Cascaded Deconvolution Filters

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ABSTRACT

There are few seismic wavelets that have an exact inverse filter for deconvolution, but one of these is an exponentially damped sinusoid. As shown by Lines and Treitel (1983) this wavelet has an exact 3-term inverse. Unlike most seismic wavelets, a Wiener deconvolution filter can be designed that shapes this wavelet to an output that is a nearly perfect spike. For the majority of seismic wavelets where a perfect spike is not achievable, one can shape the actual wavelet to an exponentially damped sinusoid prior to and then apply a Wiener filter that will spike the exponentially damped wavelet. This appears to hold some promise, as shown by a computational example.

INTRODUCTION

The Wiener wavelet deconvolution filter has been widely used in the seismic processing industry ever since the time of their introduction to the industry by Robinson (1954). The successful design of this deconvolution filter depends on desired spiking position, filter length, prewhitening level, and of course the accuracy of the wavelet estimate itself. Wiener filter design is explained in detail in several publications including Robinson and Treitel (2008). In this paper, we will focus on a wavelet that has an exact deconvolution filter – that being a damped exponential sinusoid. This wavelet and its 3-term inverse were examined by Lines and Treitel (1983) in their discussion of second moment norm filters. We review this wavelet and its exact spiking filter. Wiener filters for other wavelets will generally not have such ideal performance. However, we can design filters that can shape wavelets to a desired damped sinusoid as described by Robinson and Treitel (2008). After applying the shaping filter, we could then apply the "exact filter" for the exponentially damped wavelet. These cascaded deconvolution filters are shown here for computational examples.

METHODOLOGY AND RESULTS

In order to understand the nature of a spiking filter, we review the derivation of such filters from Lines and Treitel (1983). Consider the wavelet given by a damped sinusoid with damping coefficient α and frequency ω_0.

$$w(t) = e^{-\alpha t} \sin \omega_0 t, \ \alpha > 0, \ t \ge 0 \tag{1}$$

.

This can be written in terms of complex exponentials as:

$$w(t) = \frac{1}{2i}\left(e^{-(\alpha - i\omega_0)t} - e^{-(\alpha + i\omega_0)t}\right). \tag{2}$$

The discrete z-transform of *w(t)* is:

$$w(z)=\frac{1}{2i}\left\{1+e^{-(\alpha-i\omega_0)}z+e^{-2(\alpha-i\omega_0)}z^2+\ldots\right\}-\frac{1}{2i}\left\{1+e^{-(\alpha+i\omega_0)}z+e^{-2(\alpha+i\omega_0)}z^2+\ldots\right\}$$
(3a)

Equation (3a) can be rewritten as:

$$w(z)=\frac{1}{2i}\left[\frac{1}{1-e^{-\left(\alpha-i\omega_0\right)}z}-\frac{1}{1-e^{-\left(\alpha+i\omega_0\right)}z}\right]$$
$$w(z)=\frac{1}{2i}\left[\frac{ze^{-\alpha}(e^{i\omega_0}-e^{-i\omega_0})}{1-(e^{-(\alpha-i\omega_0)}+e^{-(\alpha+i\omega_0)})z+e^{-2\alpha}z^2}\right] \quad (3b)$$

This can be re-expressed as:

$$w(z)=\left\{\frac{ze^{-\alpha}\sin\omega_0}{1-\left(2e^{-\alpha}\cos\omega_0)z+e^{-2\alpha}z^2\right)}\right\} \quad . \quad (4)$$

Taking the reciprocal of (4), the discrete z-transform for the exact inverse filter is then given by:

$$w^{-1}(z)=\frac{1}{\sin\omega_0}\left[e^{\alpha}z^{-1}-2\cos\omega_0+e^{-\alpha}z\right] \quad (5)$$

It will be convenient to deal with a 1-unit delayed deconvolution filter, with discrete z-transform

$$h(z)=zw^{-1}(z)=\frac{1}{\sin\omega_0}\left[e^{\alpha}-(2\cos\omega_0)z+e^{-\alpha}z^2\right] \quad (5a)$$

If we convert this unit-delayed z-transform back to the time domain, we see that we obtain a causal deconvolution filter *f(t)* given by the three-term sequence:

$$f(t)=(f_0,f_1,f_2)=\frac{e^{\alpha}}{\sin\omega_0}\left[1,\ -2e^{-\alpha}\cos\omega_0,\ e^{-2\alpha}\right] \quad (6)$$

It can be shown that this causal filter is also minimum-delay whenever *both* roots (z_1, z_2) of the *z*-polynomial (5a) lie outside the unit circle in the complex plane, namely that:

$$\left\|(z_1, z_2)\right\| = e^{\alpha}\left[\cos\omega_0 \pm \sin\omega_0\right] > 1$$

We have derived an analytic form of the deconvolution filter for the exponentially damped sinusoid and have shown it to be a 3-term sequence. We can also compute a numerical form of this by computing a 3-term Wiener deconvolution filter that attempts to shape this wavelet to a spike. This computation was done in Lines and Treitel (1983) for an exponentially damped sinusoid with f=20Hz. As anticipated, the Wiener filter effectively gave a near-perfect spike as the filter output. The result expressed in equation (5a) was stated earlier by Robinson (1967, p. 343)

In this paper, we use an exponentially damped sinusoid with a frequency of 90 Hz, a 1 ms sample interval and a value of $\alpha = 100$**.** This wavelet is shown in Figure 1.

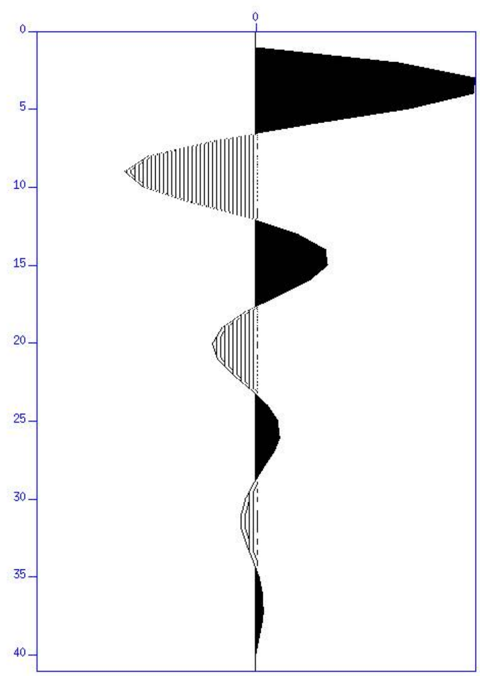

FIG. 1. Exponentially damped sinusoidal wavelet with dominant frequency of 90 Hz. Horizontal axis is in samples where sample interval is 1 ms.

The computation of a Wiener spiking filter produces a filter output in Figure 2 which is nearly a perfect spike. Treitel and Lines (1982) define this filter output as a resolving kernel. Ideally, the resolving kernel would resemble a spike (delta function).

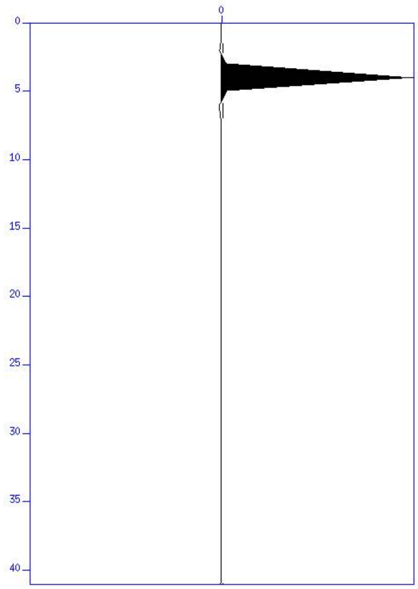

FIG.2. Filter output from application of a Wiener spiking filter converts the exponentially damped sinusoid wavelet in Figure 1 to a nearly perfect spike as anticipated by previous analysis in equations 1-5.

This idealized performance of a Wiener filter for exponentially damped sinusoid is all well and good, but how would this apply to the deconvolution of a more realistic wavelet? The shaping of most wavelets to a spike with a Wiener filter will not be so ideal. This is due to the fact that most wavelets are bandlimited and may even have holes in their spectrum that could lead to singularities in deconvolution unless prewhitening is used (Treitel and Lines, 1982).

To illustrate this, we will use a wavelet in Figure 3 from a previous study by Dey (1999) and we will examine the deconvolution result.

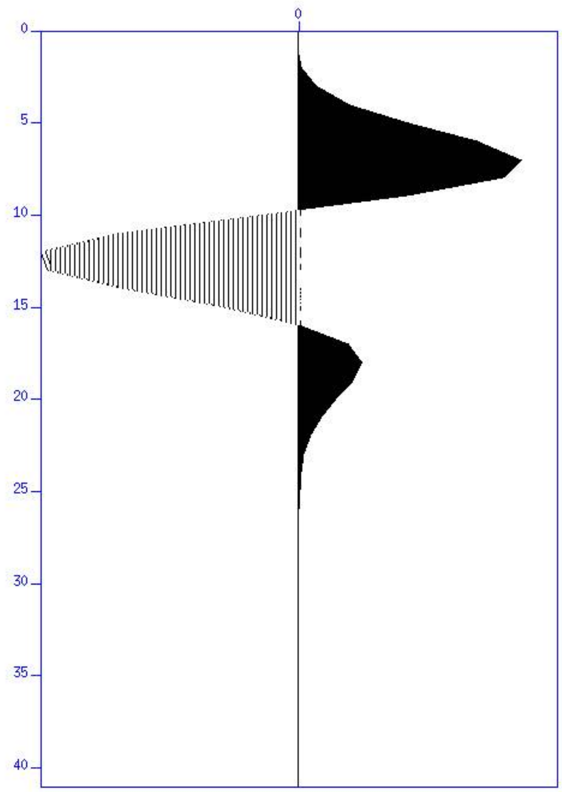

FIG.3. A seismic wavelet as used in Dey (1999).

A Wiener filter can be designed using the algorithms described in Robinson (1967) in which an optimum spiking position is chosen by comparing the difference between the actual filter output and a spike (delta function). If one compares filter output performance for different spiking positions, an optimum spiking position can be chosen. If one compares the input wavelet to the output of the deconvolution, we see that deconvolution has some desirable characteristics. The output has a narrower peak (Figure 4) than the input wavelet (Figure 3) – leading to improved resolution of arrivals. Also, the output has a symmetrical shape more closely resembling a zero phase wavelet. Therefore, following an appropriate time shift, peaks (and troughs) on the deconvolved output will coincide with peaks (and troughs) in the reflectivity.

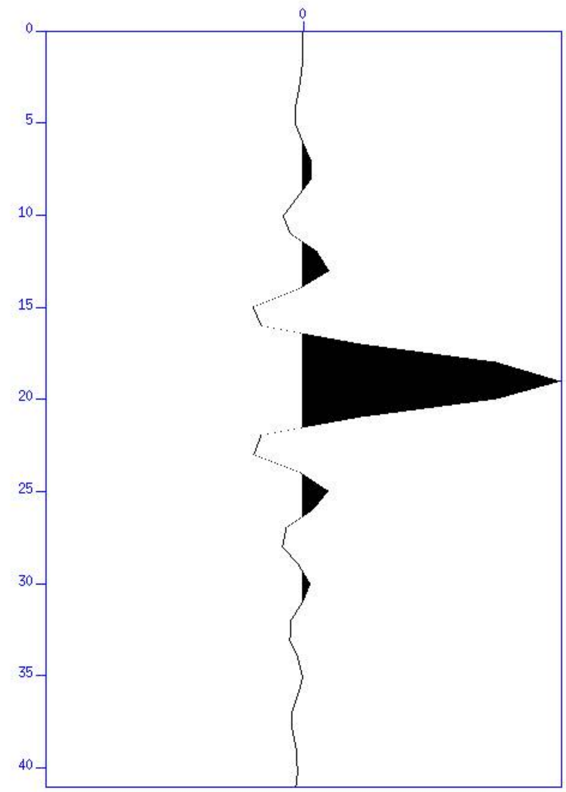

FIG. 4. The output from applying a Wiener spiking filter to the wavelet in Figure 3.

Nevertheless, deconvolution of the wavelet in the case of Figure 4 does not result in a knife-sharp output as in Figure 2. One might ask the question. Can one design a digital filter that would convert the wavelet of Figure 3 to the damped exponential wavelet of Figure 1 and thereby utilize the ideal deconvolution properties of the exponentially damped wavelet? This would require using a shaping filter to convert one wavelet to another, followed by the "exact" deconvolution filter. In other words, we would use a cascaded filter approach.

It is perhaps easiest to envision the shaping filter in the frequency domain. Let $W_1(\omega_0)$ be the Fourier transform of the actual wavelet, and let $W_2(\omega_0)$ be the Fourier transform of the exponentially damped sinusoid. The Fourier transform of the filter that will convert the wavelet to an exponentially damped sinusoid is given by:

$$F(\omega_0) = \frac{W_2(\omega_0)}{W_1(\omega_0)}, \tag{7}$$

where ω_0 =angular frequency of the exponentially damped sinusoid.

On the right side of (7), if we multiply the numerator and denominator by W_1^* we will have the frequency domain version of the Wiener filter that attempts to shape the wavelet, $w_1(t)$ into a desired output that is $w_2(t)$.

In fact, it is worth noting that $W_2(\omega_0,\omega)$, the Fourier transform of a causal exponentially damped sinusoid of frequency has an analytic expression for its Fourier transform given by Papoulis (1962, p. 23) as:

$$W_2(\omega_0,\omega) = \frac{\omega_0}{(\alpha + i\omega)^2 + \omega_0^2}, \tag{8}$$

where ω = filter angular frequency of the causal exponentially damped sinusoid of frequency ω_0

Having applied a shaping filter that shapes the wavelet into a desired output that is a damped sinusoid, we would then apply the 3-term filter that shapes the damped sinusoid into a spike.

We can examine feasibility of applying the shaping filter defined by equation (7) in order to convert the wavelet of Figure 3 into the damped sinusoid of Figure 1. In order to avoid division by zero in this equation, we add a small positive constant to the denominator of equation (7). This will mean that the application of the shaping filter will not provide a perfect reproduction of the desired wavelet. However, as Figure 5 shows, the shaping filter provides a wavelet that closely resembles the desired wavelet.

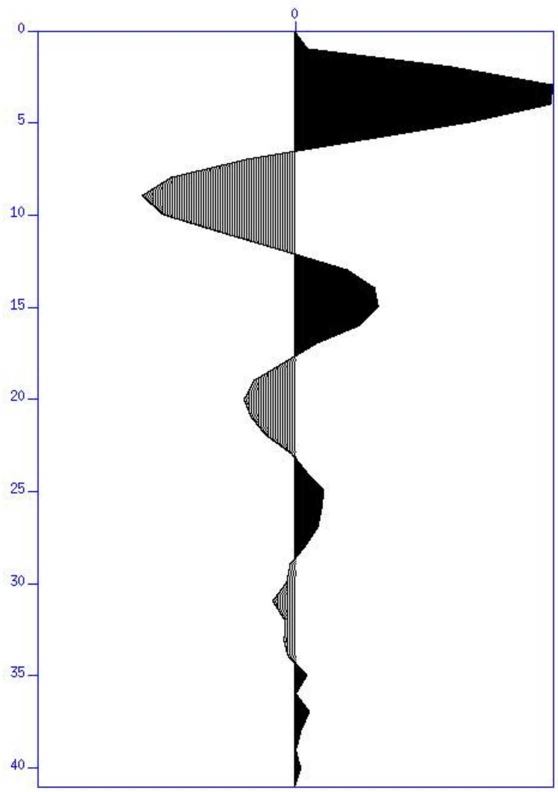

FIG. 5. The output from applying the shaping filter of equation (7) to shape the wavelet of Figure 3 to the wavelet of Figure 1.

The next step in the cascaded deconvolution process requires that we apply a deconvolution filter to spike the shaped wavelet. The application of a Wiener filter to the wavelet of Figure 5 produces an output or resolving kernel as shown in Figure 6.

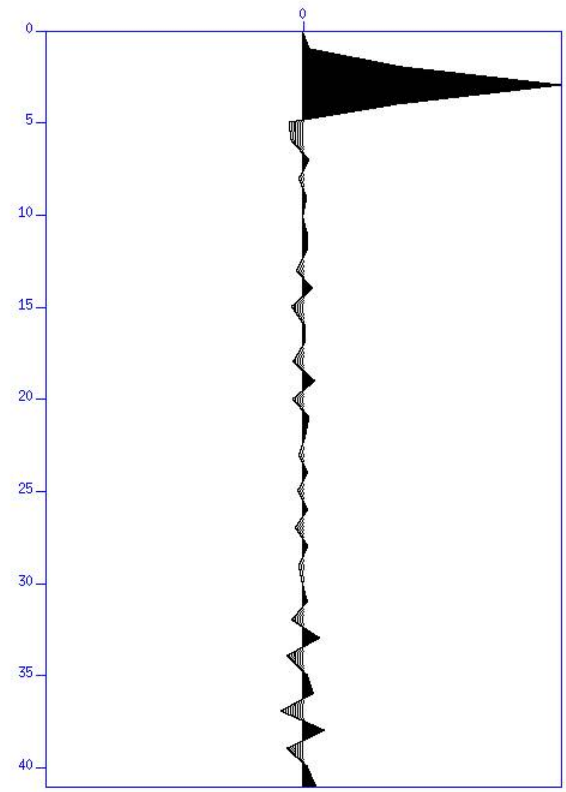

FIG. 6 The application of a Wiener deconvolution spiking filter (length=5, prewhitening =1% produces a resolving kernel that more closely resembles a spike than the direct deconvolution resolving kernel of Figure 4.

This cascaded deconvolution is very encouraging since although this resolving kernel in Figure 6 is not a perfect spike, as shown in Figure 2, it is a considerable improvement over the resolving kernel of the direct deconvolution of the wavelet, as previously shown in Figure 4. This provides some support for the use of cascaded deconvolution - with one step shaping the wavelet into a damped exponential followed by a deconvolution of the shaped wavelet.

At this stage, it is worthwhile to consider the application of this method to a data example. For this purpose, we consider a reflectivity function derived from wells in central Alberta as used by Dey (1999). This reflectivity is shown in Figure 7.

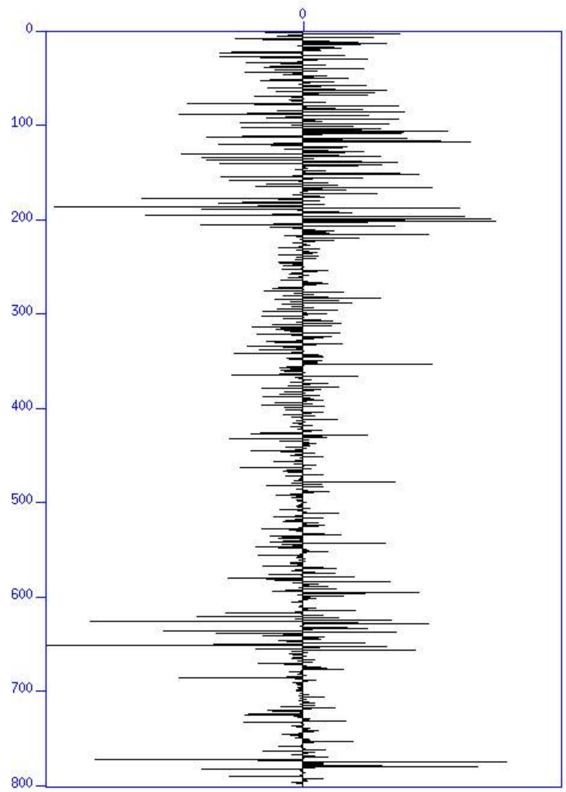

FIG. 7. The reflectivity function used in Dey (1999), as derived from well logs of a central Alberta well.

If we convolve the wavelet in Figure 3 with the reflectivity in Figure 7, we obtain a synthetic seismogram that can be used for testing our cascaded deconvolution filter. This trace is shown in Figure 8.

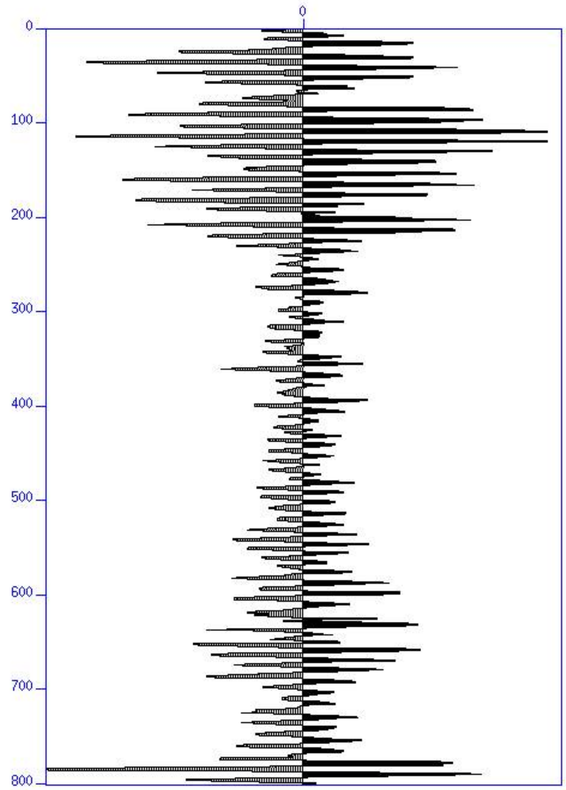

FIG. 8 The convolution of the wavelet in Figure 3 with the reflectivity in Figure 6 produces the above trace.

The first step in the cascade deconvolution is to convert the trace in Figure 6 to the trace in Figure 7 by convolving the shaping filter defined by equation (7) with the trace in Figure 8. Doing this will produce the trace in Figure 9.

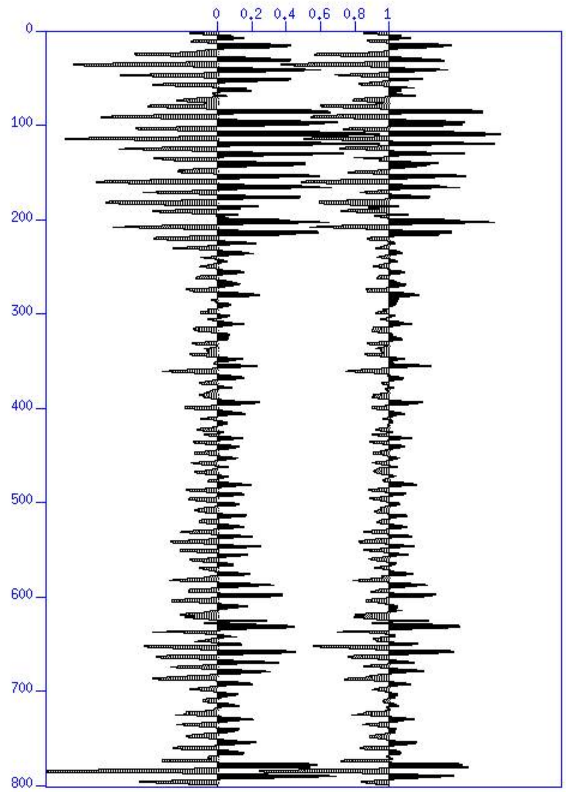

FIG. 9 The convolution of the shaping filter defined by equation (7) to the trace in Figure 9 (on the left) produces the phase-shifted trace (on the right), that is used in deconvolution .

The final step in this cascaded filtering procedure will be to apply the deconvolution filter effectively defined by equation (6) to the converted trace on the right hand side of Figure 9. This produces the final deconvolved trace. Figure 10 displays a comparison of the trace in Figure 9, with the deconvolution, and the actual reflectivity. We note a very encouraging comparison of the deconvolution (middle trace) with the reflectivity (right trace).

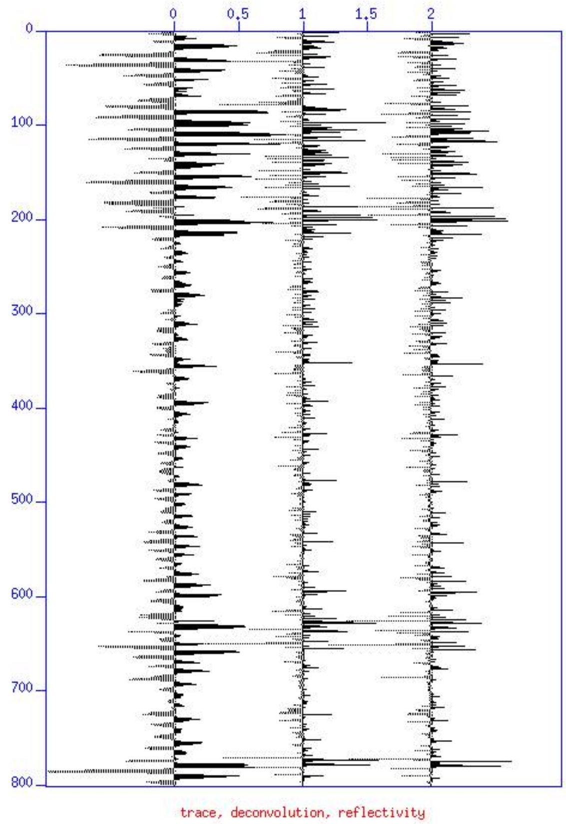

FIG. 10 A comparison of an input trace (left trace) with the cascade deconvolution (middle trace) and the actual reflectivity. The deconvolution correlates very well with the desired reflectivity function.

This encouraging result would suggest a cascade of a shaping filter to convert the wavelet to an exponentially damped sinusoid followed by a deconvolution filter to spike the damped sinusoid may prove to be a viable procedure.

Are there other possible uses for the exponentially damped sinusoid wavelet that has an exact 3-term inverse filter and can be ideally deconvolved? Actually such a wavelet should be useful in seismic modeling. If one were to use this wavelet in creating synthetic seismograms, one could apply a deconvolution filter to get an ideal representation of the reflectivity function. More tests are needed to demonstrate this usage.

CONCLUSIONS

It has been shown that the exponentially damped sinusoid has an exact 3-term spiking filter. While most wavelets do not have such an effective deconvolution, it may be advantageous to design a shaping filter that will convert the actual wavelet to a damped sinusoidal wavelet and then apply deconvolution. This has been illustrated with synthetic examples.

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