Full waveform inversion of multimode surface wave data: numerical insights

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ABSTRACT

Full waveform inversion has been demonstrated to be a powerful tool for high resolution velocity model building. However, using surface wave data in FWI presents many challenges. In particular, the dispersive nature of surface waves results in the amplification of cycle skipping problems. Here, we propose decomposing surface waves into their fundamental and higher order modes and inverting them in a sequential approach to mitigate this problem. Even though the fundamental mode amplitudes are typically larger than the higher order modes, the latter ones can travel in the deeper parts of the near-surface at higher frequencies. Therefore, we use the fundamental mode to produce an initial approximation to the near-surface S-wave velocities and then perform another step of FWI using the higher order modes to produce a more detailed velocity profile, particularly at larger depths. We also argue that although each individual higher mode is less energetic than the fundamental mode, as a group, the combination of all higher modes surpasses the energy and reach of the fundamental mode. We performed the modal separation by designing a mask in the F-K domain that roughly contains the energy of each mode. Results obtained from synthetic data demonstrate the potential of this approach to avoid cycle skipping and to improve the resolution of inverted S-wave velocity models.

INTRODUCTION

As multiparameter elastic full waveform inversion (FWI) becomes mainstream a detailed characterization of the near-surface becomes necessary. In particular, the characterization of S-wave velocities in the near-surface has remained a challenge for the imaging of the subsurface using S-wave data. S-wave refraction data are often not available in reflection seismic surveys making difficult the computation of S-wave velocities in the near-surface. Borrowing concepts and methods commonly used in earthquake seismology and engineering, practitioners in the exploration geophysics realm have turned their attention to the analysis of surface waves for the characterization of S-wave velocities in the near-surface.

In surface seismic acquisition, surface waves are inevitably excited and recorded in the acquired data. Early seismic acquisition designs attempted to remove this wave mode by carefully designing geophone arrays that would filter their energy. However, with the advent of digital geophones and the resulting increase in trace spatial density, the approach regarding surface waves changed significantly. Instead of attempting to filter them during acquisition, the newer seismic designs allowed for a better characterization of the surface-wave propagation, passing on the filtering task to the seismic processing teams.

Here, we attempt to exploit the surface wave data to obtain a better characterization of S-wave velocities in the near surface. We base our approach on the analysis of multimodal surface-wave data. We look at the characteristics of the fundamental and higher order modes in the surface-wave data, and study their individual contributions to the FWI process. We combine the ideas developed mostly in engineering for the 1D spectral inversion of multimode data with the more complete waveform-based framework provided by FWI. In contrast to conventional multichannel analysis of surface waves, our inversion is not based on the picking of dispersion curves. However, by understanding the dispersive nature of the surface-wave data we design a hierarchical multimode FWI approach. In our strategy, the fundamental mode surface-wave data are inverted first to obtain a detailed characterization of the shallowest part of the model and the low frequency components of the deeper part. Then, the resolution at the deeper parts of the model is enhanced during the inversion of the higher order modes. We tested these ideas using numerical experiments, which are presented in this study.

THEORY

In a layered medium surface wave propagation is dispersive. This means that each frequency component will travel with a different velocity. The velocities are controlled by the S-wave velocity profile of the near-surface. In addition, surface wave propagation in a layered medium is a multimode phenomenon. This means that at each frequency different modes of vibration might exist as the result of constructive interference occurring among waves undergoing multiple reflections (Foti et al., 2018).

Commonly, surface wave inversion is performed using the fundamental mode data. This mode is usually the most energetic and the easiest to identify. It contains the shortest wavelengths of the surface wave train, providing very high resolution at the shallowest part of the subsurface. However, since its amplitude decays exponentially with depth, it becomes very insensitive to velocity changes in the lower part of the near-surface.

Including higher order modes into the inversion of surface wave dispersion profiles has shown to provide higher resolution at larger depths. Ivanov et al. (2010), report that the relative error on Vs estimations dropped from 15% to 3.6% when higher order modes were jointly inverted with the fundamental mode data. However, picking dispersion curves for higher order modes can be a very challenging task due to the complex interactions between each mode, and to the lower relative amplitude of each individual mode.

Full waveform inversion of surface wave data attempts to retrieve the elastic properties of the near-surface by using all the wave modes present in the data. In its original formulation it is posed as a local optimization problem, which can get stuck at local minima if a solution close to the true subsurface parameters is not provided as an initial guess. This results in an effect commonly knonw as cycle skipping. The dispersive character of the surface wave amplifies the cycle skipping problem, making the FWI of surface wave very challenging. Alternative misfit functions have been proposed to mitigate this problem (Masoni et al., 2013; Pérez Solano et al., 2014; Yuan et al., 2015; Borisov et al., 2018). Masoni et al. (2016) follow a layer stripping approach, moving from narrow-offset high-frequency components, which contain information about the shallower parts of the near-surface, towards wide-offset low-frequency components which provide information about the lower part of the near-surface.

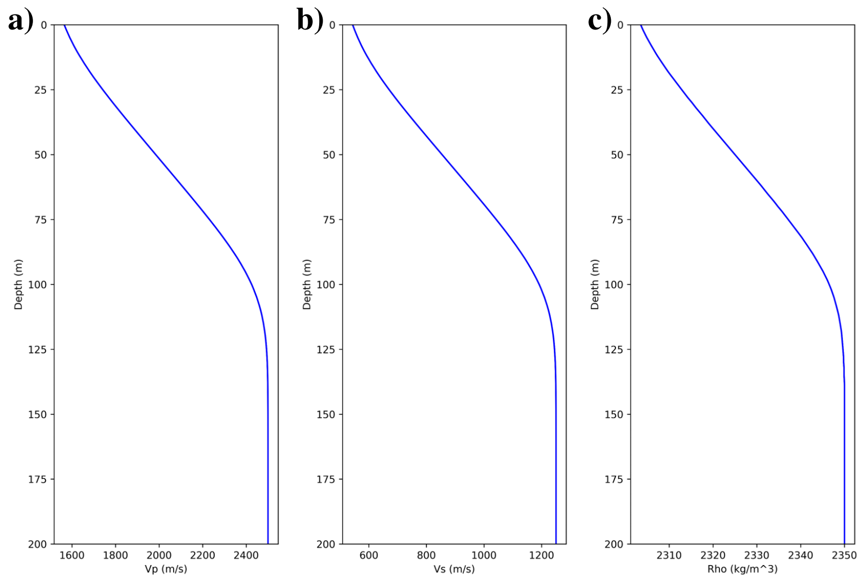

FIG. 1. (a) P-wave velocity, (b) S-wave velocity and (c) density profiles used to compute synthetic seismic data. The relatively fast velocities gradient is typical of profiles that promote the multimode propagation of surface wave.

Here, we propose using modal decomposition as a proxy for layer stripping. We perform an initial step of FWI using only the fundamental mode data, which should provide detailed information about the shallower part of the near-surface and just the low frequency component of the deeper parts. Then, we perform one additional FWI step using the high order modes to add resolution to the lower part of the model. This approach can be combined with the strategy proposed by Masoni et al. (2016), for further stability.

We use the FWI implementation developed by Pan et al. (2018), which exploits the computation of the misfit kernels and forward modelling capabilities of the *Specfem2d* package using the spectral element method (Komatitsch and Tromp, 1999).

SURFACE WAVES MODAL DECOMPOSITION

Using the velocities and density profiles in Figure 1 we compute synthetic seismic data to illustrate the mode decomposition process. The elastic parameter profiles consist of linearly increasing parameters that transition smoothly into a half-space.

The resulting particle displacements in the horizontal and vertical directions are plotted in Figure 2 (a) and (c), respectively. There we can see that most of the data consist of surface-wave energy propagating at very low velocities. A diving P-wave can also be observed at earlier times with a faster velocity. Figure 2 (b) and (d) show the dispersion spectrum of the data recorded in each component. The dashed line represents an arbitrary limit that separates the part of the spectra where the energy of the fundamental mode is contained (bottom part), from the space where higher modes energy exists (upper part). The amplitudes in the spectra have been normalized by the maximum recorded at each frequency. Notice how the fundamental mode dominates the spectrum of the data recorded on the vertical component. In contrast, the higher order modes recorded on the horizon-

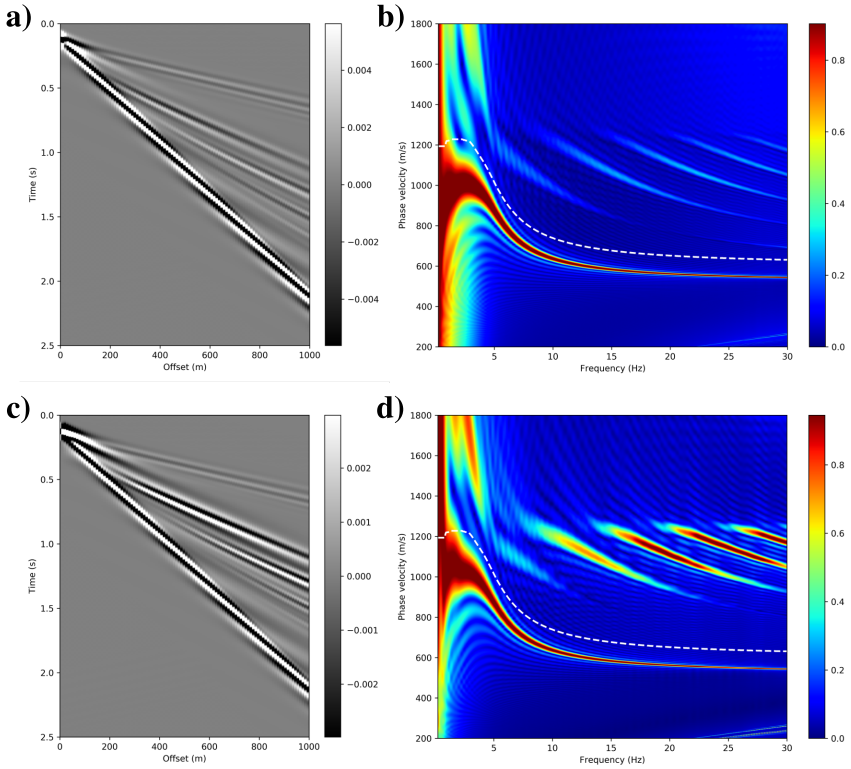

FIG. 2. Raw (a) vertical- and (c) horizontal-component data and their corresponding dispersion spectra (b) and (d), respectively.

tal component display similar levels of energy to the fundamental mode. This renders the horizontal component data a critical piece for the inversion of high order modes.

The arbitrary limit in Figure 2 can be used to design a mask in the F-K domain that filters the part of the spectrum we are interested in. Figure 3 illustrates the filtering of the fundamental mode energy from the full spectrum. The energy of the higher order modes can be filtered by just preserving the part of the spectrum that corresponds to the events with higher velocities. One additional limit can be specified to filter out refracted P-waves and reflections energy that will appear in the F-K spectrum with very high velocities.

Figure 4 shows the data and the output spectrum after filtering the fundamental mode on both components. Notice the difference in slope between the fast low-frequency signals and the slower high-frequency data. Also, the spectrum of the horizontal component data displays a slightly wider frequency band, particularly towards the low end of the spectrum. This part of the spectrum is critical for the inversion on the deeper parts of the models.

Figure 5 displays the higher order mode data recorded on both components. Notice that after removing the fundamental mode energy, the high order modes are easier to track, particularly on the vertical component data.

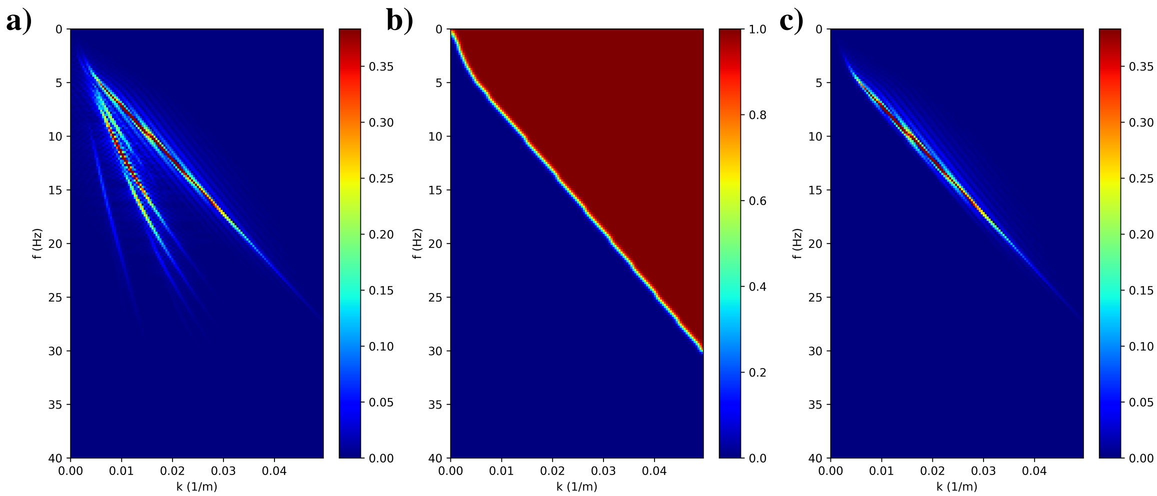

FIG. 3. (a) F-K spectrum of the horizontal component data in Figure 2a. (b) F-K mask defined to suppress the energy of the higher order modes and preserve the fundamental mode energy. (c) F-K spectrum showing fundamental mode energy only.

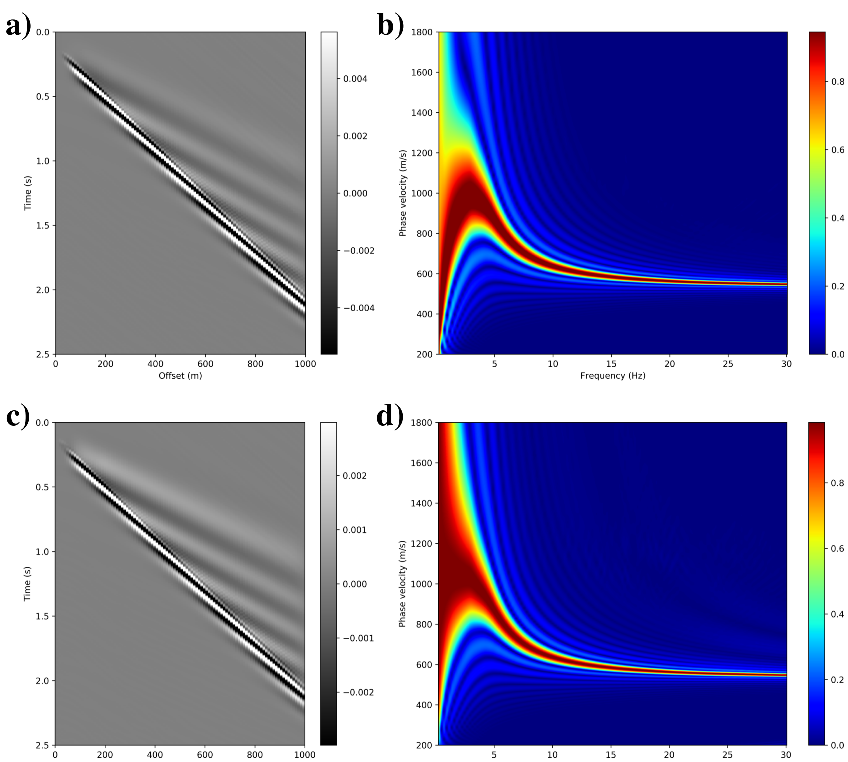

FIG. 4. Fundamental mode data recorded on the (a) vertical- and (c) horizontal components and their corresponding dispersion spectra (b) and (d), respectively.

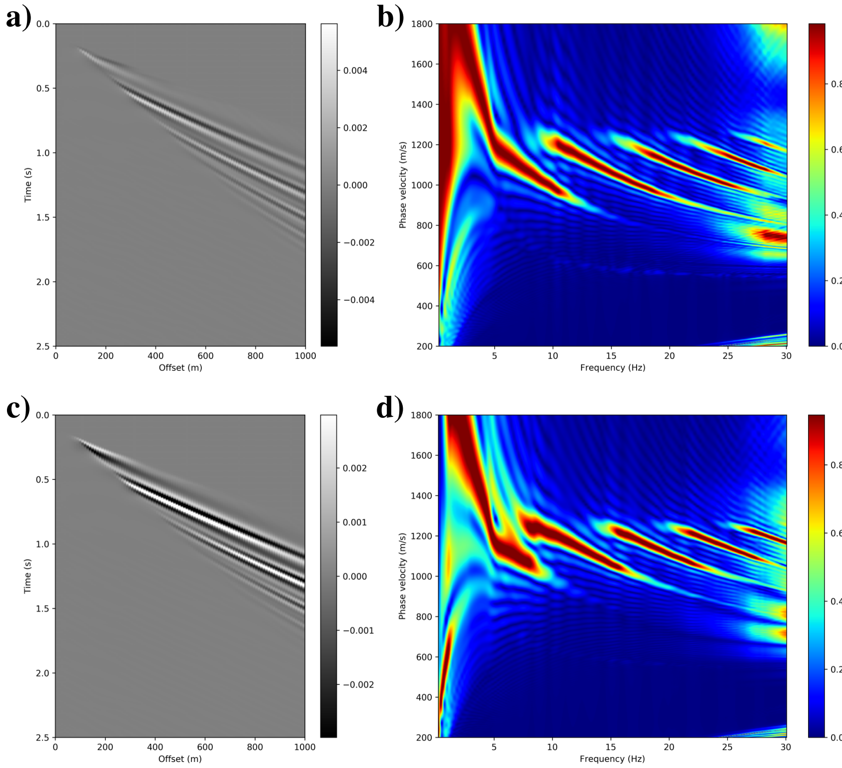

FIG. 5. Higher order modes data recorded on the (a) vertical- and (c) horizontal components and their corresponding dispersion spectra (b) and (d), respectively.

MULTIMODAL SURFACE WAVE FWI GRADIENTS

To understand the extent of the influence of each surface wave mode in an FWI iteration, we computed the FWI gradients using each wave mode separately. Figure 6 shows on the top row the unscaled gradients obtained by using the fundamental mode energy. Notice that the Vs gradient displays larger amplitudes than the ones obtained for the Vp and density models. However, this energy decays very fast with depth following the exponential decay expected according to the theory. Additionally, we can see that the fundamental mode energy seems to have very little effect on the Vp gradient.

The gradients obtained by using the data of higher order modes (Figure 6, bottom) display two very interesting characteristics. First, almost no energy appeared on the Vp gradient. This means that an FWI using higher order mode data will result in almost no change to the Vp model. Secondly, the Vs gradient covers most of the section with amplitudes that decay slowly with depth. This implies a good potential for this mode to provide details about the deeper parts of the model.

Given that most of the energy in the gradients is focused on the Vs model we plotted the contributions on the gradient coming from each particle displacement component and surface wave mode (Figure 7). The top row on Figure 7 displays the horizontal and vertical component gradients obtained with the fundamental mode data. Notice that even though they have similar magnitudes, the x-component gradient displays a deeper coverage compared to the z-component gradient. Similarly, the bottom row of Figure 7 displays the

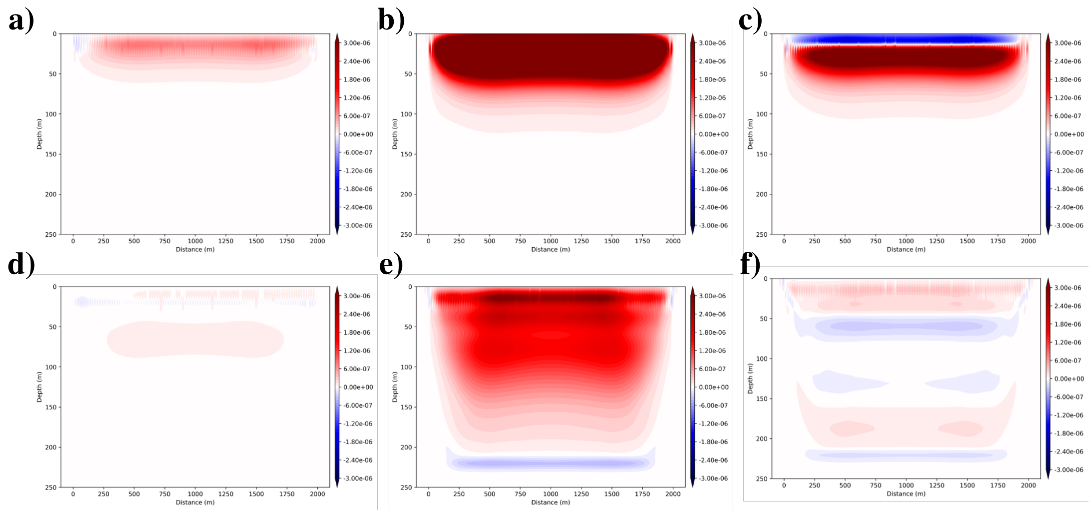

FIG. 6. (a), (b) and (c) display the output gradient of the first FWI iteration using the fundamental mode data for Vp, Vs and density, respectively. (d), (e) and (f) are the gradients obtained for the first FWI iteration using the higher order modes data.

Vs gradients obtained using the higher order modes energy recorded in each displacement component independently. Compared to the Vs gradients obtained with the fundamental mode data, the energy of the higher order modes provides a deeper coverage with slowly decaying amplitudes with depth. As before, the horizontal component data provided a slightly deeper coverage compared to the vertical component data which overall displays lower amplitude levels.

These observations reinforce the idea of the importance of the horizontal component data for the inversion of surface wave energy. In summary, they exhibit larger sensitivity to changes in Vs as experienced by the surface waves and they might contain higher order modes energy that can provide details about the deeper parts of the near-surface.

SYNTHETIC MULTIMODAL SURFACE WAVE FWI

To test the performance of this approach we created the Vp, Vs and density models displayed in Figure 8. We embedded a series of Vs anomalies in the Vs model. The Vp and density models only display linearly increasing values. These two parameters were fixed during the inversion and only the Vs model was updated after each iteration. The initial velocity models for all the three parameters consisted of models containing the exact background models.

Figure 9 summarizes the evolution of the inversion. On the top row the “observed” x- and z-component data and their corresponding dispersion spectra are plotted. Notice the backscattered surface-wave energy that is evident on the horizontal component data (Figure 9c). This produces interruptions in the continuity of the energy of the higher order modes in the dispersion spectrum (Figure 9d). Picking dispersion curves in this scenario would be a very difficult task. The second row displays the data modelled using the initial models. Notice that the dispersion spectrum for both the fundamental mode energy and the higher order modes are smooth and continuous. The data resulting from the inversion

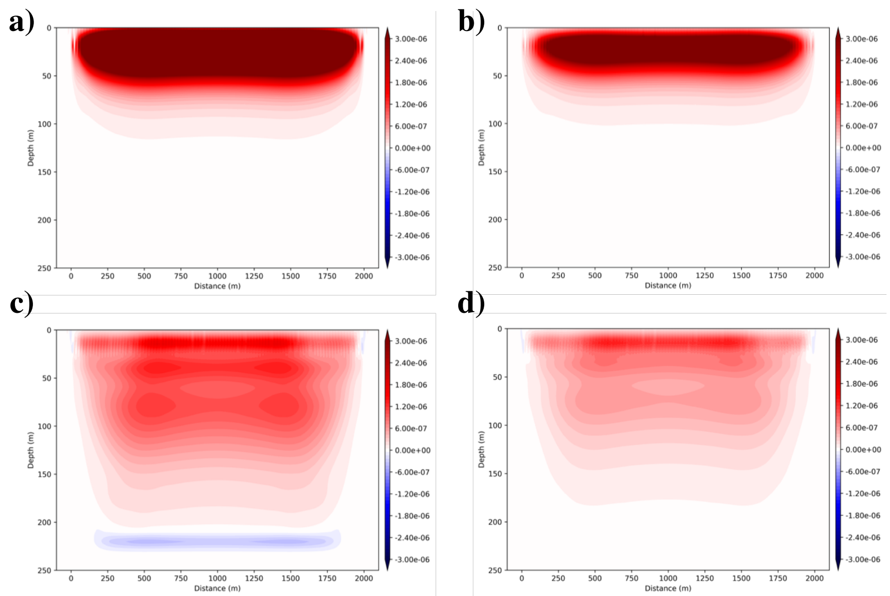

FIG. 7. Vs gradients obtained after the first FWI iteration for the (a) X- and (b) Z-component data of the fundamental mode and the (c) x- and (d) x-component data of the higher order modes.

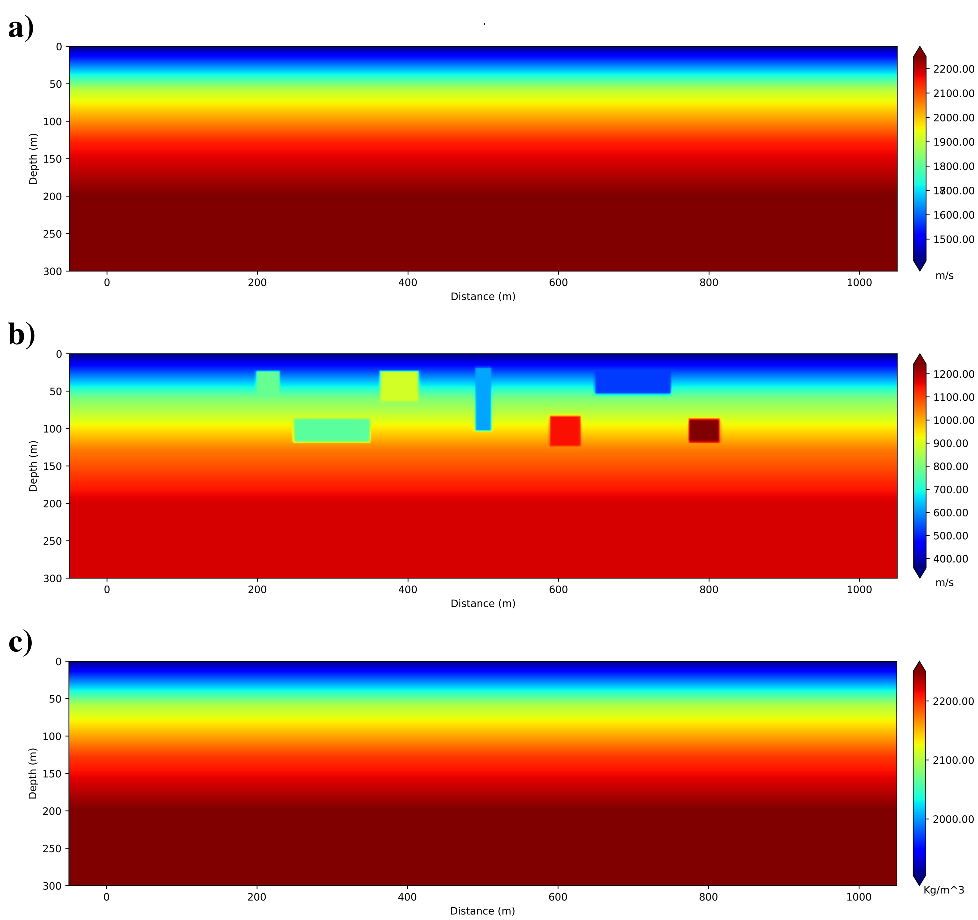

FIG. 8. True (a) Vp, (b) Vs and (c) density models used for testing.

using the fundamental mode data (Figure 9, third row), displays some perturbations on the fundamental mode data that resembles the fundamental mode energy in the observed data. However, the part of the spectra corresponding to the energy of the higher order modes does not display significant changes. Lastly, the results of the FWI after including the higher order modes are displayed on the bottom row. There we can see that both the modelled data and their frequency spectra match the observed data very closely.

Finally, Figure 10 compares the Vs model obtained after the inversion of the fundamental mode data and the one obtained after including higher order modes. Notice that the inversion of the fundamental mode data only provided low frequency updates to the model. No details for each of the anomalies can be observed in Figure 10a. On the other hand, after including the higher order modes, the location and shapes of each of the anomalies in the model were successfully retrieved.

CONCLUSIONS

Hierarchical strategies have shown to be very effective in FWI. In general, they try to first solve the most simple parts of a problem before moving to a more complete solution. In this study we have shown how this kind of approach is applicable to multimodal surface wave data. By first inverting the fundamental mode data high-resolution short-wavelength updates can be obtained in the shallowest part of the near-surface while providing long wavelength updates in the deeper parts of the model. Adding the higher order modes at a later stage improves the resolution at the deeper parts of the model. Layer stripping is then an implicit process in this approach.

We also highlight the importance of the horizontal component data in this process. Since the energy of the higher order modes is usually comparable to that of the fundamental mode in this component, a more balanced measurement can be provided to the inversion. In the case of using vertical component data only, where the fundamental mode amplitudes overwhelmes the rest of the arrivals, the inversion will be mostly driven by this wave mode.

Understanding the sensitivity of the horizontal-component data to both the fundamental and higher order modes of surface waves is important for developing FWI applications using distributed acoustic sensing (DAS) data. Despite usually providing single component measurements the extremely dense spatial sensing of the DAS data allows for an unalised recording of the surface wave energy. The application of this approach using DAS data remains to be explored.

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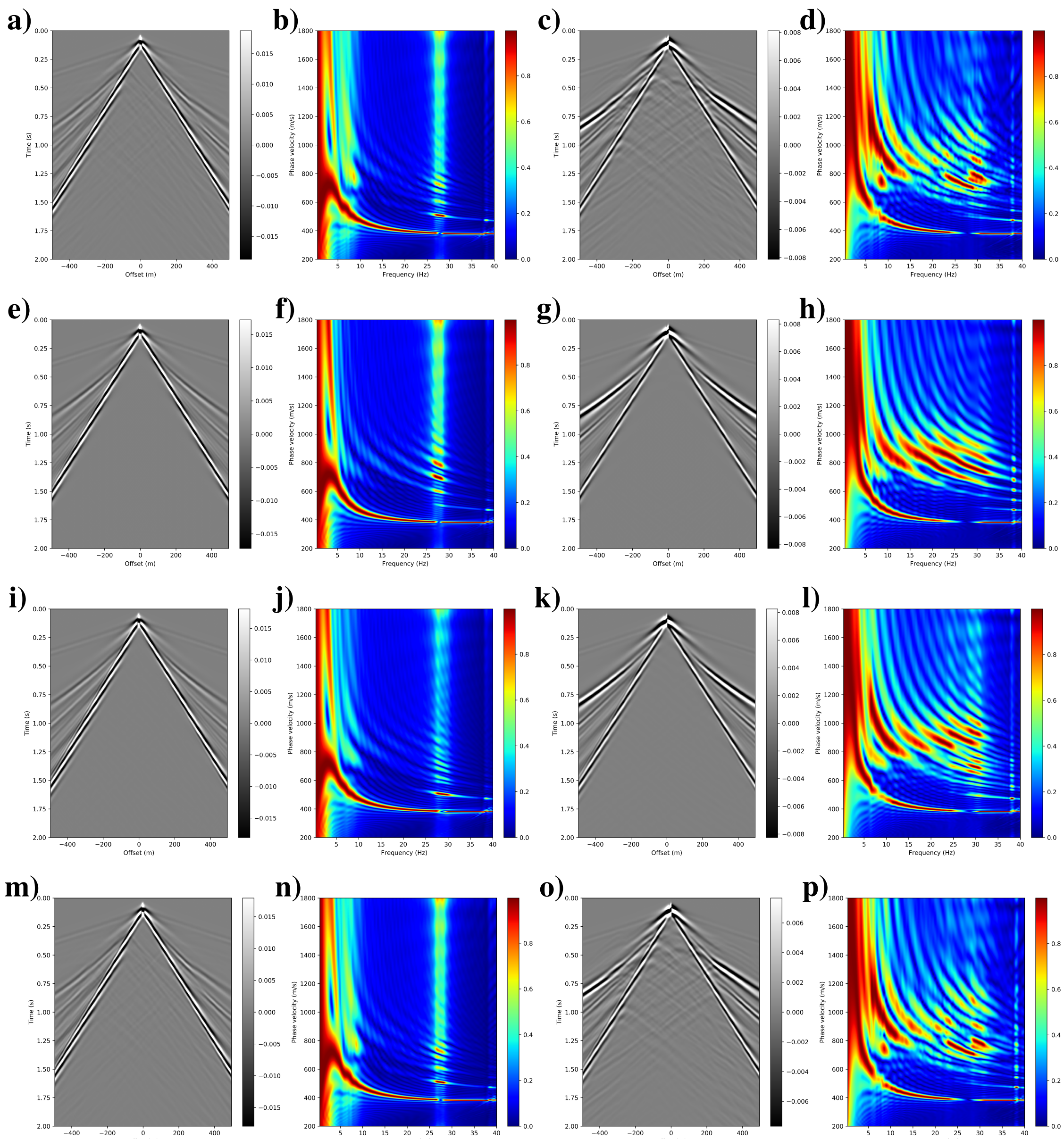

FIG. 9. (first column) horizontal and (third column) vertical-component data and their corresponding dispersion spectra at different FWI stages. (a)-(d) observed data. (e)-(h) initial model outputs. (i)-(j) results of fundamental mode data inversion. (m)-(p) Higher order modes inversion outputs.

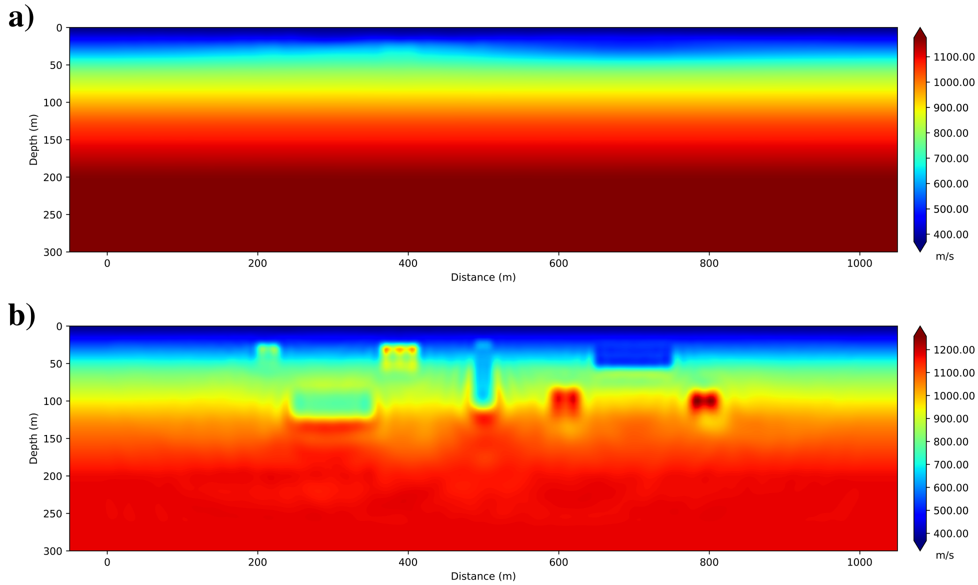

FIG. 10. Vs models obtained after FWI using (a) fundamental mode data only and (b) including higher order modes. The model in (a) is used as the starting model for the FWI including the higher order modes.

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