Phase unwrapping methods applied to DAS data

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ABSTRACT

PIMS Industrial Problem Solving Workshop in August 2018 offered a problem involving 2D phase unwrapping data acquired using distributed acoustic sensing (DAS). This report provides a summary of the work done during the week-long workshop. We begin with a description of the phase unwrapping problem. We consider the DAS dataset on which new methods were employed. We look at the results of the unwrap function found in MATLAB on the data and then provide a full description of two methods developed during the workshop. We then offer a comparison of the results from the two strategies.

INTRODUCTION

In August 2018, the PIMS Industrial Problem Solving Workshop offered a project on phase-unwrapping data acquired using distributed acoustic sensing (DAS). Phase unwrapping problems often arise when considering data collected using distributed acoustic sensing as the data shows the optical phase information. For such data, the amplitude is limited to a range of $[-\pi, \pi]$ resulting in a wrapping effect on the data. As such, the data is called the wrapped signal. In order to extract the information, the wrapped signal must be unwrapped.

In this report, we will summarize the phase unwrapping problem in 1D and 2D cases. We then focus on solving the phase unwrapping problem for DAS data and present two new strategies developed at the workshop for solving this problem. Finally, we apply both strategies to some real DAS data to show that both strategies provide better results than the MATLAB build in unwrap function.

PHASE UNWRAPPING PROBLEM

Consider the 1D noise-free problem first, suppose ϕ is the original signal and ψ is the wrapped signal. We define the phase wrapping operator $\mathcal{W}$ as:

$$\psi = \mathcal{W}[\phi] = \phi + 2k\pi,\ k \in \mathbb{Z}, \tag{1}$$

such that $|\psi| \leq \pi$. Figure 1 shows an example of (a) the original signal ϕ and (b) the wrapped signal ψ.

The goal of the phase unwrapping problem is to solve for the original signal ϕ with the wrapped signal ψ. In 1982, Itoh defined a necessary condition to solve the 1D unwrapping problem:

$$\Delta\phi_i = \mathcal{W}[\Delta\psi_i], \tag{2}$$

if $|\Delta\phi_i| < \pi$, where the operator Δ means $\Delta\phi_i = \phi_i - \phi_{i-1}$, $i = 0, 1, ..., N$ for $N \in \mathbb{N}$ (Itoh, 1982). The 1D noise-free problem is relatively straightforward to solve. Errors occur if the Signal-Noise-Ratio (SNR) is too low and if there is undersampling. Figure 2 shows the results of 1D phase unwrapping algorithm based on Itoh condition on different

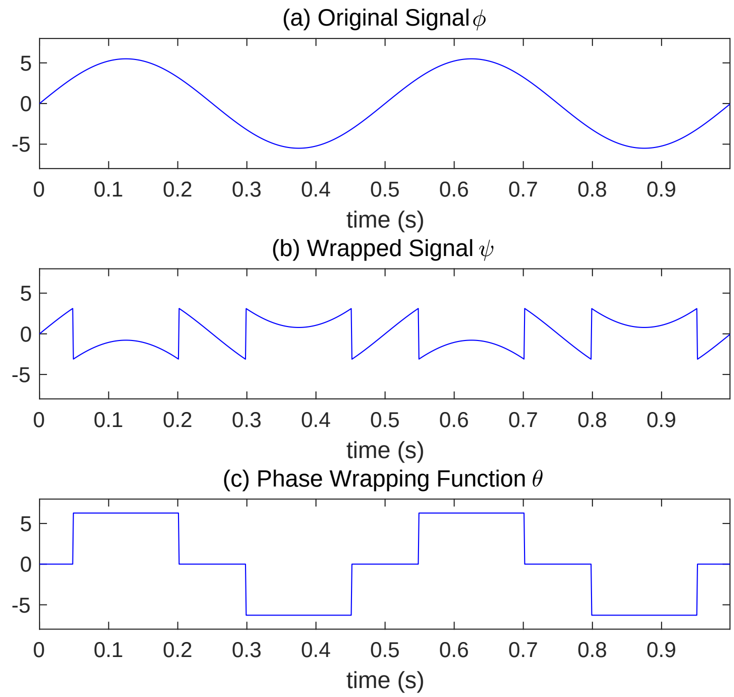

FIG. 1. An example of phase unwrapping problem. (a): Original signal ϕ. (b): Wrapped signal ψ. (c): Phase wrapping function θ.

noise levels. We can see that the false unwrapping happens in the Figure 2 (f) as the noise increases.

We suppose ϕ is a continuous function in 2D case. The Itoh condition extends to 2D as follows:

$$\Delta_x \phi_{i,j} = \mathcal{W}[\Delta_x \psi_{i,j}], \quad \Delta_y \phi_{i,j} = \mathcal{W}[\Delta_y \psi_{i,j}], \tag{3}$$

if $|\Delta_x \phi_{i,j}| < \pi$ and $|\Delta_y \phi_{i,j}| < \pi$. The 2D problem is less straightforward. We can understand some of the complexities better if we notice that we can extend Itoh condition to a continuous space via the integral

$$\phi(\vec{r}) = \phi_0 + \int_C \Delta\psi d\vec{r}, \tag{4}$$

where C is a closed path between any two points. In 1D case, the path C is fixed; however, in 2D case, there are several paths between two points. In order to obtain true phase with minimal unwrapping errors in the 2D case, the integral must be independent of the path C (Esseling, 2014). This means that no matter which path C we take between two points, the integral has the same value. Several methods exist to approach this issue in the 2D phase unwrapping problem.

During the workshop we only considered the 1D phase unwrapping algorithm and applied it to the DAS data trace by trace. The reason is the DAS data usually has large

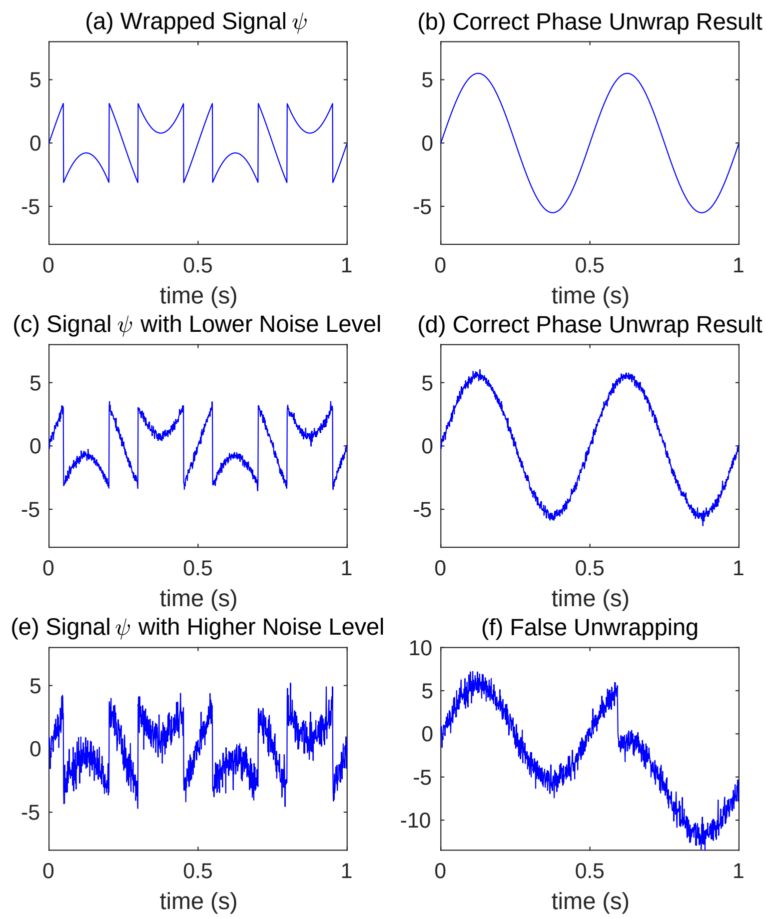

FIG. 2. The left column shows wrapped signals with different noise levels. The right column shows the results of 1D phase unwrapping algorithm. When the noise level is higher, false unwrapping errors may occur in unwrapping.

volumes and less computational complexity is required. And also the original signal of DAS data is not spatially continuous. In the subsequent sections, we will discuss the 1D phase unwrapping algorithm in detail.

DISCUSSION ON 1D PHASE UNWRAPPING ALGORITHM

Here we assume white noise is added onto the signal. Instead of using the Itoh condition globally, a traversal strategy will be used.

Let ϕ be the original signal and

$$\phi = \tilde{\phi} + \eta, \tag{5}$$

where $\tilde{\phi}$ is a continuous signal and η is the Gaussian white noise. Also, let

$$\phi = \psi + \theta, \tag{6}$$

where ψ is the wrapped signal with $-\pi \leq \psi_i \leq \pi$ for $i = 1, 2, \cdots, N$ and θ is the piece-wise constant phase wrapping function which contains the phase wrapping information. It satisfies $\theta_i = 2k\pi$ for $k \in \mathbb{Z}$ and $\Delta\theta_i = 0, \pm 2\pi$ for $i = 1, 2, \cdots, N$. Figure 1 (c) shows an example of θ.

Applying the difference operator to Eqn. 6 gives $\Delta\phi = \Delta\psi + \Delta\theta$. Then,

$$\Delta\psi = \Delta\phi - \Delta\theta = \Delta\tilde{\phi} + \Delta\eta - \Delta\theta. \tag{7}$$

Next, we define the wrapped point of the signal: for the i-th point of the wrapped signal, if $\Delta\theta_i = 0$, it is a non-wrapped point. If $\Delta\theta_i = \pm 2\pi$, it is a wrapped point.

Since $\tilde{\phi}$ is a continuous function and its central frequency is far lower than the Nyquist frequency in the DAS data, then $|\Delta\tilde{\phi}_i| \ll 2\pi$ is small. Also, we have $\Delta\theta_i = 0, \pm 2\pi$. When the SNR is high, $|\Delta\eta_i| \leq |\eta_i| + |\eta_{i-1}|$ should be small, otherwise $|\Delta\eta_i|$ might be large. For the i-th point in the wrapped signal, we consider two cases:

1. The ψ_i is a non-wrapped point.

$$\Delta\psi_i = \Delta\tilde{\phi}_i + \Delta\eta_i - \Delta\theta_i = \Delta\tilde{\phi}_i + \Delta\eta_i. \tag{8}$$

   When SNR is high, both $|\Delta\tilde{\phi}_i|$ and $|\Delta\eta_i|$ are small, we can set a tolerance tol such that $|\Delta\psi_i| < \mathrm{tol}$ is true for every non-wrapped point. When SNR is low, $|\Delta\eta_i|$ is unpredictable and a simple tolerance might not works for every non-wrapped point. That might cause false unwrapping.

2. The ψ_i is a wrapped point.

$$\Delta\psi_i = \Delta\tilde{\phi}_i + \Delta\eta_i - \Delta\theta_i = \Delta\tilde{\phi}_i + \Delta\eta_i \mp 2\pi. \tag{9}$$

   When SNR is high, both $|\Delta\tilde{\phi}_i|$ and $|\Delta\eta_i|$ is small, and $|\Delta\psi_i|$ should be close to 2π. When SNR is low, by the above reason, $|\Delta\psi_i|$ might not be close to 2π.

From the above discussion, as the SNR is high we can find all wrapped points by setting a tolerance and travelling through all points of $\Delta\psi$. The algorithm can be described as:

1. Compute $\Delta\psi$. Set a tolerance tol and travel through all points of $\Delta\psi$.
2. If $|\Delta\psi_i| < \mathrm{tol}$, then ψ_i is a non-wrapped point, let $\Delta\theta_i = 0$. If $|\Delta\psi_i| > \mathrm{tol}$, ψ_i is a wrapped point, let $\Delta\theta_i = \pm 2\pi$. The sign of $\Delta\theta_i$ is opposite to the sign of $\Delta\psi_i$.
3. Integrate $\theta_i = \sum_{j=1}^{i} \Delta\theta_j$.
4. Then $\phi_i = \psi_i + \theta_i$.

Figure 3 shows the 1D phase unwrapping algorithm applied to the signal in Figure 2 (c).

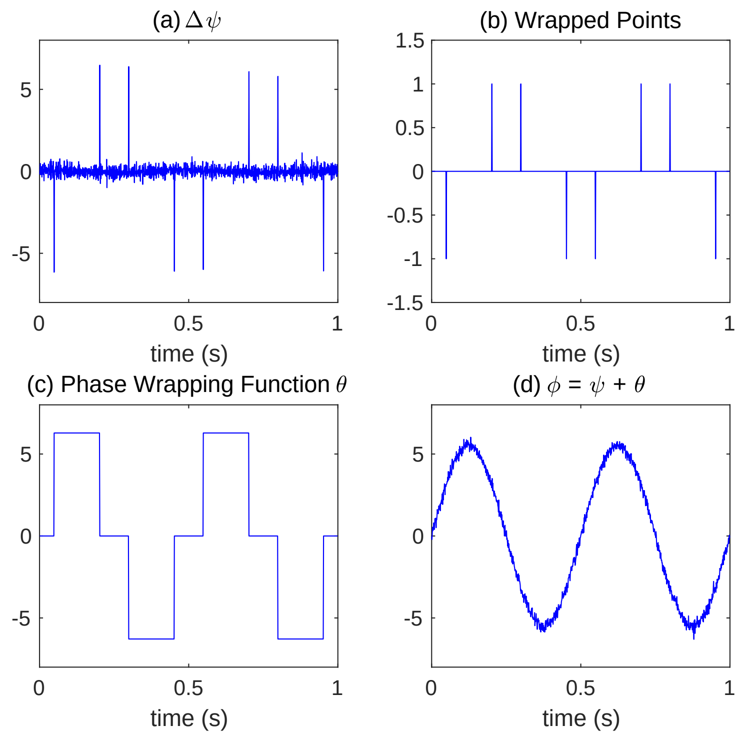

FIG. 3. A demonstration of 1D phase unwrapping algorithm.

DAS DATA

Figure 4 presents the dataset to which we applied our methods during the workshop. The data was acquired using distributed acoustic sensing. Several preprocessing techniques have been applied to the data. The sampling frequency is 20 kHz and spatial spacing of interrogator is 0.67 meter. The SNR of DAS data is not consistent for each trace and heavily depends on the location of the interrogator. The data contains two notable areas. In the upper part of the dataset, the phase wrapping occurs frequently in the signals. The high wrapping error maybe caused by the change of temperature. In the lower part of the data, phase wrapping only happens at certain locations where a peak or trough occurs. In this case, the signal oscillates around 0. Figure 5 (a) and (b) shows two locations in the DAS data seen in Figure 4. Location 1 mainly contains the first type of signals we discussed previously and location 2 contains the second type as well as a hyperbola.

MATLAB provides a 1D phase unwrapping function (MathWorks, 2018). It allows the user to set a constant tolerance for the noise level; however, since the SNR has significant variations in different traces, false unwrapping can happen and cannot be controlled with a single coefficient. We applied the MATLAB build-in unwrap function to the two different locations we discussed above. Figure 5 shows the results of different tolerance levels applied to the two locations. Specifically, Figure 5 (a) shows location 1 and (b) shows location 2. The results of MATLAB unwrap function are shown in Figure 5 (c) through (h). From the results, we can tell that errors happen at each of the noise tolerance and it is impossible

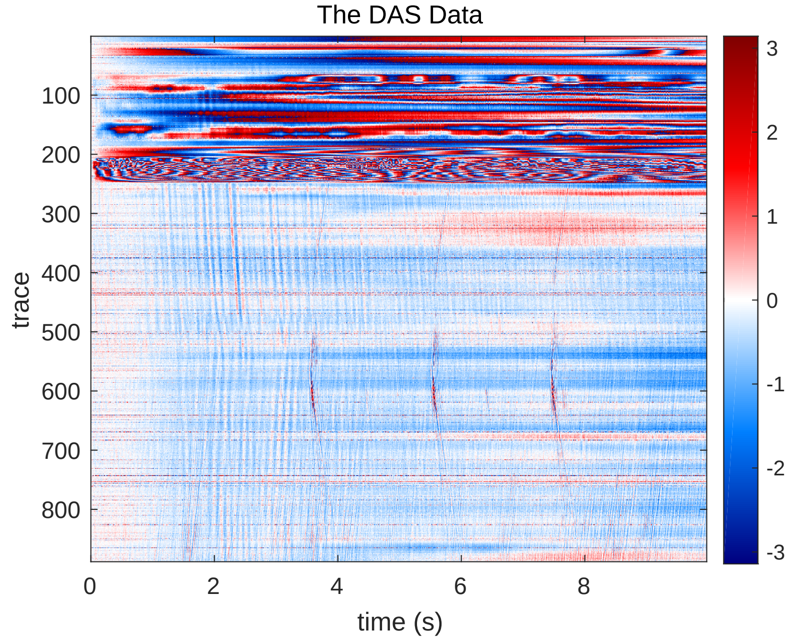

FIG. 4. The DAS data: The sampling frequency is 20 kHz and the spatial sampling interval is 0.67 meter.

to control the false unwrapping with one tolerance coefficient.

OUR WORKS

During the 2018 Industrial Problem Solving Workshop (IPSW 2018), we developed two different strategies for solving the phase unwrapping problem of DAS data. The main idea is to recognize the false unwrapping by introducing spatial information. We consider the DAS data as a two dimensional signal. Let $\phi_{i,j}$ be the original data and $\psi_{i,j}$ be the wrapped data where i is the index of trace and j is the index of sampling in time domain. We assume that the DAS data is piecewise continuous in space and the noise can be represented with a Gaussian. We set a tolerance level for noise $\mathrm{tol1}$ which is the same as in the MATLAB unwrap function and another tolerance for spatial component $\mathrm{tol2}$.

In Strategy 1, for each point $\psi_{i,j}$, we implement the 1D phase unwrapping algorithm in time domain first, then we determine whether $\psi_{i,j}$ is a wrapped point by using the information of the adjacent traces. For each trace i, the strategy can be described as:

1. Compute the signal difference in time domain $\Delta\psi_{i,j} = \psi_{i,j+1} - \psi_{i,j}$.
2. Find all wrapped points. If $|\Delta\psi_{i,j}| < \mathrm{tol1}$, $\psi_{i,j}$ is a non-wrapped point, let $\Delta\theta_{i,j} = 0$. If $|\Delta\psi_{i,j}| > \mathrm{tol1}$, $\psi_{i,j}$ is a wrapped point, let $\Delta\theta_{i,j} = \pm 2\pi$ and the sign is opposite to the sign of $\Delta\psi_{i,j}$.
3. (i) If $\psi_{i,j}$ is a non-wrapped point, then integrate by $\theta_{i,j} = \sum_{k=1}^{j} \Delta\theta_{i,k}$ and $\phi_{i,j} = \psi_{i,j} + \theta_{i,j}$.

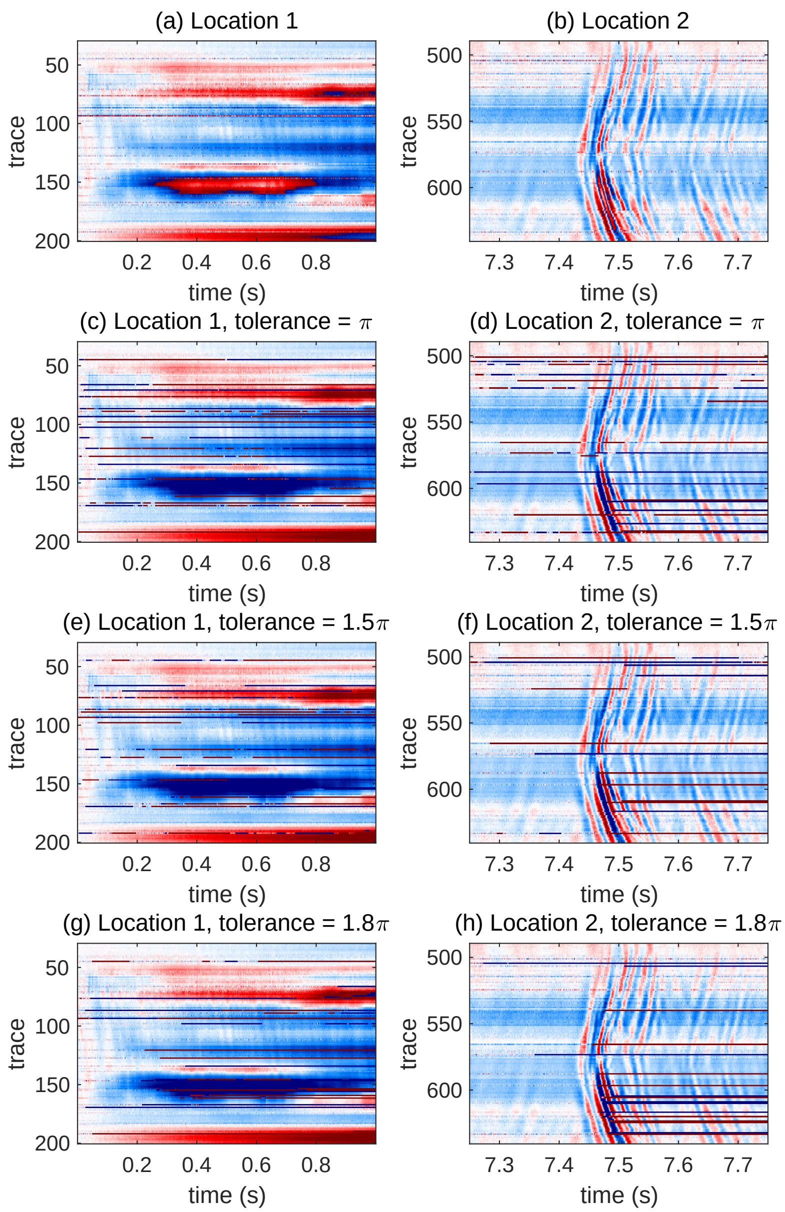

FIG. 5. Two portions of the DAS data: Location 1 mainly contains the first type of signal and Location 2 contains the second type as well as a hyperbola. Implementing MATLAB unwrap function will cause errors to occur in unwrapping.

(ii) If $\psi_{i,j}$ is a wrapped point, first integrate with $\theta_{i,j} = \sum_{k=1}^{j} \Delta\theta_{i,k}$ and $\phi_{i,j} = \psi_{i,j} + \theta_{i,j}$. Set $\bar{\phi}_{i,j} = 0.5(\phi_{i-1,j} + \phi_{i+1,j})$. If $|\phi_{i,j} - \bar{\phi}_{i,j}| < \mathrm{tol2}$, we accept the integration result. Otherwise, it is not a wrapped point. Let $\Delta\theta_{i,j} = 0$, integrate again with $\theta_{i,j} = \sum_{k=1}^{j} \Delta\theta_{i,k}$ and $\phi_{i,j} = \psi_{i,j} + \theta_{i,j}$.

In Strategy 2, first we implement the MATLAB build-in unwrap function. Then, we replace all the points where errors occur in unwrapping by the mean value of nearby points. The strategy is described as:

1. Implement the MATLAB unwrap function on each trace of DAS data.
2. For each wrapped point in $\psi_{i,j}$, if $|\phi_{i-1,j} - \phi_{i,j}| > \mathrm{tol2}$ and $|\phi_{i+1,j} - \phi_{i,j}| > \mathrm{tol2}$, then $\psi_{i,j}$ is a false unwrapping point.
3. We replace $\phi_{i,j}$ by $\bar{\phi}_{i,j} = 0.5(\phi_{i-1,j} + \phi_{i+1,j})$.

Figure 6 shows the results of Strategies 1 and 2 on Location 1 and 2. By comparing Figure 6 and Figure 5, we can see that both strategies provide fewer errors than the MATLAB unwrap function. Although there are still errors in the results in Figure 6, the output is much more stable then simply implementing the MATLAB built-in function. In the results of Strategy 1, the behaviour of low SNR traces have been kept. Strategy 2 provides more consistent results but smeared the low SNR traces; however, it might cause problems when the data changes rapidly in spatial component.

CONCLUSIONS AND FUTURE WORK

We began with an explanation of the phase unwrapping problem. We then discussed where possible errors occur in unwrapping. Then, we considered the DAS data with which we would be working in the report. We applied MATLAB unwrap function to the data. We presented two phase unwrapping algorithms developed at IPSW 2018. We noted that both strategies performed better than MATLAB unwrap function; however, some errors were still present for both algorithms.

We will consider the 2D phase unwrapping algorithms for the DAS data in the future. Specific strategies will be studied since the DAS data is spatially piecewise continuous. Also, mixed strategies, combinations of phase unwrapping algorithm and denoising techniques like Kalman filter or wavelet filter, will be studied.

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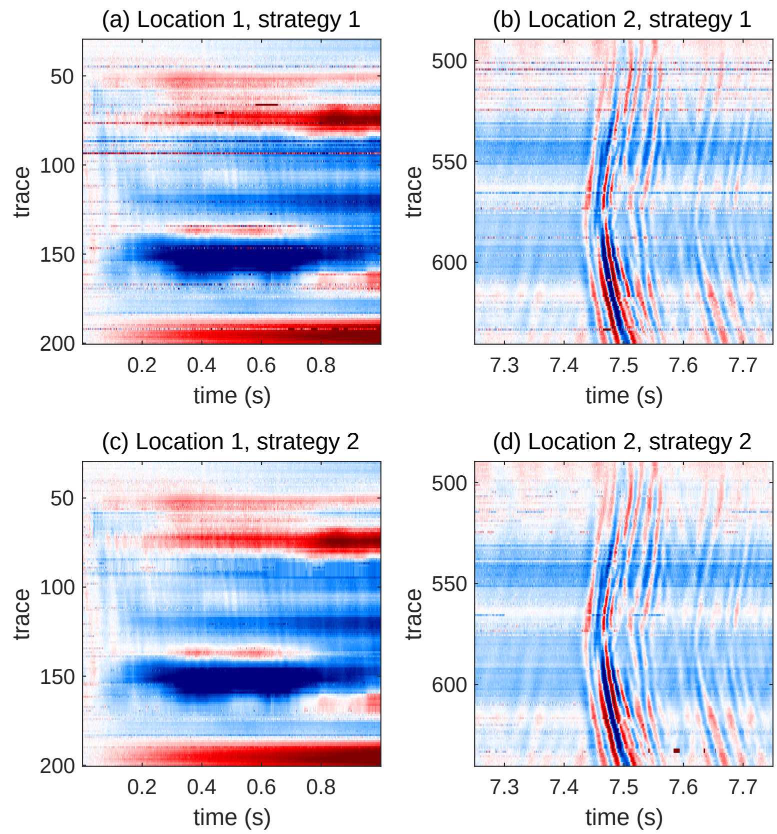

FIG. 6. Results of the two strategies for Location 1 (left) and Location 2 (right).

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