Compensating for attenuation by inverse Q filtering

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Motivation

- Assess and compare the different methods of applying inverse Q filter
- Use Q filter as a reference to assess phase restoration in Gabor deconvolution

Outline

- Anelastic attenuation: Constant Q model
- Inverse Q filter approaches
  - Inverting the Q matrix
  - Downward continuation
  - Pseudodifferential operators
- Comparison of the methods
- Conclusion

Attenuation mechanisms

(Margrave G. F., Methods of seismic data processing, 2002)

- Geometric spreading
- Absorption (Anelastic attenuation)
- Transmission losses
- Mode conversion
- Scattering
- Refraction at critical angles

Anelastic attenuation

- In real materials wave energy is absorbed due to internal friction
- Absorption is frequency dependent
- Absorption effects:
  - waveform change,
  - amplitude decay
  - phase delay
- The macroscopic effect of the internal friction is summarized by Q

Constant Q theory (Kjartansson, 1979)

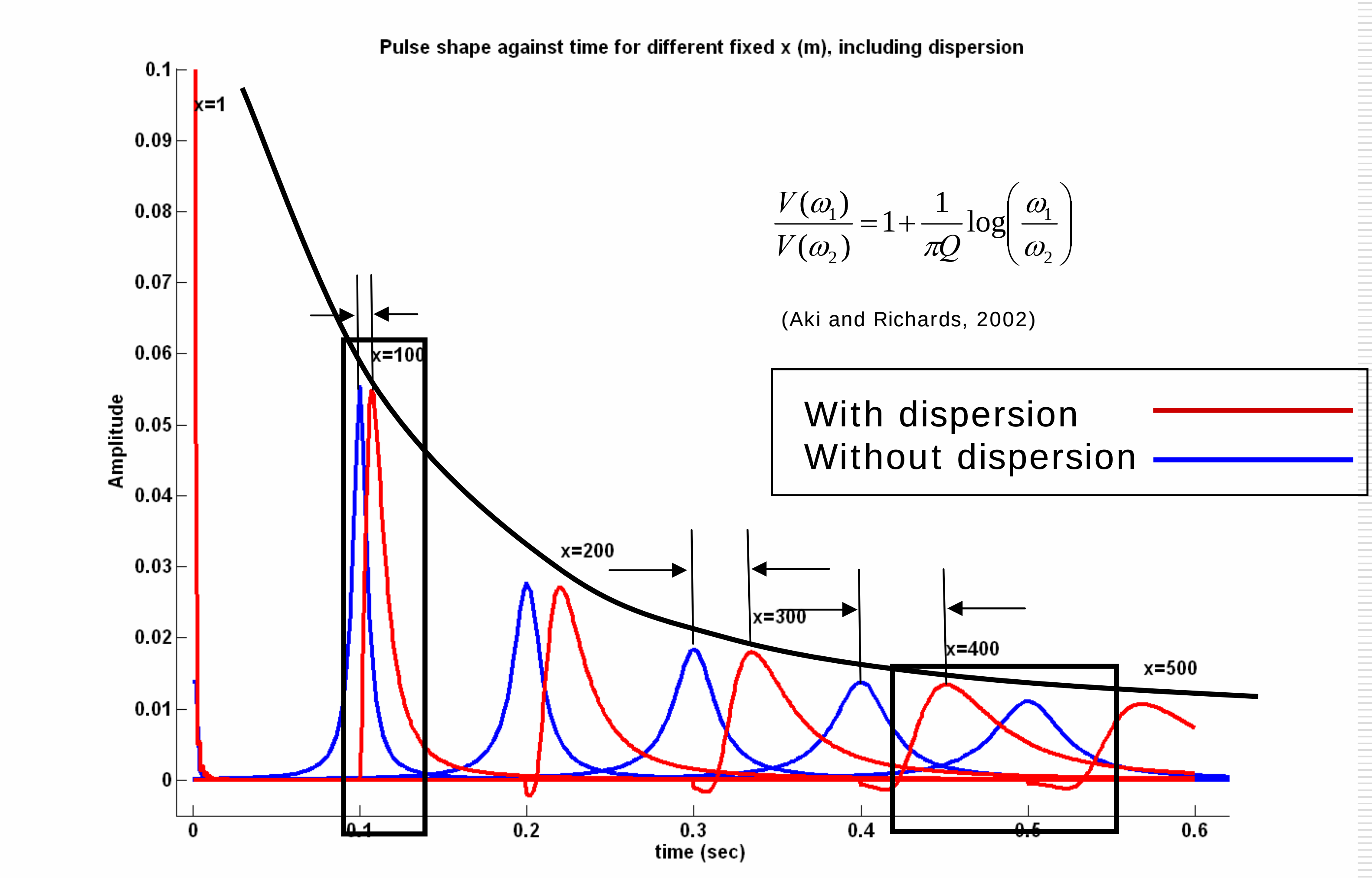
- Q is independent of frequency in the seismic bandwidth
- Absorption is linear (Q>10)
- Dispersion: each plane wave travels at a different velocity
- The Fourier transform of the impulse response is

Distance Attenuation Phase

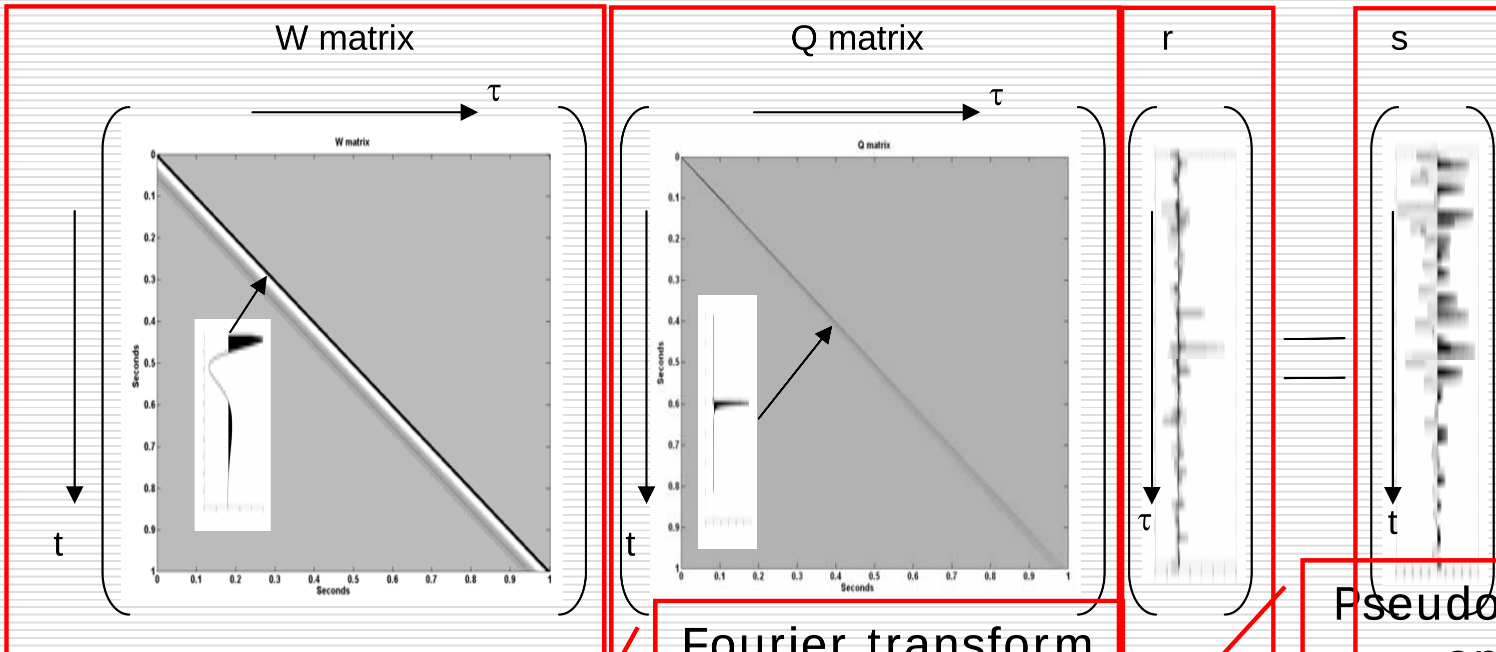
frequency Velocity

$$B(f) = \exp\left[-\frac{\pi f x}{VQ}\right] \exp\left[\frac{-2\pi i f x}{V}\right]$$

Impulse response (Q=10)



Nonstationary convolutional model



Pseudodifferential operator

Fourier transform source wavelet

Fourier transform attenuated signal

$$\hat{s}(f) = \hat{w}(f)\int_{-\infty}^{\infty}\alpha_Q(f,\tau)\,r(\tau)\,e^{-2\pi i f\tau}\,d\tau$$

$$\alpha_Q(\omega,\tau) = \exp\left(-\omega\tau/2Q + iH(\omega\tau/2Q)\right)$$

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Q filter and spiking deconvolution

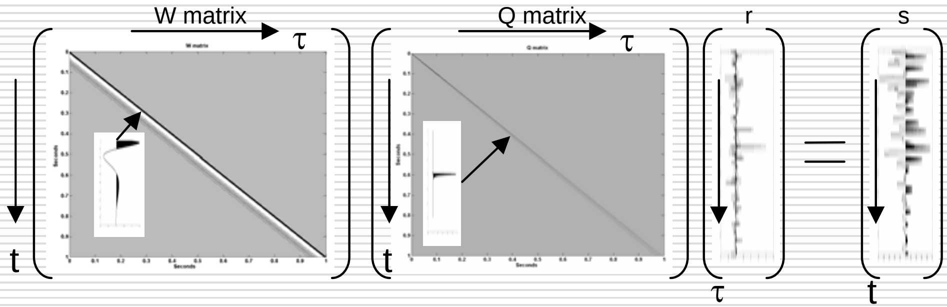
$$s = WQr$$

Exact inversion

$$r = Q^{-1}W^{-1}s$$

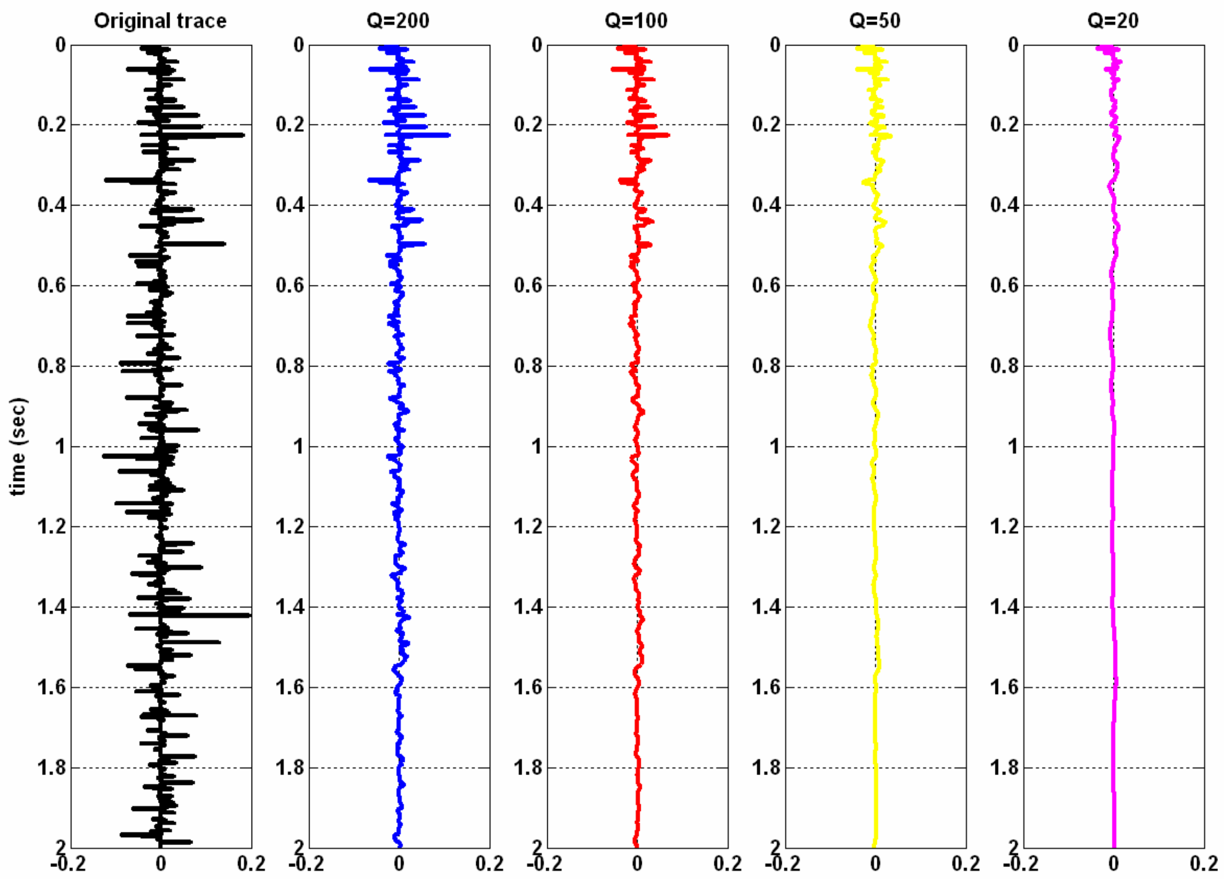
Approximate inversion

$$r \approx W^{-1}Q^{-1}s$$



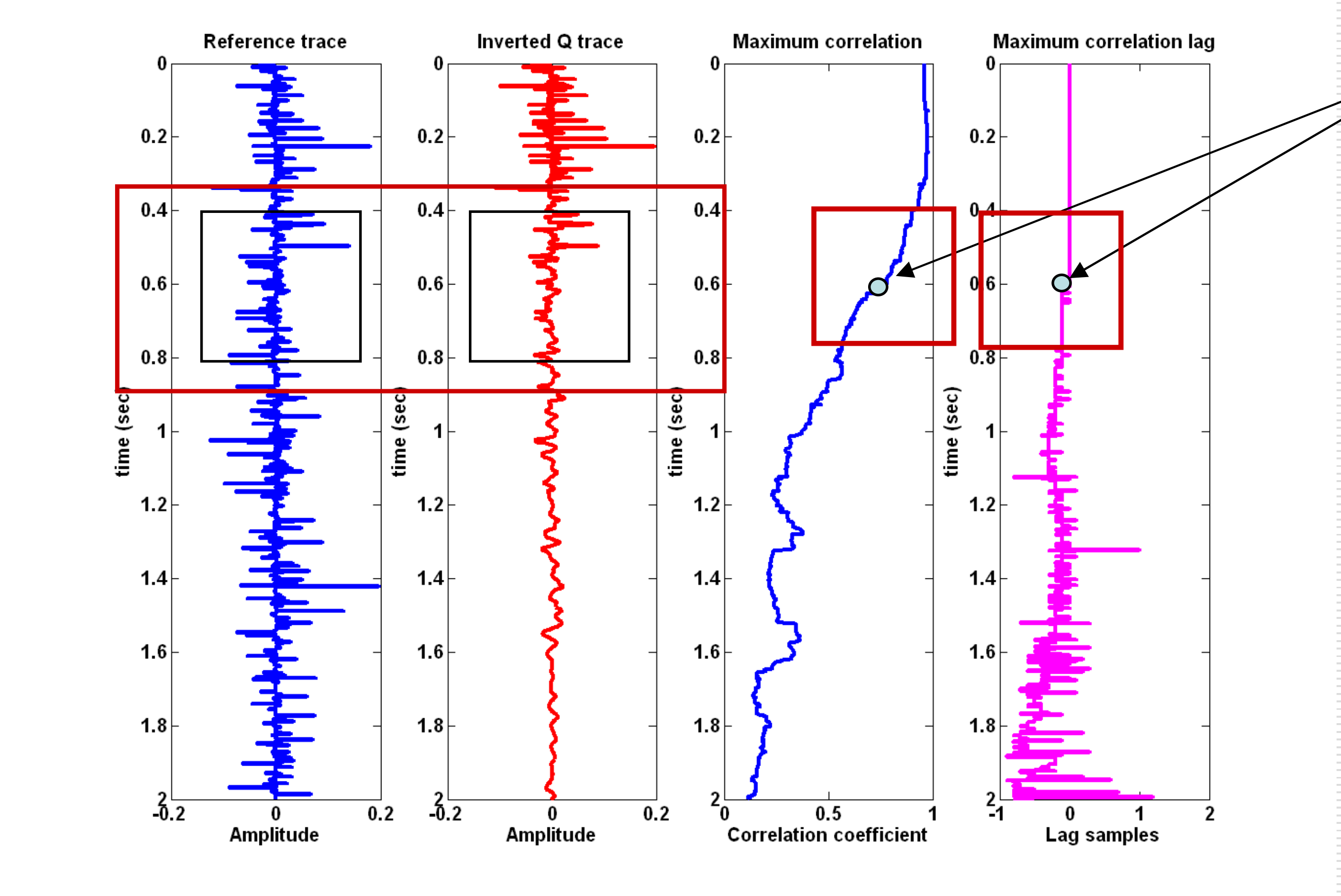
$$r = W^{-1}Q^{-1}s + \left[Q^{-1}, W^{-1}\right]s$$

Forward modeled traces

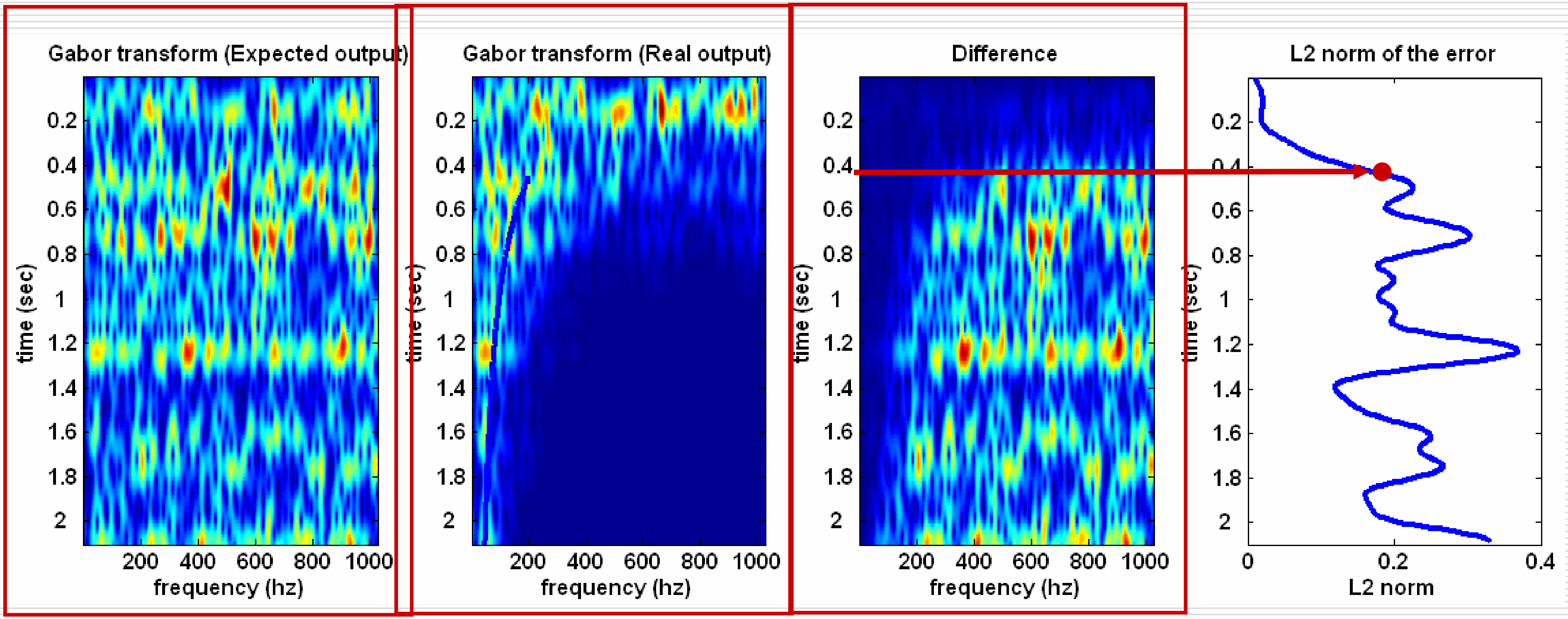


$s = WQr$ $\boxed{(W = I)}$ $\boxed{s = Qr}$

Error estimation: Crosscorrelation



Error estimation: L2 norm of error



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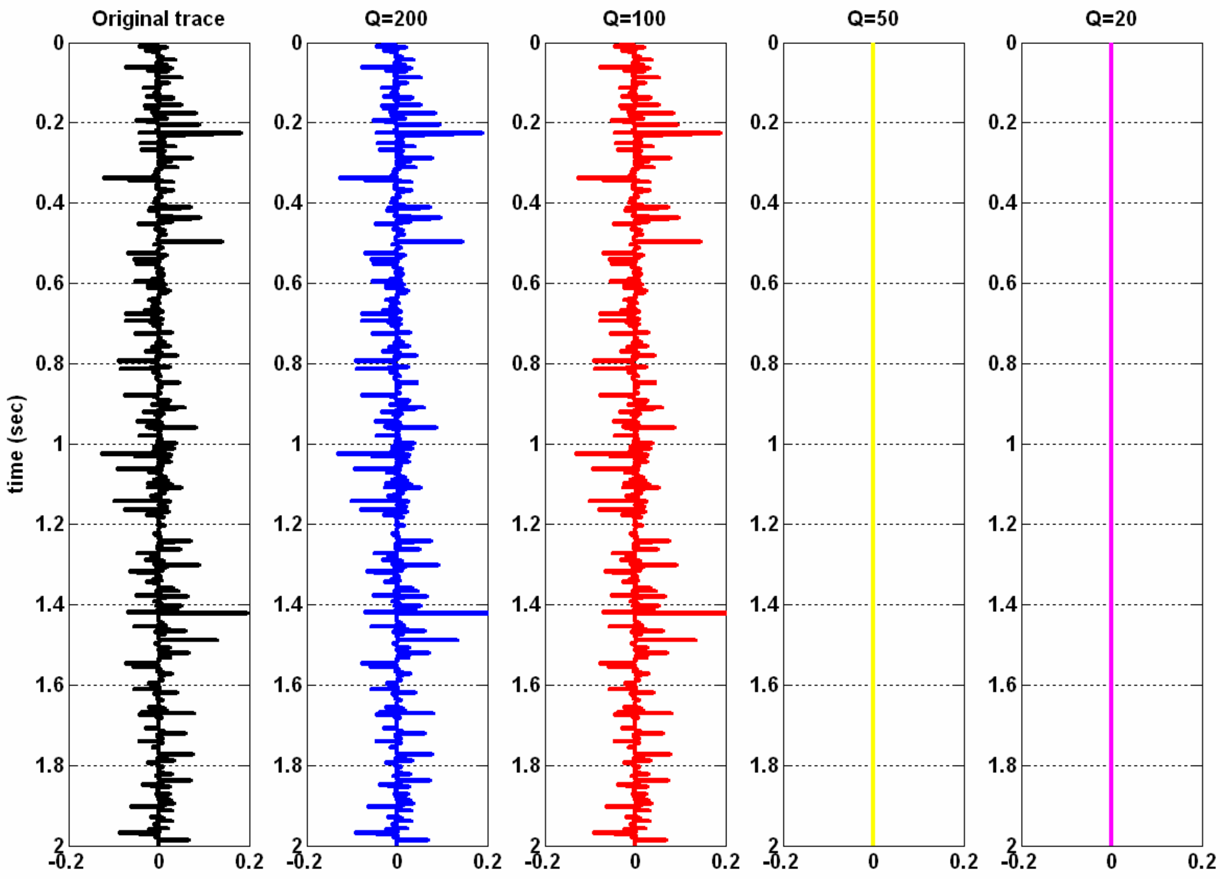
Inverting the Q matrix



$$r \approx W^{-1} Q^{-1} s$$

$$r \approx Q^{-1} s$$

A conventional matrix inversion algorithm gets unstable for Q<70

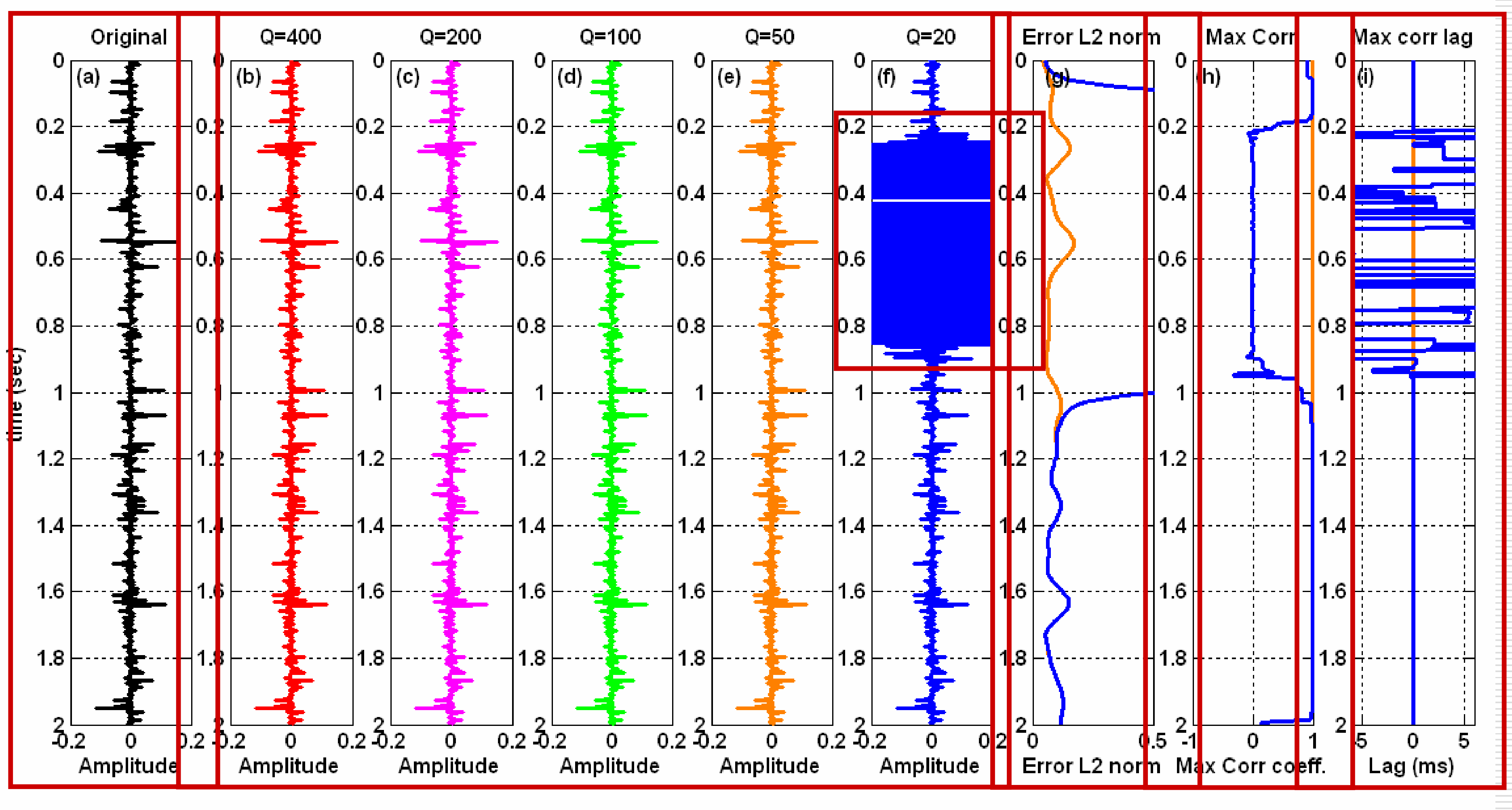
Hale's inversion matrix, Hale (1981)

$$r \approx W^{-1}(PQ)^{-1}Ps$$

Q^{-1}

- Each column of P is the convolution inverse of the corresponding column of Q.

Hale’s inversion matrix



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Downward continuation inverse Q filter, Wang (2001)

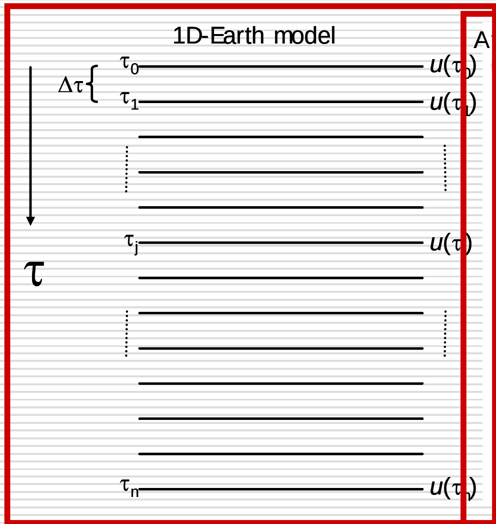
$$\frac{\partial^2 U(z,\omega)}{\partial z^2} + k^2 U(z,\omega) = 0$$

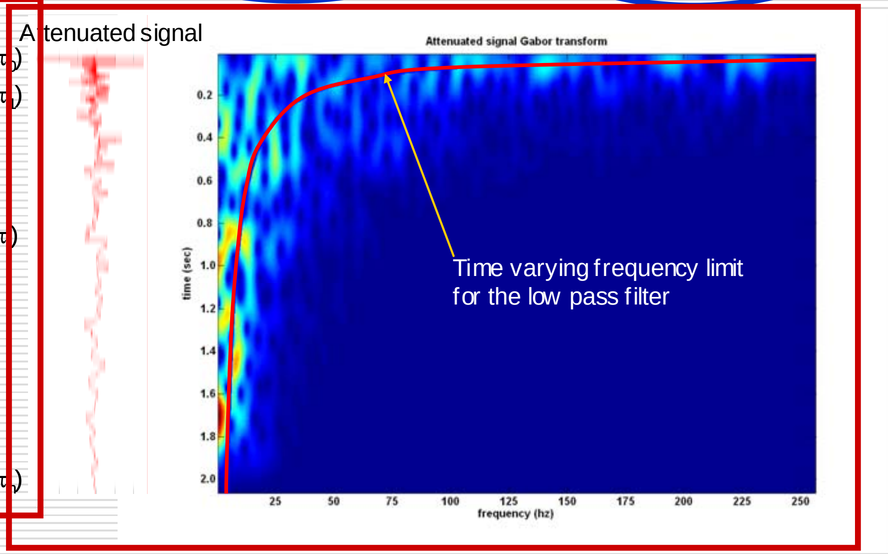
$$U(t+\Delta t) = \frac{1}{2\pi}\int U(t+\Delta t,\omega)\,d\omega$$

Phase correction

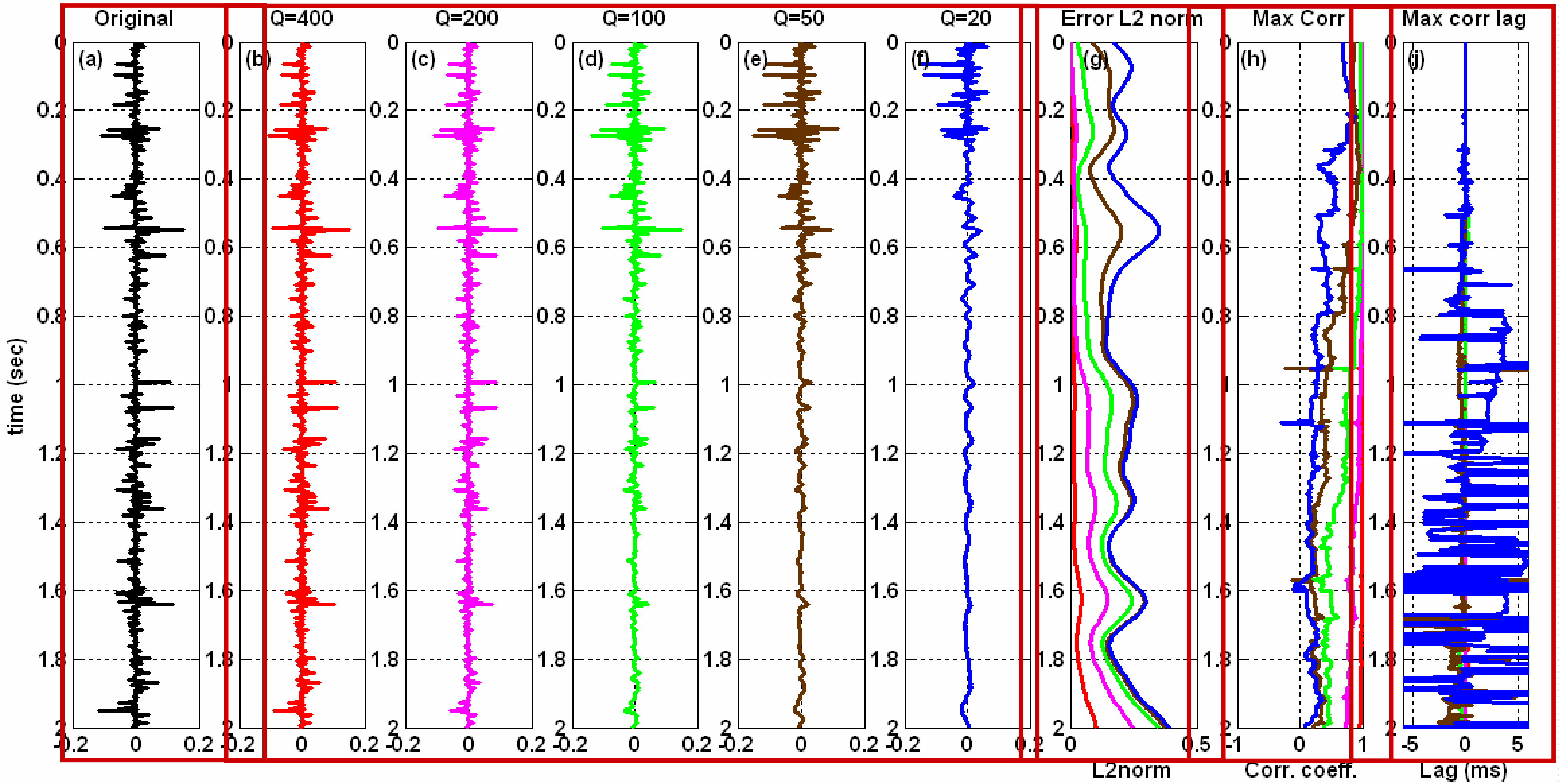
Amplitude correction

$$U(t+\Delta t,\omega) = U(t,\omega)\exp\left(\frac{i\omega V(\omega_0)}{V(\omega)}\Delta t\right)\exp\left(\frac{\omega V(\omega_0)}{2QV(\omega)}\Delta t\right)$$





Downward continuation inverse Q filter



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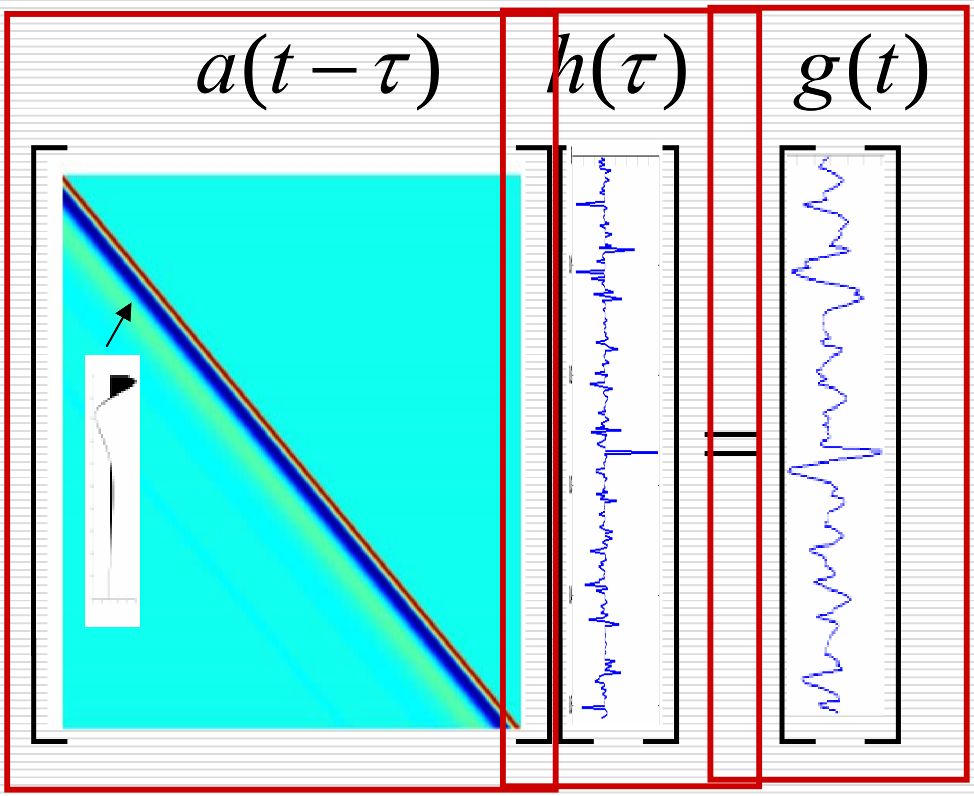
Inverse Q filtering by pseudo-differential operators, Margrave (1998)

Stationary linear filter theory

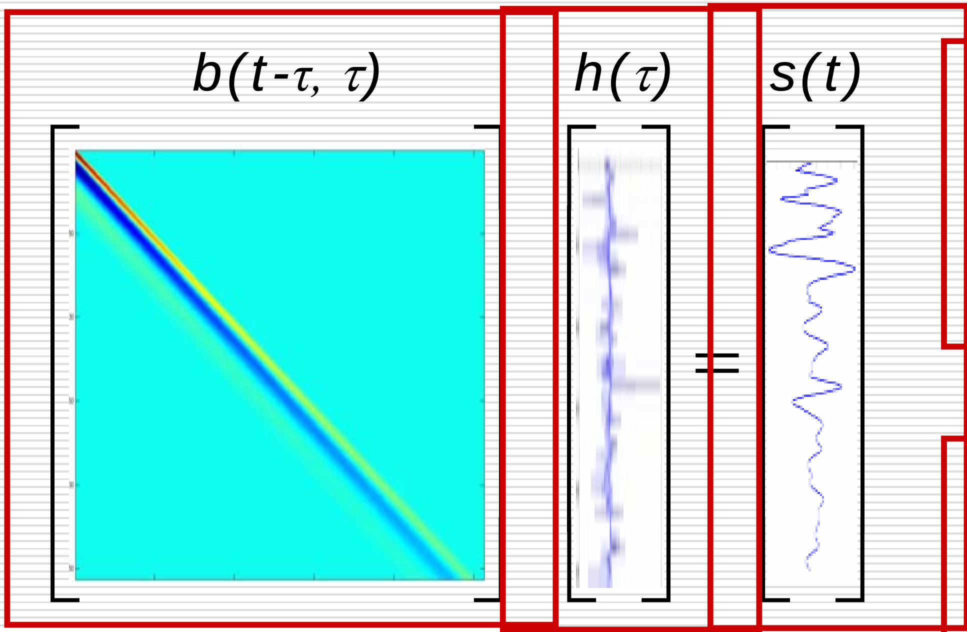


$$g(t) = \int_{-\infty}^{\infty} a(t-\tau)h(\tau)d\tau$$

In a stationary linear filter the output can be obtained by convolving an arbitrary input with the impulse response

Inverse Q filtering by pseudodifferential operators

Nonstationary linear filter theory



Nonstationary convolution

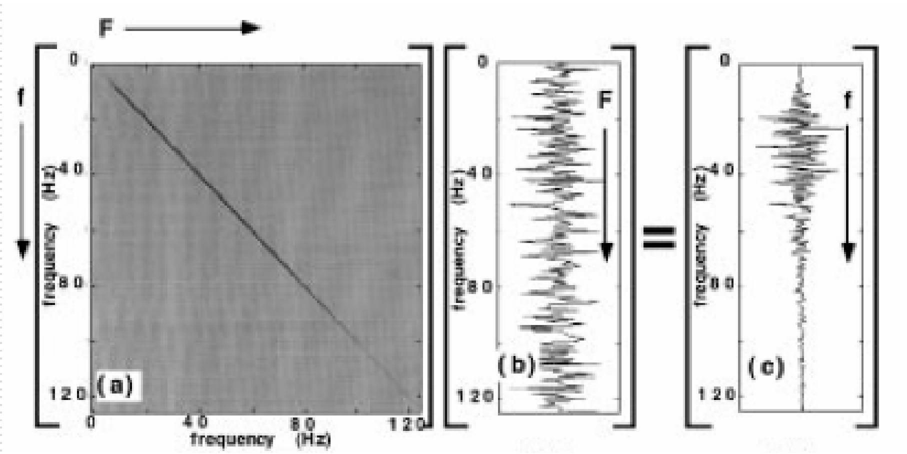
$$s(t) = \int_{-\infty}^{\infty} b(t-\tau,\tau)h(\tau)d\tau$$

Nonstationary combination

$$\tilde{s}(t) = \int_{-\infty}^{\infty} b(t-\tau,t)h(\tau)d\tau$$

Inverse Q filtering by pseudodifferential operators

Nonstationary convolution and combination in the frequency domain



From Margrave (1998)

Nonstationary convolution

$$S(\omega) = \int_{-\infty}^{\infty} B(\omega - \Omega, \Omega) H(\Omega) d\Omega$$

Nonstationary combination

$$\tilde{S}(\omega) = \int_{-\infty}^{\infty} B(\omega - \Omega, \omega) H(\omega) d\Omega$$

These integrals are the nonstationary extension of the convolution theorem

Inverse Q filtering by pseudodifferential operators

$$S(\omega) = \int_{-\infty}^{\infty} \beta(\omega,\tau) h(\tau) e^{-i\omega\tau} d\tau$$

Generalized Fourier integral (direct)

Anti-standard pseudo-differential operator

Nonstationary convolution in mixed domains

$$\tilde{s}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \beta(\Omega,t) H(\Omega) e^{i\Omega t} d\Omega$$

Generalized Fourier integral (inverse)

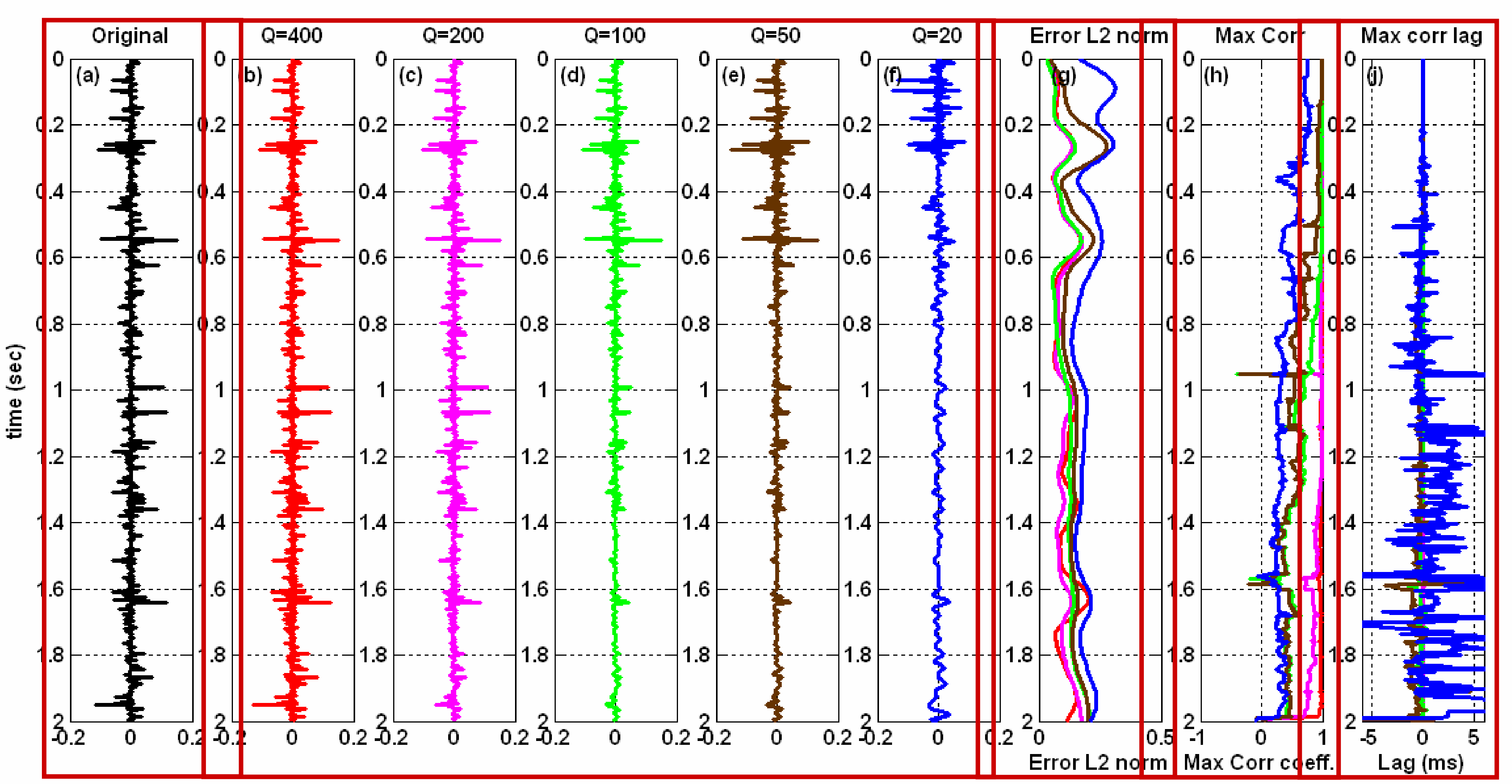
Standard Pseudo-differential operator

Nonstationary combination in mixed domains

Forward Q filter: $\beta(\omega,\tau) = \alpha_Q(\omega,\tau) = \exp\left(-\omega\tau/2Q + iH(\omega\tau/2Q)\right)$

Inverse Q filter: $\beta(\omega,\tau) \approx \alpha_Q^{-1}(\omega,\tau) = \exp\left(\omega\tau/2Q - iH(\omega\tau/2Q)\right)$

Inverse Q filtering by using pseudodifferential operators



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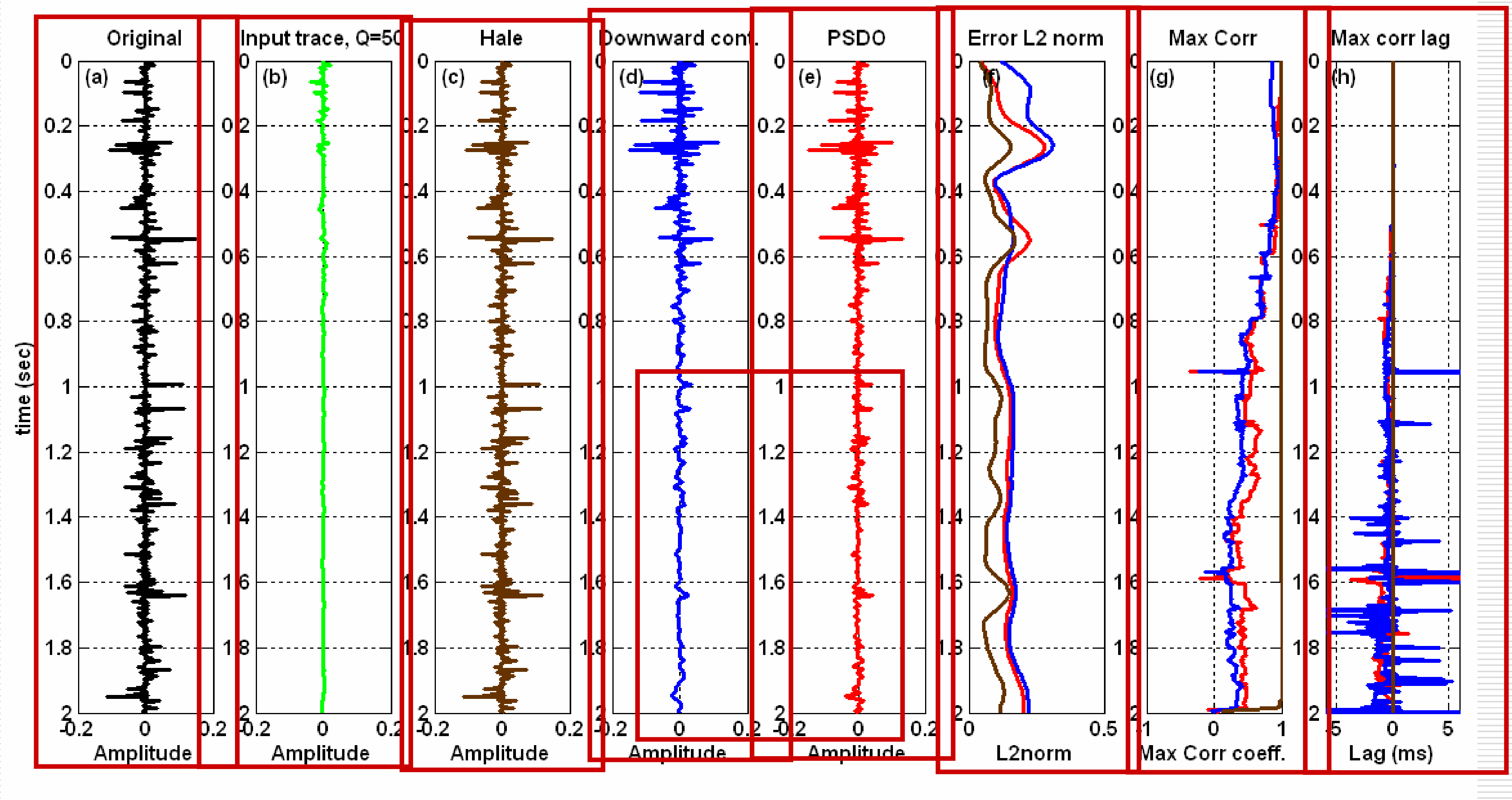
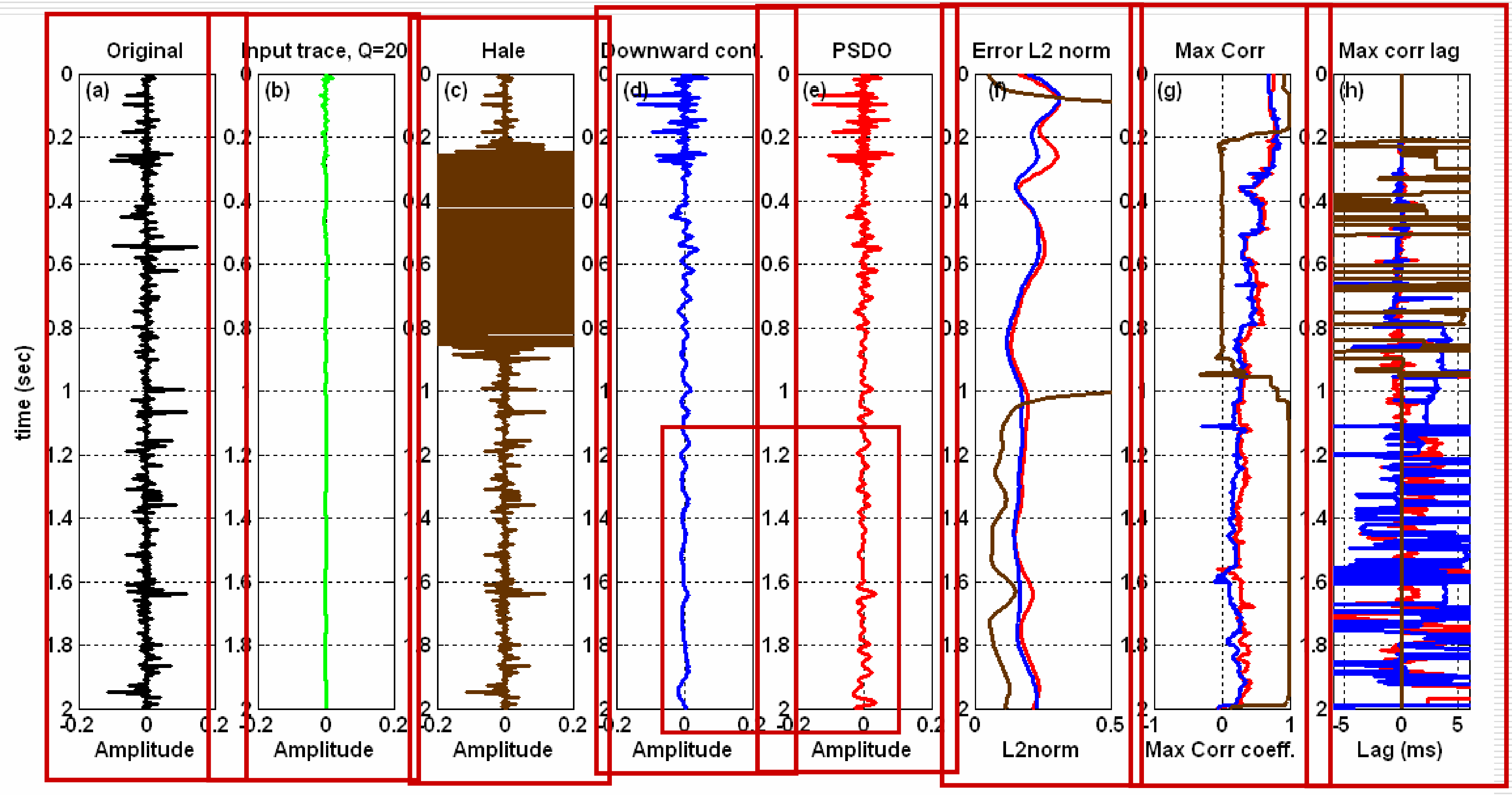
Comparison for Q=50



Comparison for Q=20



Conclusions

	Amplitude recovery	Phase recovery	Numerical stability	Computat. cost
Hale's matrix inversion	A+ (Q>40) D (Q<40)	A+ (Q>40) D (Q<40)	A+ (Q>40) D (Q<40)	C
Downward continuation	C	A-	A+	A+
Pseudo-differential operator	C+	A	A+	A

Future work

- Consider as variables
  - Uncertainty in Q estimation
  - Variations of Q with depth
  - Noise
- Improve amplitude recovery and efficiency in pseudodifferential operator Q filtering

References

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