# Comparison of different surface wave dispersion inversion methods in engineering scale

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## ABSTRACT

Multichannel Analysis of Surface Wave (MASW) inverts the S wave velocity using the Rayleigh dispersion curve. It has three steps: field data acquisition, dispersion analysis, inversion analysis. Due to its robustness and simplicity, it is the most widely used method to characterize the near surface velocity. Factors influencing the method's accuracy are analyzed, and the determination method for the initial input parameters is given. Advantages and limitations of MASW are introduced in this paper. For modeling tests, we used SPECFEM to generate synthetic data, using the misfit of the generated dispersion curve and picked dispersion curve of the synthetic data to update S wave velocity. The least square Gauss-Newton inversion method is adopted to minimize the misfit. The damping parameter and stability parameter of the inversion are tested. In addition, global optimization methods like Simulated Annealing and Monte Carlo using Markov Chain were applied to simple layered models.

## INTRODUCTION

### Near surface

The region of the Earth referred to as the 'near surface', in which most geotechnical and environmental applications are carried out, occupies depths of less than 30m (Richards and Aki, 1980). Near surface research could bring benefits to exploration seismic problems also, by reducing the required drilling and sampling cost, detecting localized anomalies and determining static corrections for reflection signals. Near-surface characterization using elastic waves is conducted at frequencies varying from a few Hz to a few kHz, and the corresponding wavelength ranges between a fraction of a meter to tens of meters. Thus, the long wavelength condition is applied to do near-surface soil characterization, since the wavelength is much larger than the particle size. Elastic wave propagation in the near surface is affected by elastic properties including mass density $\rho$, bulk stiffness $B$ and shear stiffness $G$.

Shallow layer materials exhibit diverse mechanical properties with different soil and rock compositions. In general, the stress in near surface areas increases rapidly with depth. Meanwhile, pore pressures generated by fluids and gases in pores with different shapes and scales impacts mechanical properties. The near surface seismic wavefield is determined by all these characteristics combined with the seismic source nature.

Conventional seismic wave propagation theory is developed in the background of linear elasticity. In current research, the research involves other factors like dissipative medium and spatial attenuation.

## Surface waves

Surface waves, like the name, are waves which travel along surfaces. Due to dispersive nature, surface waves carry a lot of information about the media through which they propagate. In near-surface seismology, Rayleigh waves and Love waves are the two major surface wave types which could be distinguished by their displacement characteristics (Figure 1).

Love waves are generated by continual reflection off the earth's free surface and the subsequent interference of the down-going SH waves with those turned back toward the surface (Haskell, 1953). It means Love waves are formed through constructive interference of the higher order SH surface multiples. Love waves do not have vertical particle motion. Their displacements lie in the horizontal plane, orthogonal to the propagation direction and surface normal direction. In contrast, high energy Rayleigh waves are easily observed in the common

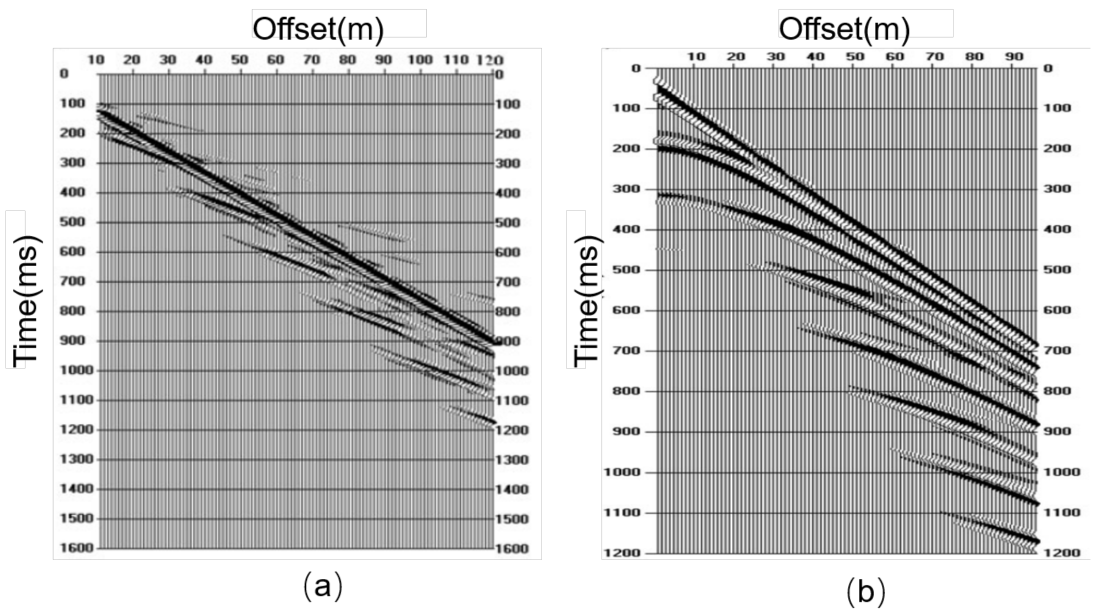


FIG. 1. a. Synthetic P-Sv data in a two-layered model. b. Synthetic SH wave data in a two-layered model.

vertical component seismograms. They are generated by a more complicated interaction of the P and SV waves with the free surface. With its rolling nature, particles follow a counterclockwise rotational orbit in the vertical plane. Its amplitude decreases exponentially with increasing depth. Ground Rayleigh waves speed is less than the phase velocity of shear wave, usually ranging from 50 to 300m/s, the most agreed penetration depth for low frequency Rayleigh waves is 30m. The Rayleigh wave and Love wave in seismograms are shown in Figure1. Since velocity may change at different depths, the velocity of Rayleigh wave is related to wavelength (and therefore frequency). It shows evident dispersion, smaller velocity with higher frequency. Another important characteristic of Rayleigh wave is its dispersion curve is multimoded for layered structures when geophones are placed under the surface (Figure 2). The higher order modes appear after the cut-off frequency as the Rayleigh wave has different velocities with the same frequency.

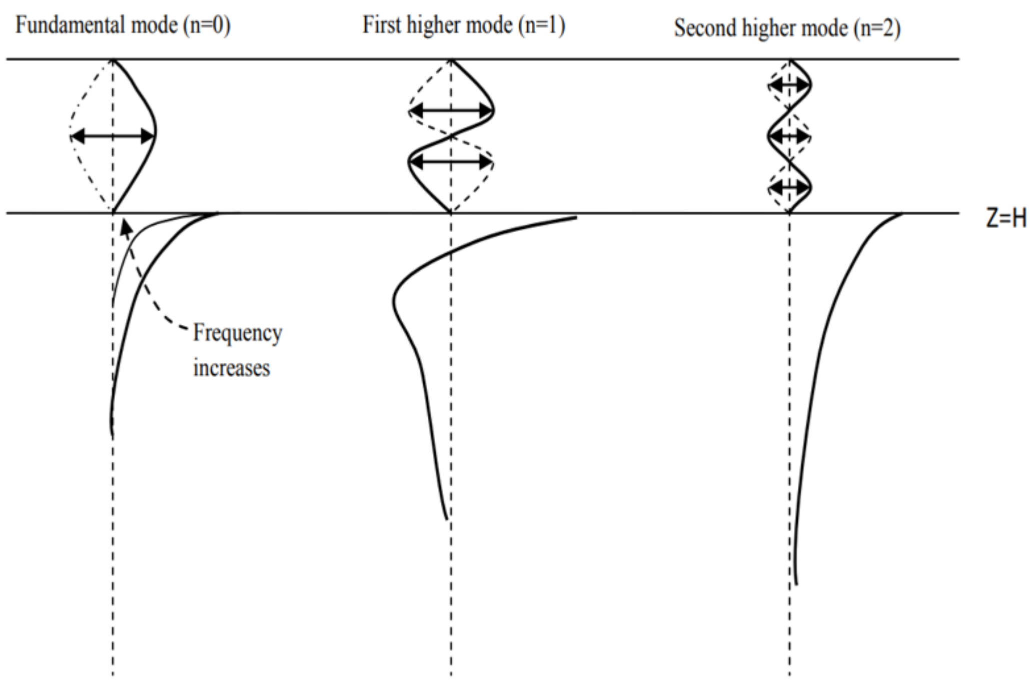


FIG. 2. Multimode principle of the Rayleigh wave

## Surface wave inversion methods

Inverting surface waves for S wave velocity has two major directions, dispersion curve inversion and full waveform inversion. The classical dispersion curve inversion method could invert S wave velocity robustly with correct layer thickness information under the assumption of a layered model (Park et al., 1998). But it is less accurate when applied in laterally heterogeneous subsurface media (Xia et al., 1999). As well the layer number and thickness should be determined in advance, otherwise it will influence the accuracy of inversion result. An alternative method uses Monte Carlo or simulated annealing methods to invert the S wave velocity and thickness simultaneously, but the computation cost is dozens or hundreds of times as high. Luo and Schuster (1991) developed the Wave-equation Dispersion inversion method, using scattered theory and connective function to find the derivative of the dispersion curve with respect to S wave velocity, in which a layered initial model is not required. Besides the advantage in resolving lateral heterogeneity, it avoids the local minimum problem.

Compared with most dispersion curve inversion methods, FWI deals well with lateral heterogeneous media, but easily falls into local minima. In order to avoid the cycle skipping problem due to shorter wavelength and inaccurate initial model, alternative objective functions could be adopted, like an envelope-based objective function or a waveform-difference objective function (Zhang et al., 2016). To improve the resolution, methods like a wavelet multi-scale adjoint method (Yuan et al., 2015) for joint inversion of body wave and surface wave was proposed. Other attempts to mitigate local minimum like layer stripping FWI, wavelet decomposition and frequency continuation approach were also developed in recent years.

## THEORY

The classical surface wave inversion has four parts: surface wave generation, dispersion curve picking, dispersion curve generation and dispersion curve inversion.

### Surface wave generation

The surface wave simulation is usually conducted through solving wave equations with free surface boundary conditions at $z = 0$. The trivial boundary condition is that the $\delta zz$ and $\delta xz$ stress components are zero on the free surface. In nature, the field behavior is the result of the physical properties of the surface only. Therefore, the method using real elastic properties of density, bulk modulus and shear modulus of free surface could generate the same result. Here, SPECFEM developed by Computational Infrastructure for

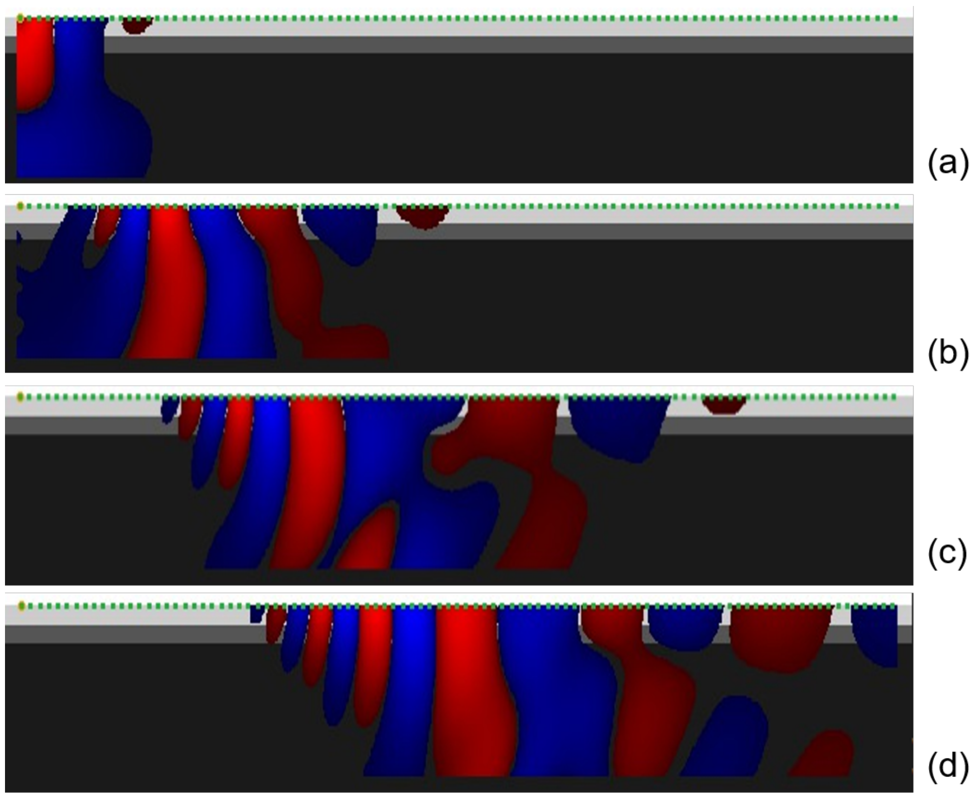


FIG. 3. Snapshots of forward modeling using SPECFEM at times a=0.5s, b=1.0s, c=1.5s, d=2.0s.

Geodynamics (CIG), using spectral element method, is adopted to generate Rayleigh wave records. Compared with finite difference methods, it has better performance in forward modeling with irregular interfaces, generating fewer undesired reflected and refracted waves. This method uses the continuous Galerkin spectral-element method, which can be seen as a particular case of the discontinuous Galerkin technique with optimized efficiency owing to its tensorized basis functions, to simulate forward and adjoint coupled acoustic-(an)elastic seismic wave propagation on arbitrary unstructured hexahedral meshes. The snapshots of surface wave modeling using SPECFEM are shown in Figure 3, and the record is shown in Figure 4. If we focus on the study of fundamental mode, some preliminary processing like

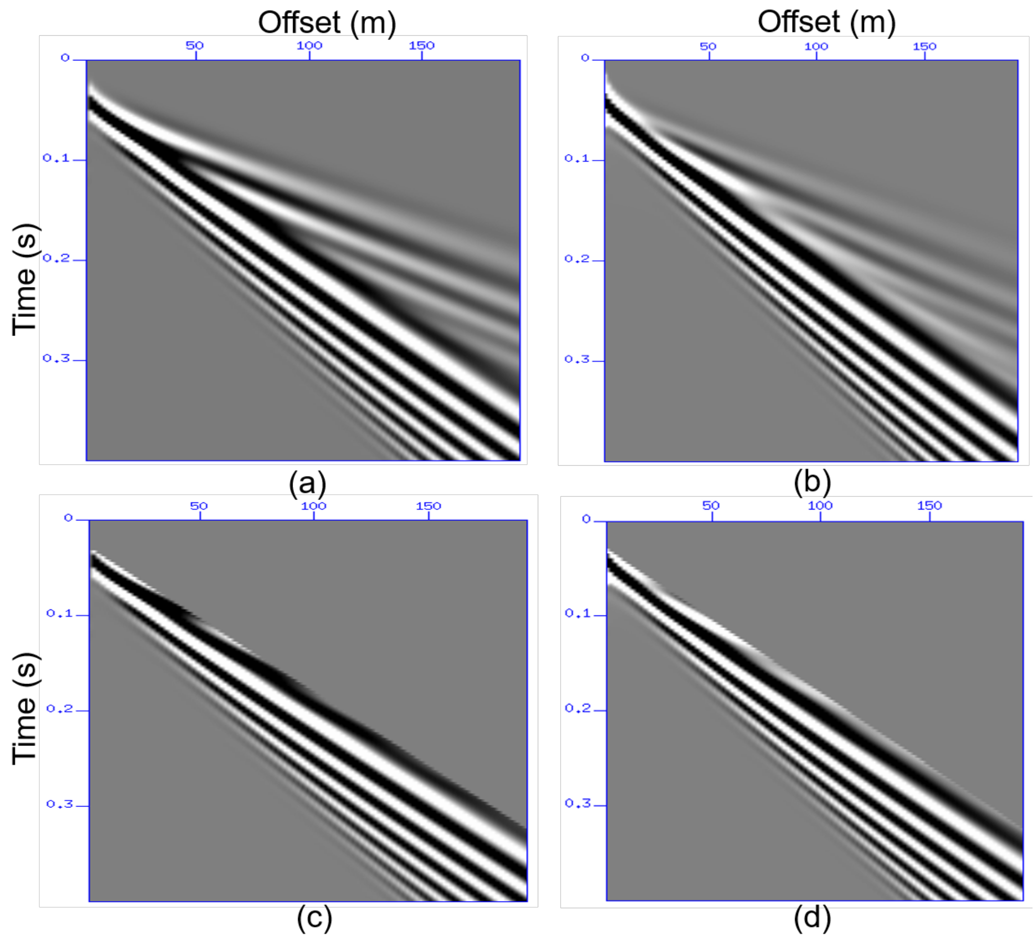


FIG. 4. Seismic records of a three-layered model, a. Ux, b. Uz, c. muted Ux, d. muted Uz.

muting of refraction wave,reflection waves and higher modes could be done to the raw data generated by SPECFEM for better analysis later.

## Multichannel analysis of surface wave(MASW)

MASW method (Figure 5) can be broken into three steps: field data acquisition, dispersion analysis and inversion analysis.

### *field parameter setting*

The Rayleigh wave frequency range or the penetration depth is related to the source wavelet and geophones; thus low frequency geophones are recommended. Some empirical relationship for real test is:

$$Zmax \approx 0.5\lambda_{max} \tag{1}$$

The maximum investigation depth is half the length of the longest Rayleigh Wave. The optimum receiver spread length is suggested to lie within the range of $Z_max$ to three times $Z_max$.

Receiver spacing $Dx$ is related to the shallowest resolvable depth ($Z_min$),

$$Dx = n * Z_{min}, 0.3 =< n <= 1.0. \tag{2}$$

With other considerations and limitations, a summary of field parameters is given below. It

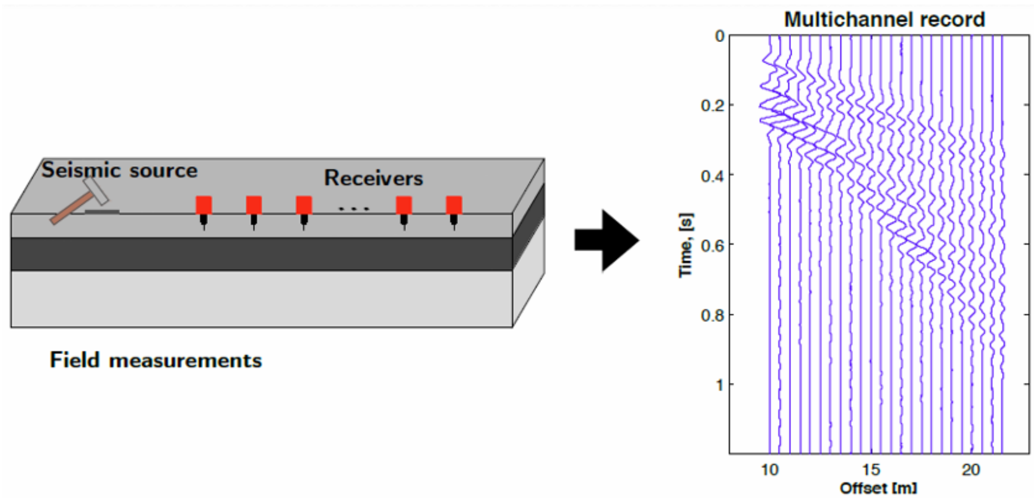


FIG. 5. Field measurement.

could serve as a guideline for field measurements.

| Depth $z_{max}$ [m] | Maximum wavelength $\lambda_{max}$ [m] | Length of receiver spread $L$ [m] | Source offset $x_1$ [m] | Receiver spacing (24-channel) $dx$ [m] |
|---|---|---|---|---|
| 5 | 10 | (5-15)<br>10 | (1-15)<br>5 | (0.2-0.7)<br>0.4 |
| 10 | 20 | (10-30)<br>20 | (2-30)<br>10 | (0.4-1.3)<br>0.9 |
| 20 | 40 | (20-60)<br>40 | (4-60)<br>20(*) | (0.9-2.6)<br>1.7 |
| 30 | 60 | (30-90)<br>60 | (6-90)<br>30(*) | (1.3-3.9)<br>2.6 |

FIG. 6. Field measurement parameters.

*dispersion analysis*

To extract dispersion curve from the record, methods of swept-frequency, phase-shift or linear $\tau - p$ transform can be used. Here the widely used $\tau - p$ transform and interpolation method is used to get the dispersion spectrum (Figure 7) of the generated data by SPECFEM2D. Then, the fundamental mode and the higher order mode dispersion curves can

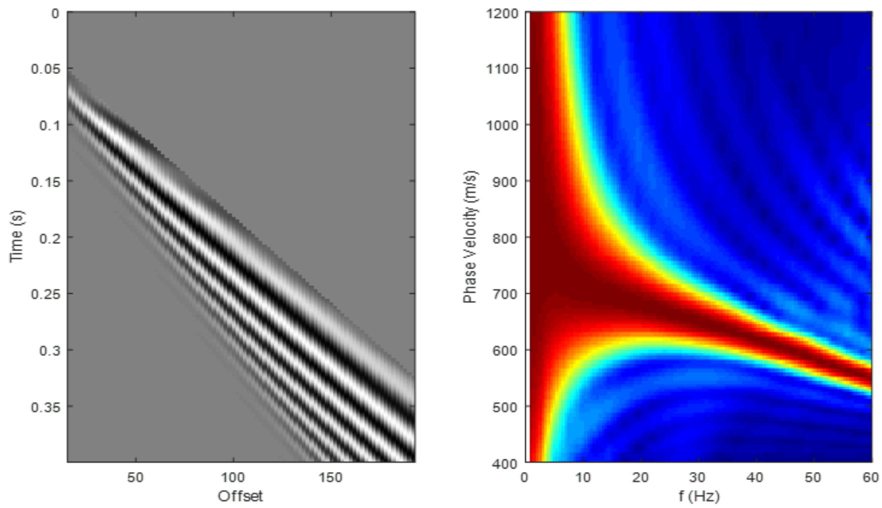


FIG. 7. Dispersion spectrum of the synthetic data.

be extracted from the spectrum. They can be picked manually similar to velocity analysis, or could be picked (Figure 8) automatically by getting the maximal energy point of each frequency. Usually the fundamental mode is of the most interest since the inversion methods using the fundamental mode are most robust and widely used. To prove the feasibility of

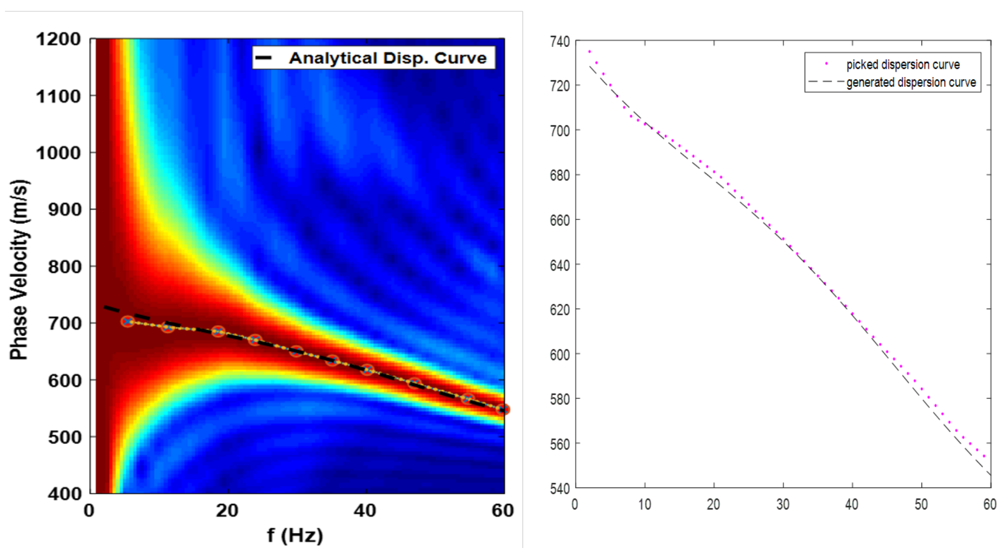


FIG. 8. Dispersion curve picking of the synthetic data.

this method, we used the accurate input parameter to generate a dispersion curve. These two dispersion curves match well.

*dispersion curve generation and inversion*

In conventional surface wave inversion methods, instead of using full waveform information, the dispersion curve is used for inversion as an important feature for surface wave.

This method is conducted by updating velocity parameters to generate synthetic dispersion curves matching the dispersion curve of real data best. Thus, when the dispersion curves match well, the accurate S wave velocity can be obtained.

Now, the first question is how to generate a dispersion curve using known parameters?

The characteristics of the dispersion curve are related to several measurable media parameters, layer thickness($h$), S wave velocity($Vs$), P wave velocity($Vp$) and density($\rho$). In MASW, a layered earth model is assumed for synthetic modeling and inversion. Therefore, every layer has the same properties, which makes it a 1D problem.

By substituting the harmonic Rayleigh wave displacement solution Equation 3 into the Navier's Equation 4, a matrix to calculate the displacement and stress eigenfunctions is obtained.

$$\mathbf{u}:\{u_1 = r_1(y,k,\omega)e^{i(\omega t-kr)}, u_2 = 0, u_3 = ir_2(y,k,\omega)e^{i(\omega t-kr)}\} \tag{3}$$

$$\frac{d}{dz}\begin{pmatrix} r_1 \\ r_2 \\ r_1^\sigma \\ r_2^\sigma \end{pmatrix} = \begin{pmatrix} 0 & -ik & 0 & \frac{1}{\mu} \\ \frac{-i\lambda}{\lambda+2\mu} & 0 & \frac{1}{\lambda+2\mu} & 0 \\ 0 & -\omega^2\rho & 0 & -ik \\ \frac{4k^2\mu(\lambda+\mu)}{\lambda+2\mu} & 0 & \frac{-ik\lambda}{\lambda+\mu} & 0 \end{pmatrix}\begin{pmatrix} r_1 \\ r_2 \\ r_1^\sigma \\ r_2^\sigma \end{pmatrix}, \tag{4}$$

where $r(k,z,w) = \left\{r_1(k,z,w); r_2(k,z,w); r_1^\delta(k,z,w); r_2^\delta(k,z,w)\right\}$. $r_1$,$r_2$ are displacement eigenfunctions, $r_1^\delta, r_2^\delta$ are stress eigenfunctions, in the form of $r_1^\delta = i\left((\lambda+2\mu)\frac{dr_2}{dz} + k\lambda r_1\right)$, $r_2^\delta = \mu\left(\frac{dr_1}{dz} - kr_2\right)$.

With boundary conditions for surface waves,

$r_1 \to 0$ and $r_2 \to 0$, when $z \to \infty$, $r_1^\delta = r_2^\delta = 0$ at the free surface $z = z_0$.

Particular values of $k$ for a certain $\omega$ can be found through searching techniques (Thomson, 1950). Due to the theoretical simplicity and ease of implementation, the propagator matrix method based on the Thomson-Haskell algorithm is commonly used to solve this linear eigenvalue problem. The non-trivial solutions of this problem are obtained by finding the roots of the Rayleigh dispersion equation $F_R\left[\lambda(z), \mu(z), \rho(z), k_i, w\right] = 0$ . Once the roots are found, the eigenfunctions can be calculated at specific depth. Rayleigh secular functions or root-bracketing techniques combined with bisection could be used based on the complex of the media (Kennett, 1974).

Every pair $\{\mathrm{k_i}, \mathrm{r_i}\,(\mathrm{y}, \mathrm{k_j}, \omega)\}$ of the result corresponds to a dispersion mode.

For $P - SV$ plane waves, the amplitudes of down-going (and for $P$ and $S$-waves, respectively) and up-going (and for $P$ and $S$-waves, respectively) waves travelling across an homogeneous half space are function of the motion-stress vector at the top of the half space ($z = z_n$). The motion-stress vector is propagated to $z_0$ by the means of the propagator matrix in Equation 5.

$$\begin{pmatrix} \acute{P}_n \\ \acute{S}_n \\ \grave{P}_n \\ \grave{P}_n \end{pmatrix} = T_n^{-1} \begin{pmatrix} r_1(z_n) \\ r_2(z_n) \\ r_1^{\sigma}(z_n) \\ r_2^{\sigma}(z_n) \end{pmatrix} = T_n^{-1} G(z_n, z_{n-1}) \ldots G(z_1, z_0)\, r(z_0), \tag{5}$$

where,

$$T_n^{-1} = \frac{-V_{sn}^2}{2\mu_n \hat{h}_n \hat{k}_n \omega^2} \begin{pmatrix} 2i\mu_n k \hat{h}_n \hat{k}_n & \mu_n l_n \hat{k}_n & \hat{h}_n \hat{k}_n & ik\hat{k}_n \\ -\mu_n l_n \hat{k}_n & 2i\mu_n k \cdot \hat{h}_n \hat{k}_n & ik\hat{h}_n & -\hat{h}_n \hat{k}_n \\ 2i\mu_n k \cdot \hat{h}_n \hat{k}_n & -\mu_n l_n \hat{k}_n & \hat{h}_n \hat{k}_n & -ik\hat{k}_n \\ \mu_n l_n \hat{k}_n & 2i\mu_n k \hat{h}_n \hat{k}_n & -ik\hat{h}_n & -\hat{h}_n \hat{k}_n \end{pmatrix}. \tag{6}$$

The matrix propagator methods have been developed into several forms ranging from the original Thomson-Haskell algorithm, the dynamic stiffness matrix method to the reflection and transmission coefficient method. The limitation of all these methods is they are only applied to layered model (VTI). The result is kind of the averaging feature of one certain horizontal layer. Therefore, a long spreading geometry is not recommended. Merging the boundary conditions with Equation 5,

$$\begin{pmatrix} 0 \\ 0 \\ \grave{P}_n \\ \grave{S}_n \end{pmatrix} = T_n^{-1} G_n \cdots G_1 r(z_0) = \begin{pmatrix} r_{11} & r_{12} & r_{13} & r_{14} \\ r_{21} & r_{22} & r_{23} & r_{24} \\ r_{31} & r_{32} & r_{33} & r_{34} \\ r_{41} & r_{42} & r_{43} & r_{44} \end{pmatrix} \begin{pmatrix} r_1(z_n) \\ r_2(z_n) \\ 0 \\ 0 \end{pmatrix}. \tag{7}$$

This equation is always true when the sub-determinant $(r_{11}r_{22} - r_{12}r_{21})$ vanishes. The problem is thus reduced to a root search along the slowness or the velocity axis for a given frequency.

There is much research concerning the root finding of secular functions and construction of secular functions. Here, the root of the secular function, which is the wavenumber k, is obtained by means of root-bracketing techniques combined with bisection (Lai and Rix, 1998). Then the Rayleigh wave velocity will be obtained given certain frequency. Substituting the k into the differential equations, the eigenfunctions of displacements and stresses can be calculated. The secular functions can be built in different forms based on the propagator matrix used and simplification methods. Thus, most work in dispersion curve modeling is done on the construction of secular functions and the root finding of secular functions. As the formula in this part is complex and tedious, we will focus on the inversion part here.

The first question in inversion is why we can invert S wave velocity using the dispersion curve? This requires a sensitivity test of dispersion curve relative to the input parameters. The sensitivities of the dispersion curve to different input parameters are different. Through synthetic tests, we can find that the shear wave velocity affects the dispersion curve most, followed by the layer thickness(Figure 9). That's also the reason why shear wave velocity can be inverted by dispersion curve. Since the Rayleigh dispersion is not sensitive to density, the density can be given by empirical relationship based on the velocity parameter. The primary wave velocity can be assumed to be related by the poission ratio to shear wave velocity. The range of shear wave velocity can be calculated from the maximum and minimum value of the dispersion curve with a ratio 1.14, which is a constant, based on the common cases of real media. The objective function for the inversion is the L2 norm of

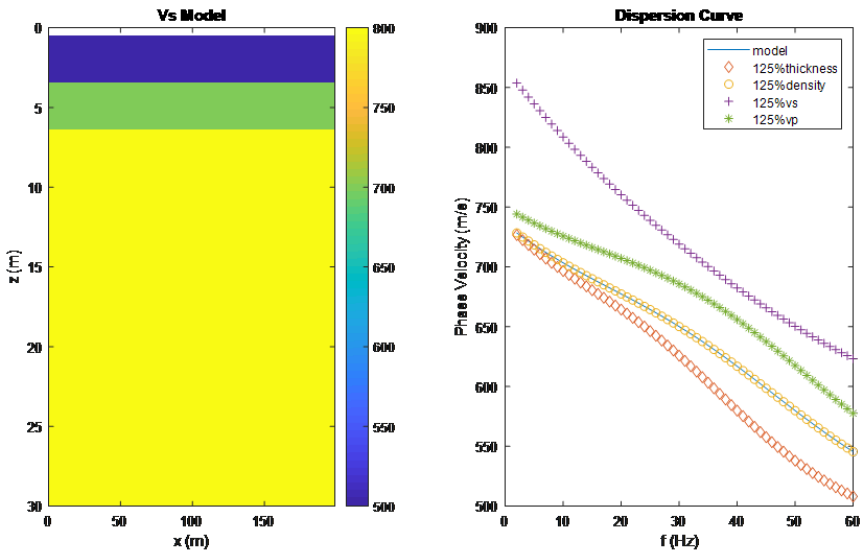


FIG. 9. Sensitivity analysis for different parameters.

the misfit between the picked fundamental mode of the observed dispersion spectrum and predicted dispersion curve. The goal is updating the shear wave velocity to generate the same dispersion curve as the observed one. Then the shear wave velocity at the near surface can be obtained. To speed the inversion and improve the stability of the result, damping parameters and regularizations should be incorporated into the inversion as well.

For the determinations of the initial model parameters, some empirical formula and assumptions are adopted. Based on the sensitivity analysis, the dispersion curve is not sensitive to the density. Therefore, the density for different layers could be set using the Gardner equation. The P velocity can be set as ratio of S wave velocity, given an appropriate Poisson value. The layer number and layer thickness can be assumed based on the shape and spread of the dispersion curve. Then the effect of thickness can be minimized by subdividing certain thinner layers within each constant S wave velocity slice or determined based on the shape of dispersion curve. Thus, the only unknown parameter left is the S wave velocity.

One important step in inversion is to calculate the gradient for inversion. The iterative gradient can be calculated through a perturbative method or nonperturbative method. For the perturbative method, the gradient is calculated in equation (10) and (11).

$$D_{syn} = D_{syn0} + J * \Delta Vs, \tag{8}$$

where $D_{syn}$ is the updated model's phase velocity vector, $D_{syn0}$ is the initial model's phase velocity vector, $J$ is the Jacobian matrix of partial derivatives, and $\triangle Vs$ is the parameter change representing the changes or perturbations to the S-wave velocity in the model.

$$J = \frac{\Delta D_{syn}}{\Delta Vs} = \frac{D_{syn}(Vs + \Delta Vs) - D_{syn} Vs}{\Delta Vs}. \tag{9}$$

Here, by adding a perturbation on $Vs$, and calculating the derivative of the dispersion curve relative to perturbation of $Vs$, the Jacobian matrix can be obtained.

The normal equation of the Gauss-Newton inversion is:

$$J^T J * \Delta Vs = J^T \left( D_{obs} - D_{syn} \right). \tag{10}$$

However, when the matrix $J_T J$ is singular, the $\triangle Vs$ could be a large value, even negative, which makes the inversion unstable. Therefore, some constraints should be added to the inversion.

Hence, one damping parameter and one stability factor are added into the inversion; the damping parameter $\alpha$ is the parameter controlling scaling of the update,

$$V_S = V_S + \alpha * \Delta V_S. \tag{11}$$

$\alpha$ ranges from (0,1), and it prevents drastic changes of the updating term. The stability

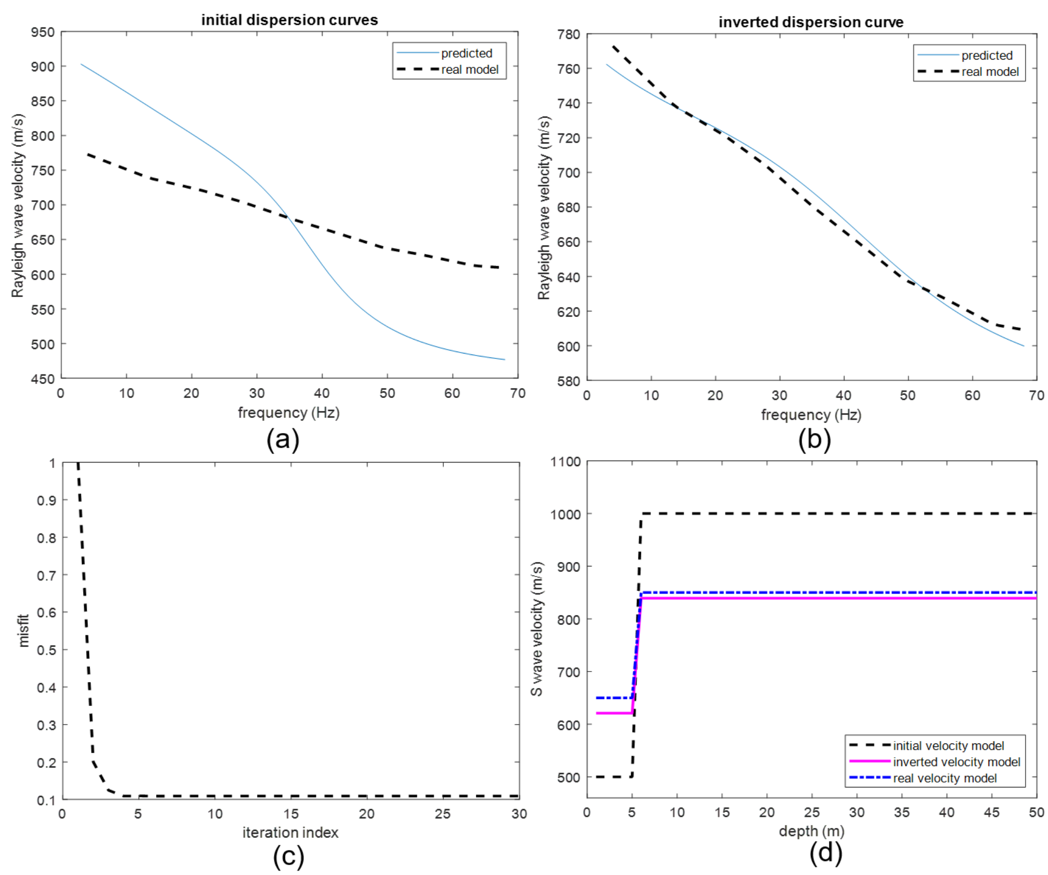


FIG. 10. Inversion results for a two-layered model. a,b,c,d are for accurate thickness information.

factor $\beta$ is added to the Jacobian matrix to prevent it from becoming singular, so that the update will be stable, rather than a large or negative value,

$$\Delta Vs = \left( J^T J + \beta * I \right)^{-1} J^T \left( D_{obs} - D_{syn} \right). \tag{12}$$

Therefore, the update term becomes

$$Vs = Vs + \alpha * \left(J^T J + \beta * I\right)^{-1} J^T \left(D_{obs} - D_{syn}\right). \tag{13}$$

The damping parameter $\alpha$ and stability factor $\beta$ will influence the convergence speed, therefore choosing the smallest parameters as long as the inversion is stable. Since the convergence speed becomes slower with larger iteration index, the update becomes more stable in the later stages of updating. Therefore, the damping parameter $\alpha$ and stability factor $\beta$ could change gradually with iterative index to speed the converging.

Based on testing, the initial value of $\beta$ could be 5, multiplying by a factor 0.8 in every iteration until it reaches the minimum number 0.5. The initial value of $\alpha$ could be 0.5, multiplying by a factor 1.2 in every iteration until it reaches the maximum number 1.0. Other regularizations like smoothness could be applied to the inversion, but it does not greatly improve the inversion result, and are not considered further.

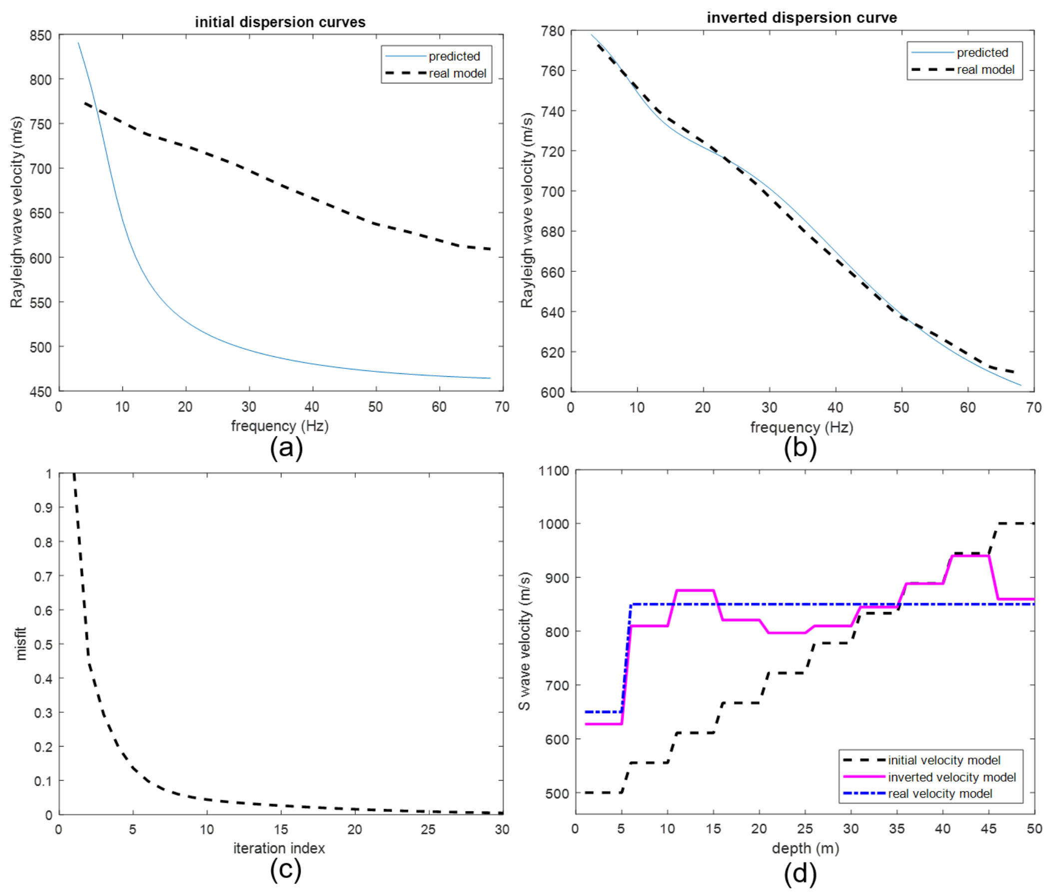


FIG. 11. Inversion results for a two-layered model. a,b,c,d are for ten thin layers.

## SIMULATION RESULTS

Several synthetic tests were conducted with layered models, including two-layered model, three-layered model with accurate thickness information or inaccurate thickness information.

## Two-layered model

First, we did a simple test for two-layered model with accurate thickness information but inaccurate initial S wave velocity, as shown in Figure 10. From the test result, we find

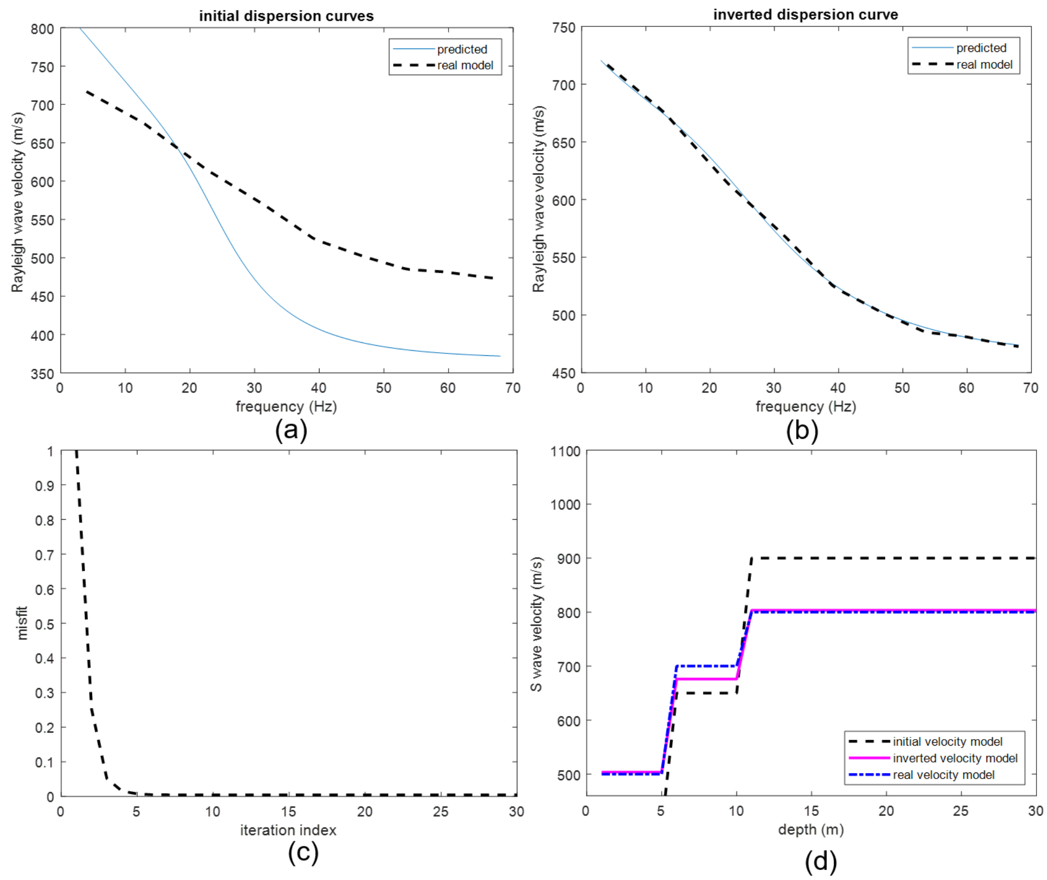


FIG. 12. Inversion results for a three-layered model. a,b,c,d are for accurate thickness information.

that with accurate thickness information, the S wave velocity could be inverted accurately for each specific layer. In this case, we do not need to add damping or stability parameters in the inversion process, since the calculation is stable. It converged well in less than 5 iterations(Figure 10-c). However, when we use thinner layers to conduct inversion, the calculation becomes unstable, and the inverted results are not that accurate due to solution nonuniqueness(Figure 11).

## Three-layered model

Another synthetic test is done on a three-layered model, as is shown in Figure 11. The initial velocity using inaccurate velocity, accurate thickness and inaccurate thickness are tested on this model.

From the above results, we can see that the thickness information is influential on the inversion results. Adding more layers in inversion can not guarantee a better inversion result, since the derivative may have a larger error for more layers(Figure 13 and Figure

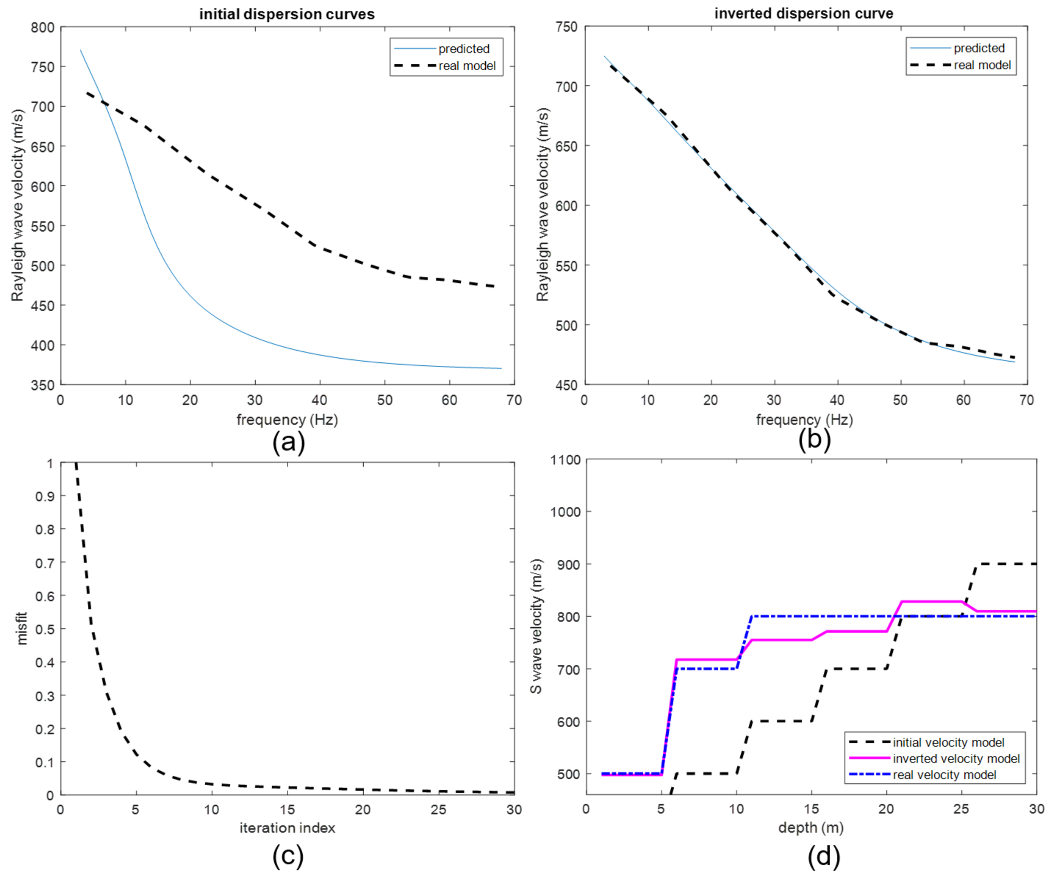


FIG. 13. Inversion results for a three layered model. a,b,c,d are for six thin layers assumption.

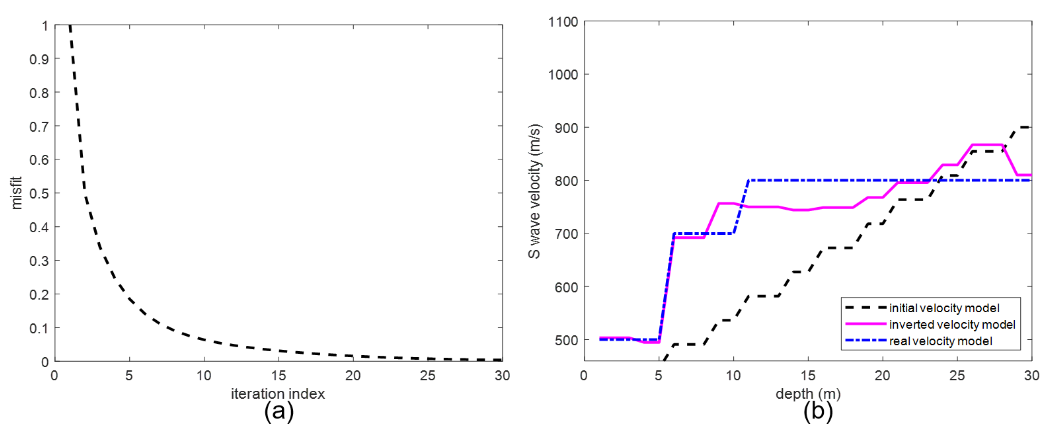


FIG. 14. Inversion results for a three-layered model. a,b are for ten thin layers assumption.

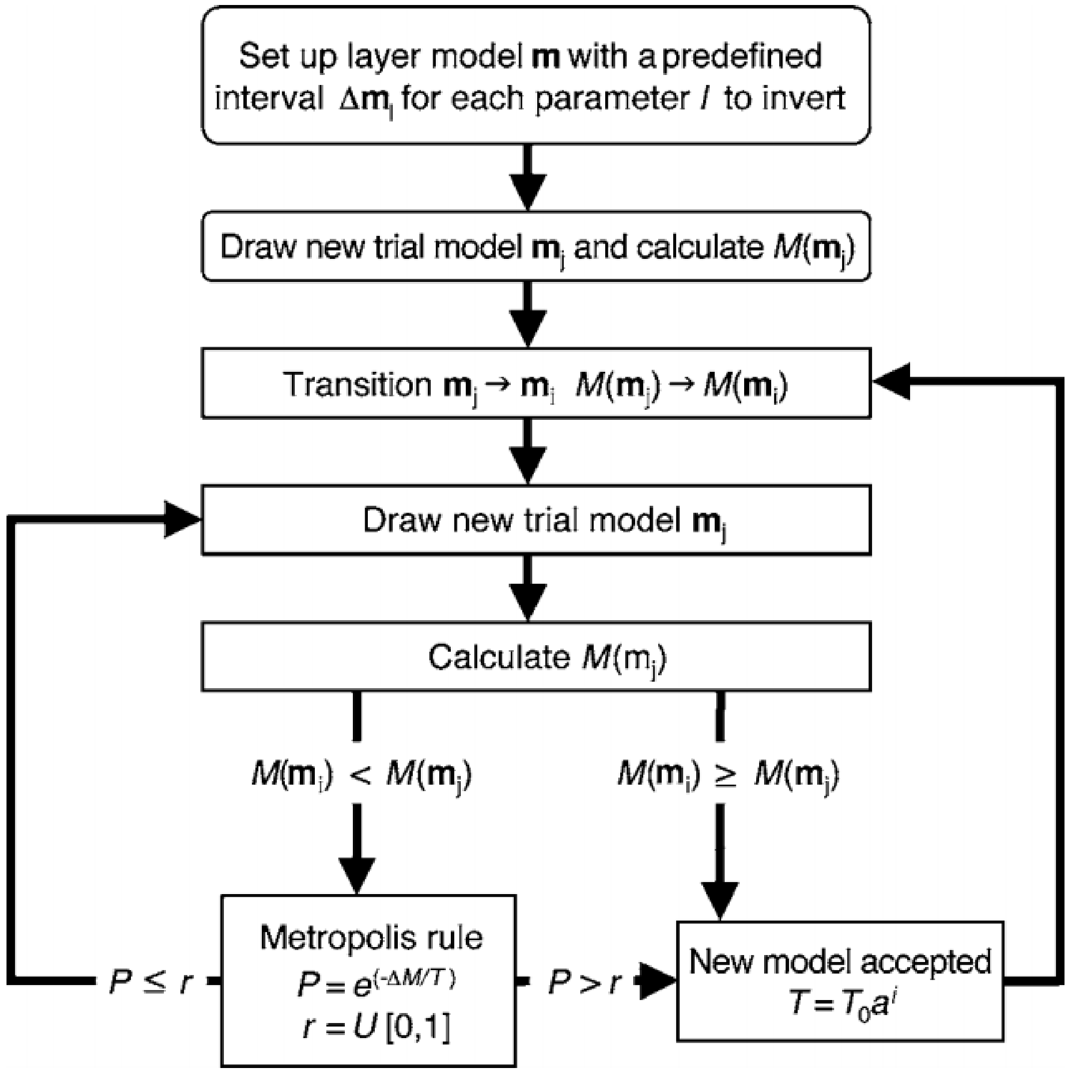


FIG. 15. Workflow of Simulated Annealing.

14). The error of the inversion results mainly results from two causes. The first is the error in fundamental mode picking in the dispersion spectrum. The picking is manual, and it has slight difference each time especially for the low frequency part. The second cause is the non-uniqueness of the inversion solution. The same dispersion curve can be generated by different velocity models. Thus, bounds or addition of initial information about the underground velocity can improve the inversion result, which makes the determination of accurate thickness a valuable research point in this method.

**global optimization methods**

Since the unknown input parameters here are only the S wave velocity of different layers and layer thickness and the model is 1D with only a few input parameters, global optimization methods like simulated annealing and Monte Carlo method using Markov Chain can also be adopted.

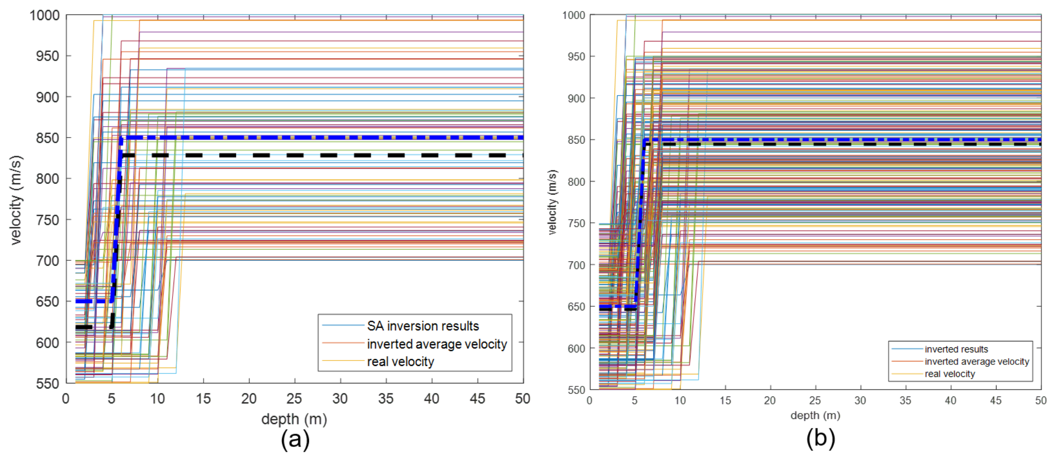


FIG. 16. Inversion results using Simulated Annealing with different temperature time and perturbation time. a. 100 temperature times and 50 perturbation times, b. 200 temperature times and 50 perturbation times.

Due to time limit, a simple two-layered model test using the Simulated Annealing method result is shown in Figure 16. For the simulated annealing, the perturbative parameter

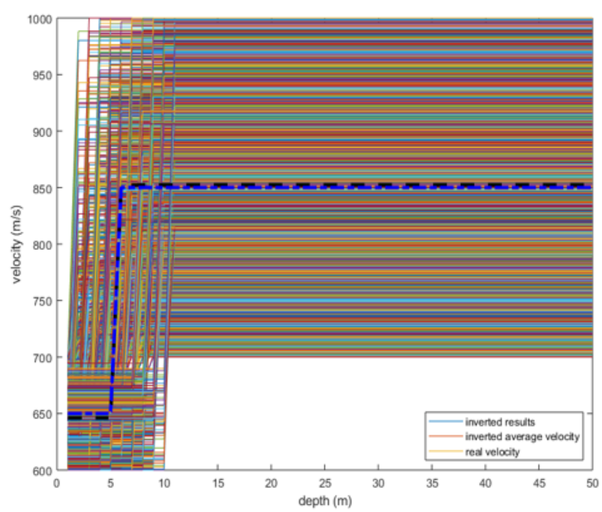


FIG. 17. Inversion results using Monte Carlo with Markov Chain.

here is the S wave velocity of each layer, and the layer thickness. As the total length in z direction is fixed, the unknown number of layer thickness is always one less than the unknown number of S wave velocity. This means that, if the model is two-layered, unknown parameters are three, S wave velocity of the first layer and second layer, and the first layer thickness. If the model is three layered, unknown parameters are five, including three S wave velocity and two layers' thickness. When the perturbative parameters are found, the other process is just like the common simulated annealing algorithm.

The work flow of simulated annealing inversion for surface wave is displayed in Figure 15. The basic process is defining a plausible range for all the unknown parameter, and perturbing the unknown parameters one by one, accepting the perturbed parameter with a probability depending on the comparison of forward modeling results or comparison of the "Energy".

Given plausible parameter range and enough testing times, an accurate result could be inverted. One parameter that still needs to be assumed is the layer number. This method is more applicable when layer number is small, otherwise the computation cost is very high.

The MCMC method has slight differences with the simulated annealing. The difference is that the Monte Carlo method is randomized sampling used for generating test cases while simulated annealing is a probabilistic metaheuristic for reaching a global minimum.

A simple two layered model inversion using the MCMC method is displayed in Figure 17.

The problem with global optimization methods is their computation cost. In the meantime, when the sampling number is not large enough, the results are influenced by the initial bounds determined. To get the optimum result often requires hours or days. The advantages of these methods are the layer thickness does not need to be assumed in advance. As in conventional inversion, layer thickness is a crucial factor influencing the accuracy of inversion result, simulated annealing or MCMC method is more advisable given enough time.

## CONCLUSIONS

The most important character of a surface wave in vertical heterogeneous media is velocity dispersion. Under certain circumstances, multimode propagation can be observed in a dispersion spectrum. To calculate the propagation velocity for different modes, we need to calculate the solution of a differential eigenproblem with certain boundary conditions for surface wave. For a particular frequency, only a few special wavenumber values have a nontrivial solution. The corresponding wavenumber is the eigenvalue for the matrix. When wavenumbers are found, the corresponding Rayleigh wave velocity, displacements could be calculated.

To invert for accurate S wave velocity, some initial conditions are required, like accurate thickness information. Otherwise, the result may not be satisfactory. When the inversion layer number is big, the derivative calculated may be spurious. Therefore, appropriate damping parameters and bounds should be added in the inversion process.

Using thinner layers can be a solution to missing knowledge of layer thickness, but the results may suffer from error generated by manual picking and nonunique solutions. Global optimization methods like Simulated Annealing and MCMC could be adopted since input parameters are limited under the layered model assumption, but they have the disadvantage of computation cost, and the layer number must also be assumed in advance.

MASW method is a robust method to characterize near surface area. However, some crucial problems need to be solved, like the determination of layer thickness, the resolution of lateral heterogeneity, picking of accurate fundamental mode dispersion curve, usages of the higher mode dispersion curves. Further study needs to be done in these aspects. One limitation for all these methods is that a layered model assumption is required. Thus, they can not deal with lateral heterogeneous media, which leads to the developments of other surface wave inversion methods like wave-equation dispersion inversion and full waveform inversion methods with appropriate misfit functions.

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