi Q}\log\left(\frac{\omega}{\omega_0}\right) - \frac{i}{2Q}\right] \quad , \tag{2}$$

where $c(\omega)$ is the wave velocity and ω_0 is a reference frequency.

Another model type sometimes used for seismic modeling is the standard linear solid (SLS) model. This standard linear solid is a viscoelastic model with a constitutive relation linear in stress, strain and their derivatives (Casula and Carcione (1992), Liu et al. (1976)). In effect, it models a viscoelastic material as consisting of spring and dash-pot in series, in parallel with a second spring. The Q value given by this model is not constant or nearly so, but is instead given by

$$Q(\omega) = \frac{1 + \omega^2 \tau_\epsilon \tau_\sigma}{\omega(\tau_\epsilon - \tau_\sigma)} \quad , \tag{3}$$

where τ_ϵ and τ_σ are relaxation times related to the constants of the effective springs and dash-pot of the model (Casula and Carcione (1992), Liu et al. (1976)). This function is sharply peaked at $\omega = \tau^{-1}$, where $\tau = \sqrt{\tau_\epsilon \tau_\sigma}$. The real part of the phase velocity for this model is given by

$$c(\omega) = c(\omega_0) \left[1 + \frac{(\omega\tau)^2}{Q(1 + (\omega\tau)^2)} - \frac{i}{2Q(\omega)} \right] \quad . \tag{4}$$

Many physical processes which could have significant impact on seismic wave attenuation are well modeled by the standard linear solid (Liu et al. (1976)). It is pointed out in Liu et al. (1976) that the standard linear solid and nearly constant Q models are not necessarily at odds with one another. A general standard linear solid can be introduced by considering several standard linear solid systems arranged in parallel. This introduces several relaxation mechanisms, and so several attenuation peaks. If the amplitudes and peak frequencies of these individual SLS components are chosen correctly, a general SLS with approximately constant Q over a given bandwidth can be constructed. In this case the dispersive behaviour of the velocity reduces to equation 2 over the nearly constant Q frequency band.

While most empirical evidence supports a Q model type that is independent of frequency, or nearly so (Liu et al. (1976), Aki and Richards (2002), Ursin and Toverud (2002)), there is little physical justification for the nearly constant Q model types. While Liu et al. (1976) do show that a nearly constant Q model type can be generated using the more physically motivated SLS model, a combination of SLS relaxation mechanisms will not in general produce a nearly constant Q model, and the case in which they do so is fairly pathological. As such, it is not unreasonable to assume that whichever Q model type is chosen for use in FWI, there is a significant chance that it does not exactly reflect the attenuation mechanisms at play in the subsurface. It remains a question of interest how the FWI performs in such circumstances.

In this research report an extreme case is investigated: that in which the Kolsky-Futterman model type is assumed and the true mechanism in the subsurface is the standard linear solid. In reality, there is relatively little empirical evidence to suggest that the SLS model type is accurate for real earth scenarios, but it offers an example of a model type significantly different from the Kolsky-Futterman and nearly constant Q model types, which are not themselves certain. In addition, it is likely given the strong capacity for the SLS system to describe real attenuation mechanisms that the true subsurface can be described by some combination of SLS relaxation mechanisms. A comparison of the two model types is shown in figure 1.

ANACOUSTIC FWI

Assumed attenuation model type

Keating and Innanen (2016) discusses the implementation and challenges associated

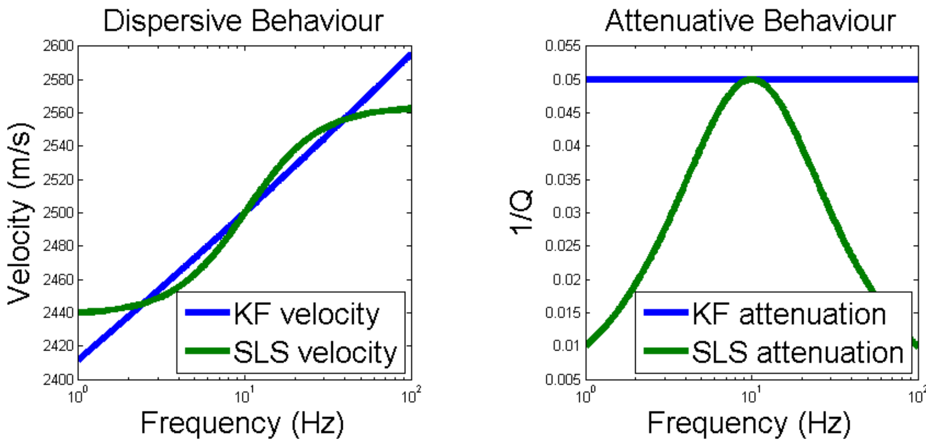

FIG. 1. Comparison of SLS and KF models for velocity 2500m/s at 15Hz and Q=20. Left: Velocity compar-ison. Right: Attenuation comparison. Note the semilog scale.

with an anacoustic FWI using a KF attenuation model. Among the important characteristics of anacoustic FWI identified in this report was the necessity of employing a range of frequencies in calculating the gradient and Hessian in order to prevent cross-talk. This makes intuitive sense, at least insofar as discrepancies in the velocity model are indistinguishable from those in the dispersion term at a single frequency. The FWI update, then, identifies attenuation and dispersion characteristics at least in part by comparing the measured and synthetic data at multiple frequencies. To understand how FWI will behave with an incorrect attenuation model, it is important to identify how the algorithm will interpret the measured data. A glance at the comparison in figure 1 identifies the fact that no KF model will be very successful in reproducing the SLS physics locally. Any choice of Q in the KF model type will fail to match the true SLS behaviour on multiple fronts. Neither the Q nor the dispersion are similar functions of frequency between models, and the relation between attenuation and dispersion is different between models, that is, an increase in Q in one model has a different effect on dispersion than an equivalent increase in another.

The significant differences between attenuation models outlined above mean that an FWI which attempts to locally match the attenuative and dispersive behaviour of the true subsurface will face significant challenges. FWI minimizes the mismatch between measured and predicted data, so it is conceivable that the model which is recovered is not that which best matches the true model locally, but rather one which effectively emulates the regional model behaviour with small scale variations. In such a scenario, the important question is whether the velocity information can still be accurately recovered.

Flexible model type

In the face of an unknown attenuation model type it may be prudent to allow the recovered model more flexibility than would be desirable with a known attenuation model type. One promising approach is to define Q and $c(\omega_0)$ only for a small frequency band, and to recover independent values for these parameters on each band. The idea here hinges on the fact that on a sufficiently small frequency band, any model type which can vary attenuation and velocity individually can approximate well any other model type that defines a velocity and attenuation. By defining these parameters only on very small bands, an approximation

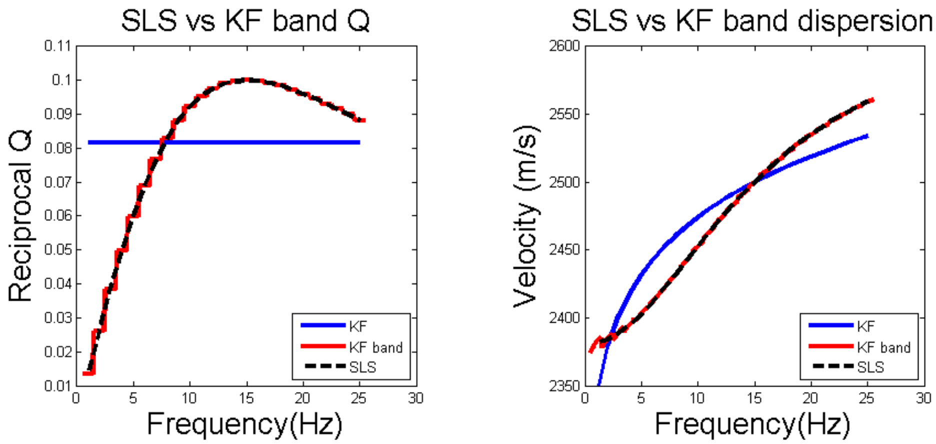

FIG. 2. Comparison of SLS with best fitting KF, and band-defined KF. Due to the highly dissimilar behaviour of the model types, the KF result is a poor approximation of the SLS behaviour. The band-defined KF is capable of matching the SLS behaviour much more closely, though still differs in dispersive behaviour on each band.

of one model type's behaviour in terms of another can be obtained. This idea is illustrated in figure 2, where the SLS and KF behaviours are shown. If the KF model is defined on a small frequency band, it can approximate the SLS behaviour much more closely.

While the flexible strategy outlined above offers the capacity to match an unknown anacoustic model type, it is unclear whether this potential can be realized in an FWI approach. Two significant challenges may present themselves in inversion using this strategy. Firstly, while the overall dispersive character of an ideal recovered model will closely match the true model, these behaviours may differ significantly over the small bands on which the inversion occurs. This is evident in figure 2, where at low frequencies the dispersive behaviour on a given band differs significantly between the SLS and band KF model types. This means that insofar as the inversion considers the dispersive character of the observed data, it may lead the estimated model away from the best approximation. The second problem is associated with the independent recovery of the behaviour on each frequency band. This may lead to an inability to recover features on a scale different from that being considered. For example, at high frequencies, long wavelength features may be difficult to recover. It is difficult to address the impact of these concerns without the aid of synthetic examples, which are considered in the next section.

NUMERICAL EXAMPLES

In order to test the efficacy of the anacoustic FWI strategies described in the previous section, simple numerical tests were carried out. In these examples, an SLS true model was used, and each of the anacoustic FWI strategies outlined in the previous section was tested. The velocity model investigated was very simple, with the aim of identifying the challenges faced in each strategy in an otherwise straightforward case. The true model consisted of two circular velocity anomalies of 2200m/s separated by an elliptical attenuation anomaly of minimum Q 10 at 15 Hz on a homogeneous background of velocity 2500m/s and infinite Q. This model is shown at two example frequencies in figures 3 and 4. The reference frequency used in the KF model type for inversion was 30 Hz. Other details of the generation of the synthetic data and optimization approach are given in Keating and Innanen (2016).

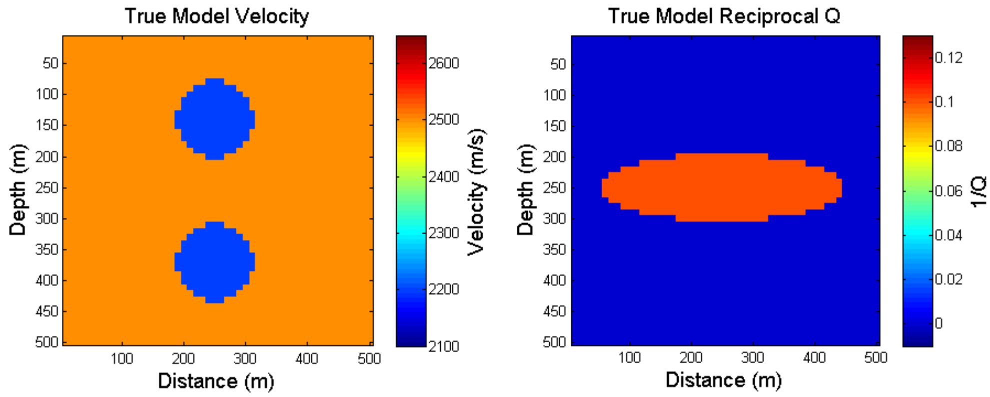

FIG. 3. True model velocity (left) and Q (right) for SLS model type at 15Hz.

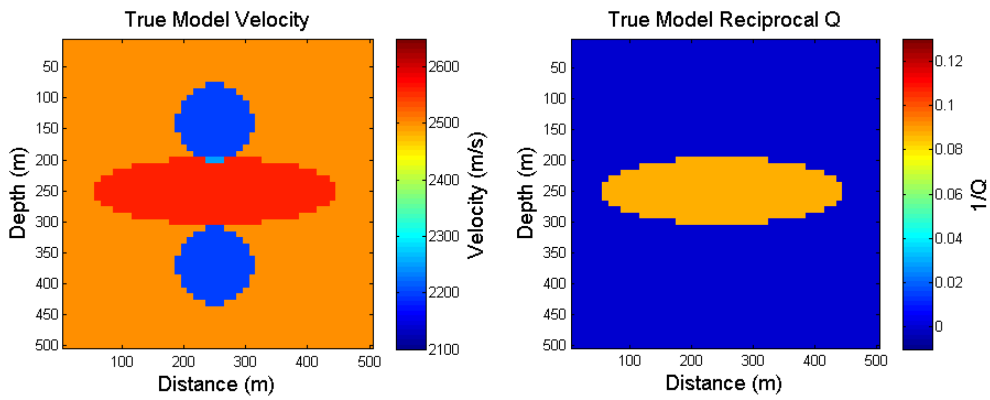

FIG. 4. True model velocity (left) and Q (right) for SLS model type at 25Hz.

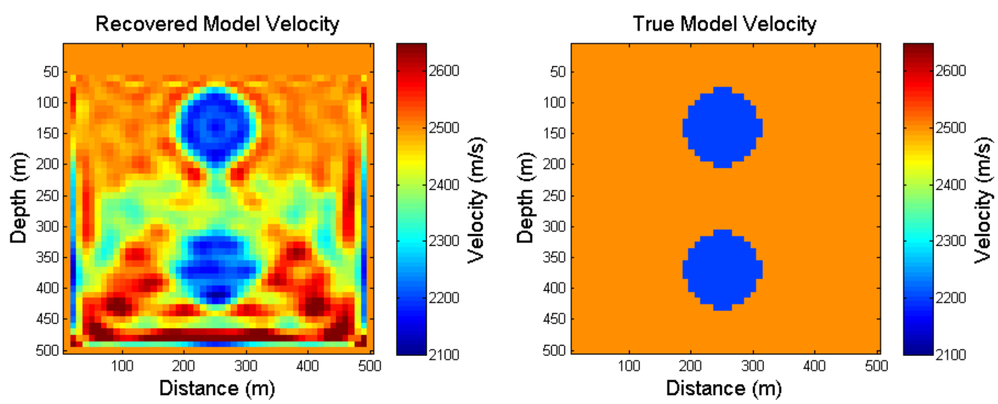

FIG. 5. Recovered velocity (left) and true velocity (right) using acoustic FWI.

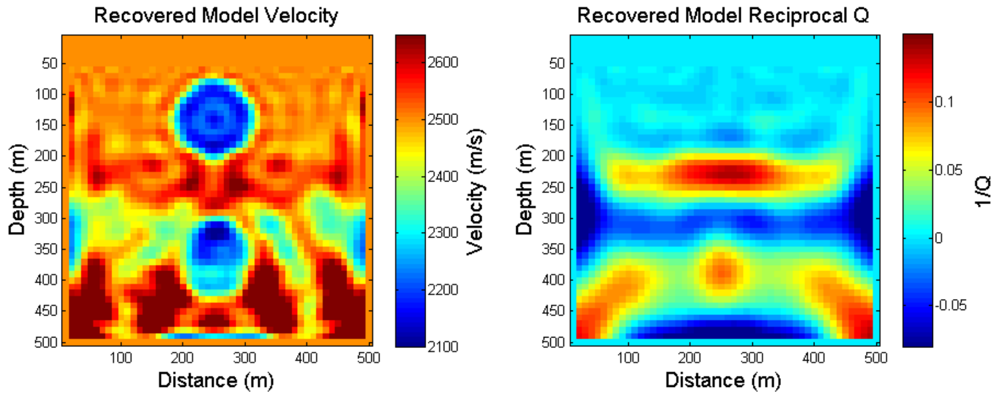

FIG. 6. Recovered velocity (left) and Q (right) using KF anacoustic FWI. Recovered velocity is inferior to that using acoustic FWI (figure 5).

Assumed attenuation model type

Figure 6 shows the result of anacoustic FWI where the output model strictly obeys the KF constant Q behaviour. There are clearly significant problems with the result shown. The recovered Q model does not correctly identify the spatial extent of the true Q anomaly, and in the absence of constraints recovers unphysical negative Q values. Additionally, the recovered velocity model is badly corrupted in this case. This illustrates the failure of the KF anacoustic FWI in this situation; not only does the attenuation model recovered tell us little about the true attenuation (as might have been anticipated with differing attenuation models), but the inclusion of inclusion of attenuative and dispersive effects has resulted in a poor recovered velocity. Indeed, a comparison to the result using a purely acoustic FWI (figure 5) demonstrates that the consideration of anacoustic effects did not aid the inversion when the wrong model type was assumed. This significant failure, even in a simple setting, motivates the transition to more flexible attenuation models that may be able to better cope with unknown attenuation behaviour.

Flexible model type

By relaxing the requirement that Q and $c(\omega_0)$ be frequency independent in the Kolsky-Futterman model, more freedom to match the observed attenuation behaviour is allowed. To implement a frequency dependent Q and $c(\omega_0)$, small bands of frequencies were inverted sequentially, and the effective model stored for each band. The starting model for each band was given by the final model of the previous one. As described in Keating and Innanen (2016), it is necessary to use a frequency band of nonzero width to prevent crosstalk. Using a frequency band of 2Hz, this flexible KF FWI yielded the results shown in figures 7 and 8. Two significant improvements are evident in comparison to the constant Q results in figure 6; the velocity model is significantly improved over the acoustic result, closely resembling the true velocity model at both example frequencies, and the effective Q model, while displaying some differences from the true model, correctly identifies the position and approximates the amplitude of the true Q anomaly. The lower Q artifacts occur in a poorly constrained region, and are likely not a cause for concern.

A more detailed measure of the recovered and true model behaviour can be obtained by

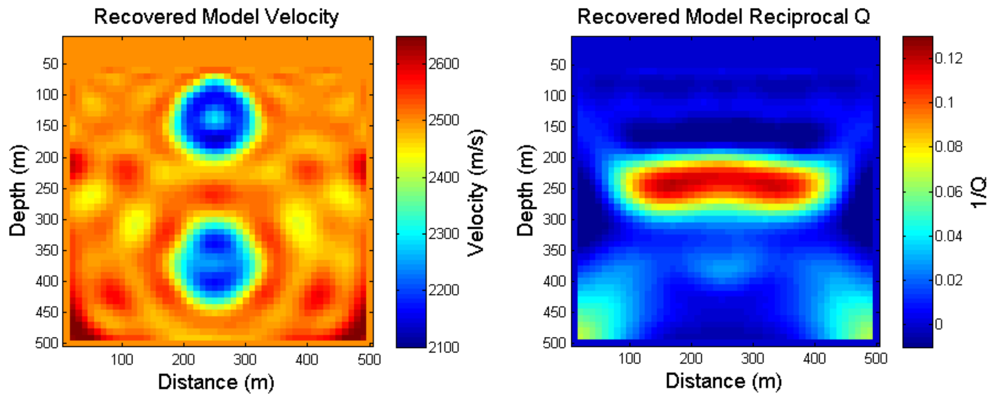

FIG. 7. Recovered velocity (left) and Q (right) using flexible KF FWI at 15Hz. Considerable improvements are evident in comparison to figure 6. Results compare favourably to true model (figure 3).

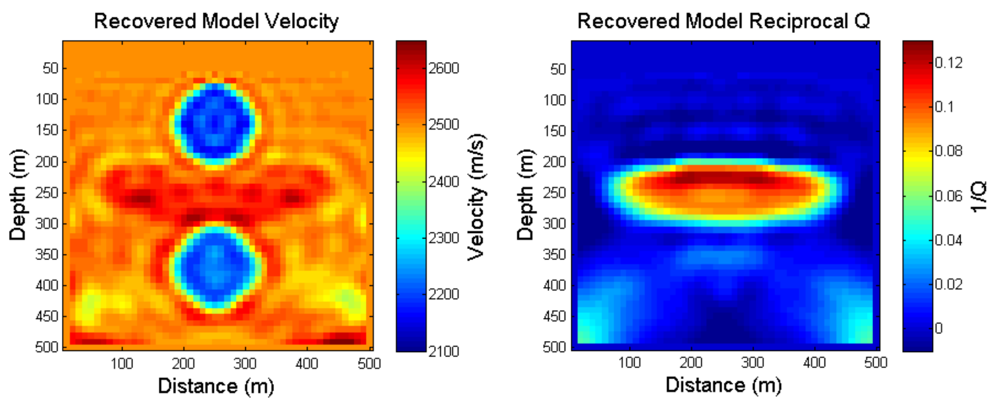

FIG. 8. Recovered velocity (left) and Q (right) using flexible KF FWI at 25Hz. Results compare favourably to true model (figure 4).

comparing the average reciprocal Q and velocity over the region of nonzero Q in the true model. This is plotted in figure 9. It can be seen in this plot that the overall behaviour of the true SLS model type is being approximated to a much greater extent than possible with frequency invariant Q and velocity. Significant failures do occur at low frequencies, especially in the recovered velocity. This is also the region in which the dispersive behaviour differs most strongly from the ideal recovered band behaviour (figure 2).

DISCUSSION

The flexible approach to dealing with model type uncertainty seems to offer significant advantages, but limitations do exist. As described previously, disagreements in the local dispersive behaviour between the ideal and observed models, as well as difficulties recovering wavenumbers far from the considered frequency could hamper successful inversion. Indeed, the failure in recovering velocities at low frequencies in figure 9 occurs where the problems with local dispersion would be most pronounced. Strategies for coping with these problems still need to be developed. Adopting a flexible model which allow both the local dispersion and attenuation to vary independently is a topic of ongoing research.

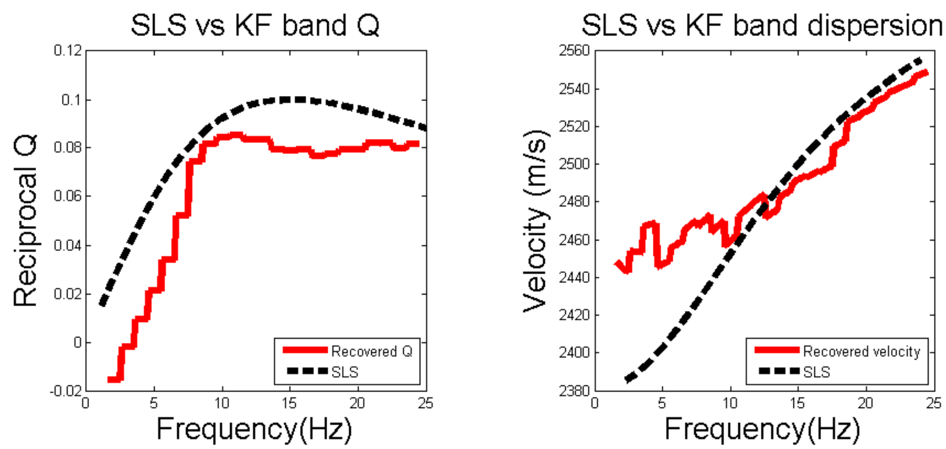

FIG. 9.
Average reciprocal Q and velocity over the non-zero Q region of the true model. Recovered values using flexible KF method reproduce trend of true values much more accurately than a true KF model can (figure2). Velocity is poorly recovered at low frequency.

CONCLUSIONS

In this report, anacoustic FWI with an unknown attenuation model type was investigated. Numerical experiments were performed using a SLS true model type. Assuming an incorrect known Q model type, such as the Kolsky Futterman constant Q model, led to very poor results, arguably worse than the results of an acoustic FWI. By relaxing the requirement of constant Q and recovering Q over small frequency bands the result can be dramatically improved. While the approach implemented in this report leaves room for considerable improvement, the strategy of recovering assumed model type parameters over local frequency ranges to cope with unknown model type seems a highly promising strategy for coping with unknown attenuation model type.

ACKNOWLEDGMENTS

The authors thank the sponsors of CREWES for continued support. This work was funded by CREWES industrial sponsors and NSERC (Natural Science and Engineering Research Council of Canada) through the grant CRDPJ 461179-13. Scott Keating was also supported by the Queen Elizabeth II scholarship.

REFERENCES

Aki, K., and Richards, P. G., 2002, Quantitative seismology: University science books, 2nd edition.

Casula, G., and Carcione, J., 1992, Generalized mechanical model analogies of viscoelastic behaviour: Bolletina di Geofisica Teorica ed Applicata, **34**.

Futterman, W., 1962, Dispersive body waves: Journal of Geophysical Research, **67**.

Keating, S., and Innanen, K., 2016, Challenges in anacoustic full waveform inversion: CREWES Annual Report, **28**.

Kolsky, H., 1956, The propagation of stress pulses in viscoelastic solids: Philosophical Magazine, **1**.

Liu, H., Anderson, D., and Kanamori, H., 1976, Velocity dispersion due to anelasticity; implications for seismology and mantle composition: Geophys. J. Roy. Astr. Soc, **47**.

Ursin, B., and Toverud, T., 2002, Comparison of seismic dispersion and attenuation models: Stud. geod. geoph., **46**.