“Exploring” spherical-wave reflection coefficients

Chuck Ursenbach
Arnim Haase

Research Report: Ursenbach & Haase, “An efficient method for calculating spherical-wave reflection coefficients”

Outline

- Motivation: Why spherical waves?
- Theory: How to calculate efficiently
- Application: Testing exponential wavelet
- Analysis: What does the calculation “look” like?
- Deliverable: The Explorer
- Future Work: Possible directions

Motivation

- Spherical wave effects have been shown to be significant near critical angles, even at considerable depth

See poster: Haase & Ursenbach, “Spherical wave AVO-modelling in elastic isotropic media”

- Spherical-wave AVO is thus important for long-offset AVO, and for extraction of density information

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Spherical Wave Theory

- One obtains the potential from integral over all *p:*

$$\phi_{PP}(\omega) = Ai\omega \exp(-i\omega t)\int_0^\infty \boxed{R_{PP}(p)}\, J_0(\omega pr)\exp[i\omega\xi(h+z)]\frac{p}{\xi}\,dp$$

- Computing the gradient yields displacements

$$u(\omega) = \nabla\phi(\omega)$$

- Integrate over all frequencies to obtain trace

$$u(t) = \int_{-\infty}^{\infty} f(\omega)u(\omega)\,d\omega$$

- Extract AVO information: $R_{PP}^{\text{spherical}}$

  Hilbert transform → envelope; Max. amplitude; Normalize

Alternative calculation route

$\phi_{\mathrm{PP}}(\omega)$

$\downarrow$ gradient

$u(\omega)$

$\downarrow$ p-integration

$u[\{\omega_i\}]$

$\downarrow$ inverse FFT

$u(t)$

$\downarrow$ Hilbert transform

$|u(t)+iH[u(t)]|$

$\downarrow$ locate maximum

envelope$[u(t=t_{\max})]$

$\downarrow$ normalize

R_{PP}

$$\int_0^\infty R_{\mathrm{PP}}\left[\int_0^\infty f(\omega)i\omega\,\mathrm{e}^{-i\omega t}\,J_0(\omega pr)\,\mathrm{e}^{i\omega\xi(h+z)}\,d\omega\right]\frac{p}{\xi}\,dp$$

$\downarrow$ analytic for $f(\omega)=\omega^n\exp(-|s\omega|)$

$$\int_0^\infty R_{\mathrm{PP}}\,I(p,t)\frac{p}{\xi}\,dp$$

$\downarrow$ gradient

$$\int_0^\infty R_{\mathrm{PP}}\frac{\partial I(p,t)}{\partial R}\frac{p}{\xi}\,dp$$

$\downarrow$ $t=R/\alpha;$ normalize with $R_{\mathrm{PP}}=1$

$$\int_0^\infty R_{\mathrm{PP}}\,W_n(p)\frac{p}{\xi}\,dp$$

$\downarrow$ numerical integration

$R_{\mathrm{PP}}^{\mathrm{spherical}}$

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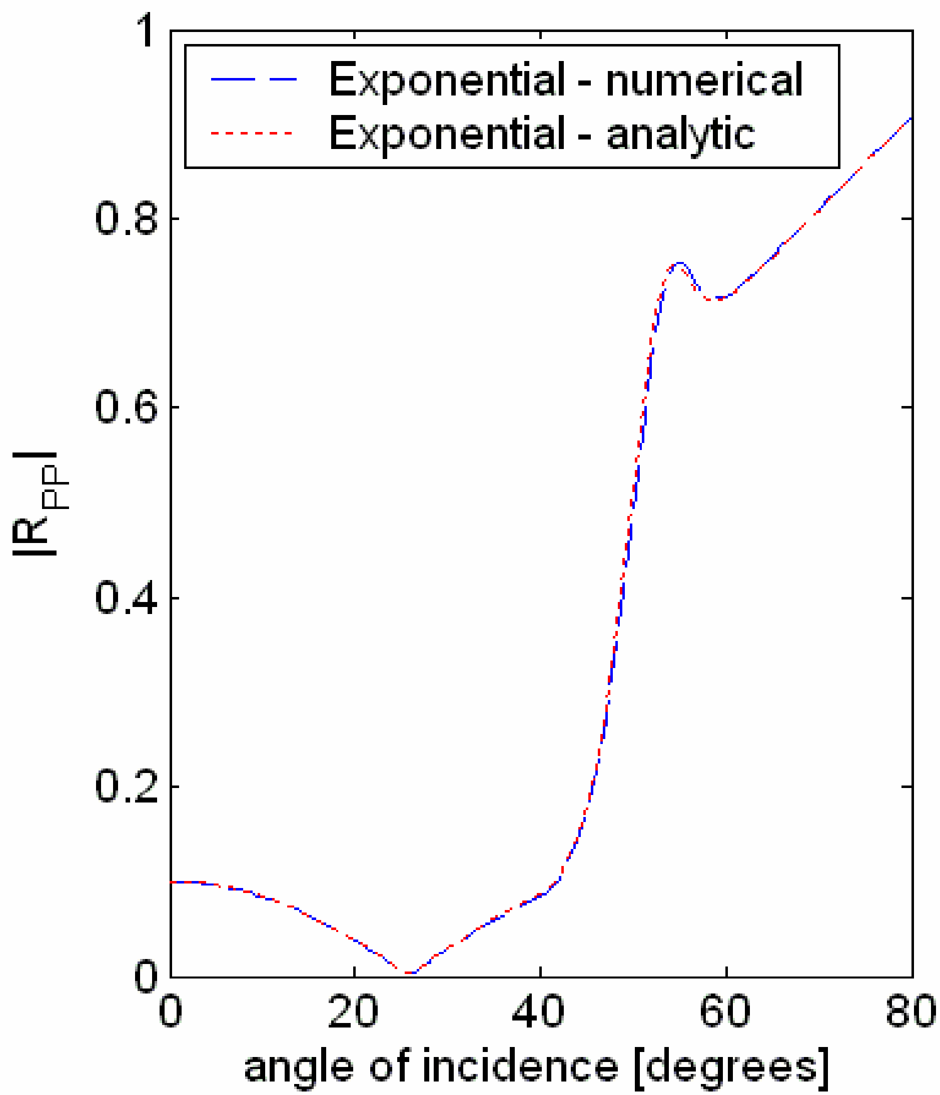
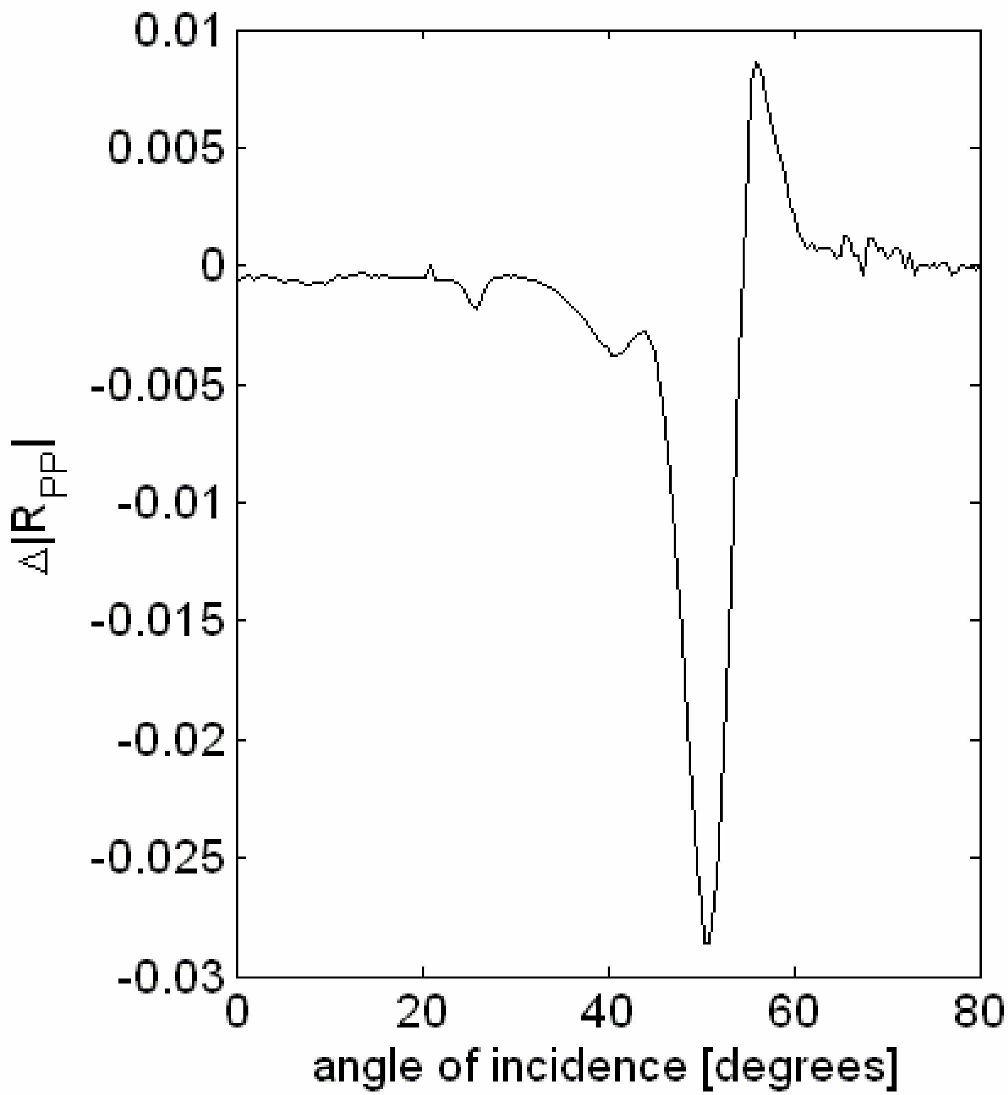
Class I AVO application

- Values given by Haase (CSEG, 2004; SEG, 2004)
- Possesses critical point at ~ 43°

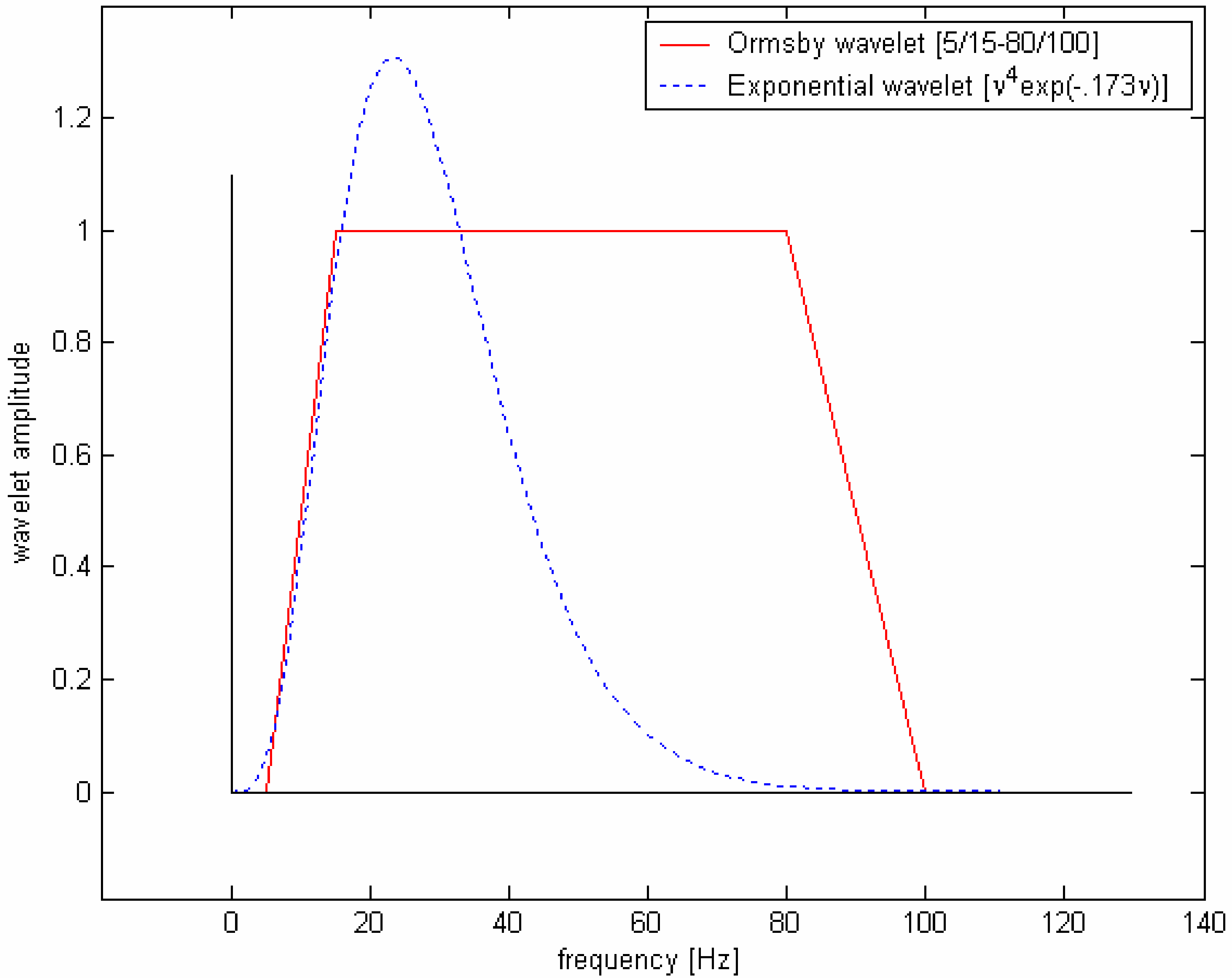
	Upper Layer	Lower Layer
V_P (m/s)	2000	2933.33
V_S (m/s)	879.88	1882.29
ρ (kg/m^3)	2400	2000

Test of method for $f(\nu) = \nu^4 \exp(-[.173\ \mathrm{Hz}^{-1}]\nu)$

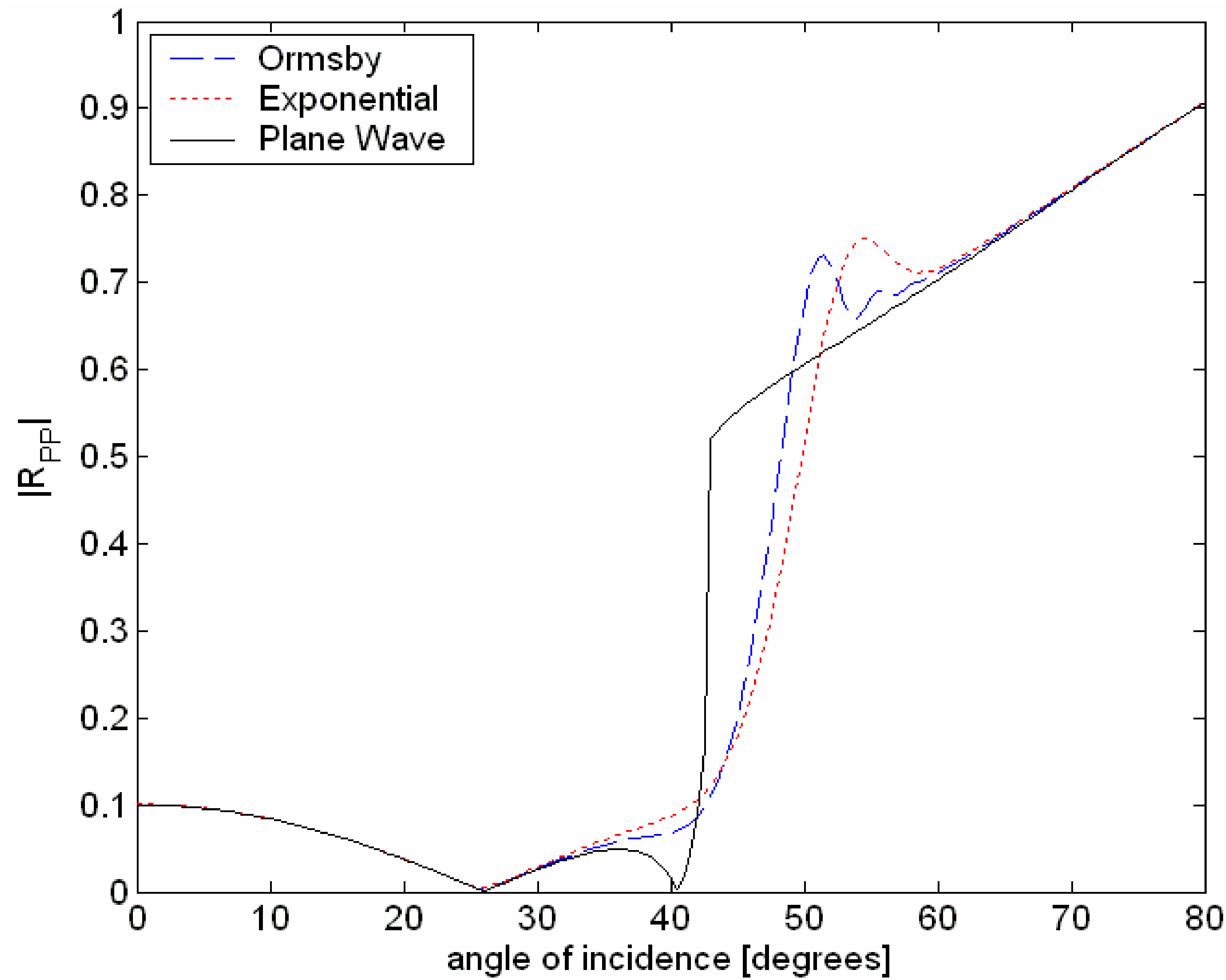




Wavelet comparison



Spherical R_{PP} for differing wavelets



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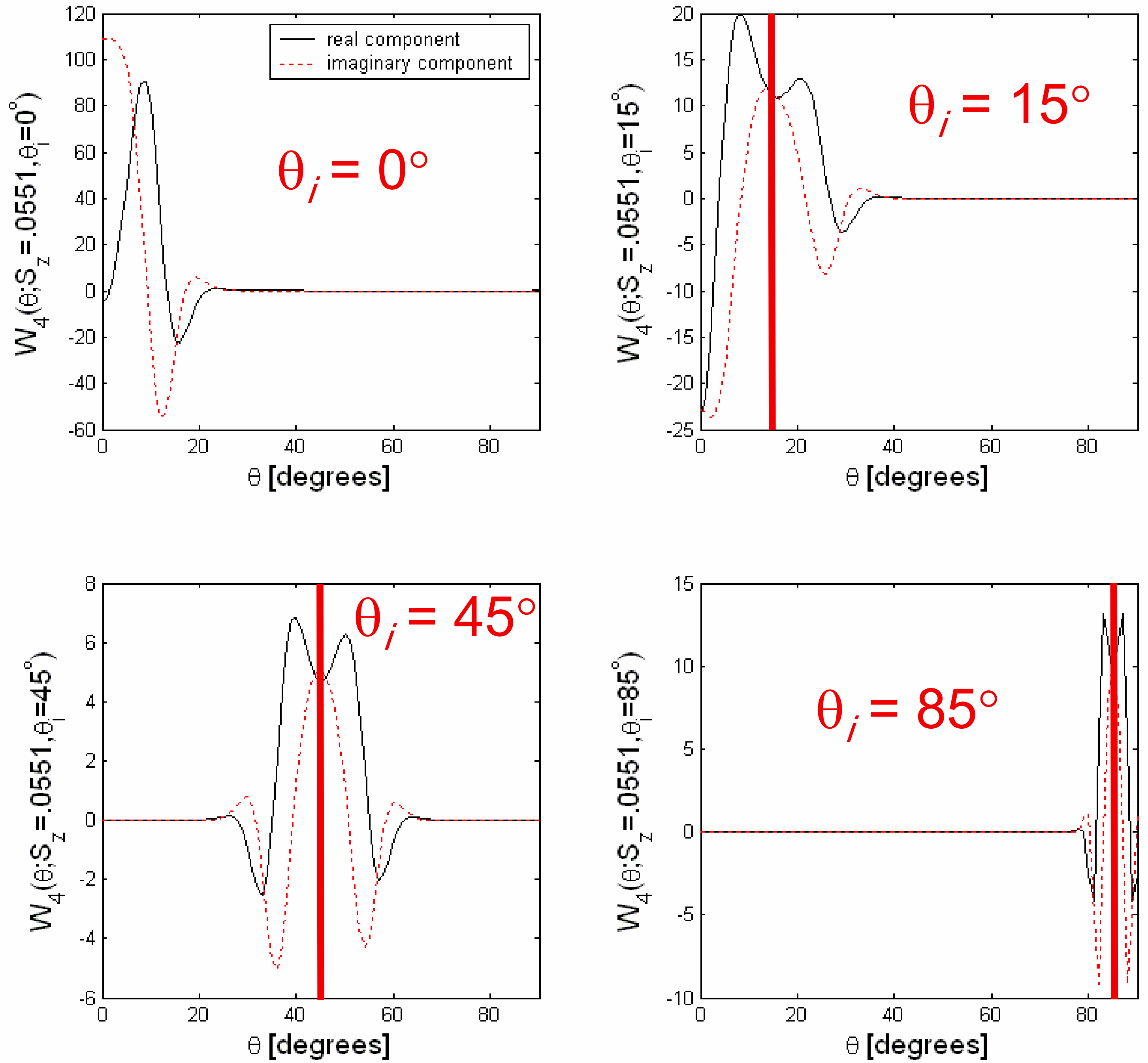
Behavior of W_n

$$R_{\mathrm{PP}}^{\mathrm{spherical}} =$$

$$\int_0^\infty R_{\mathrm{PP}} W_n(p) \frac{p}{\xi} dp,$$

$$p = \frac{\sin\theta}{\alpha}$$



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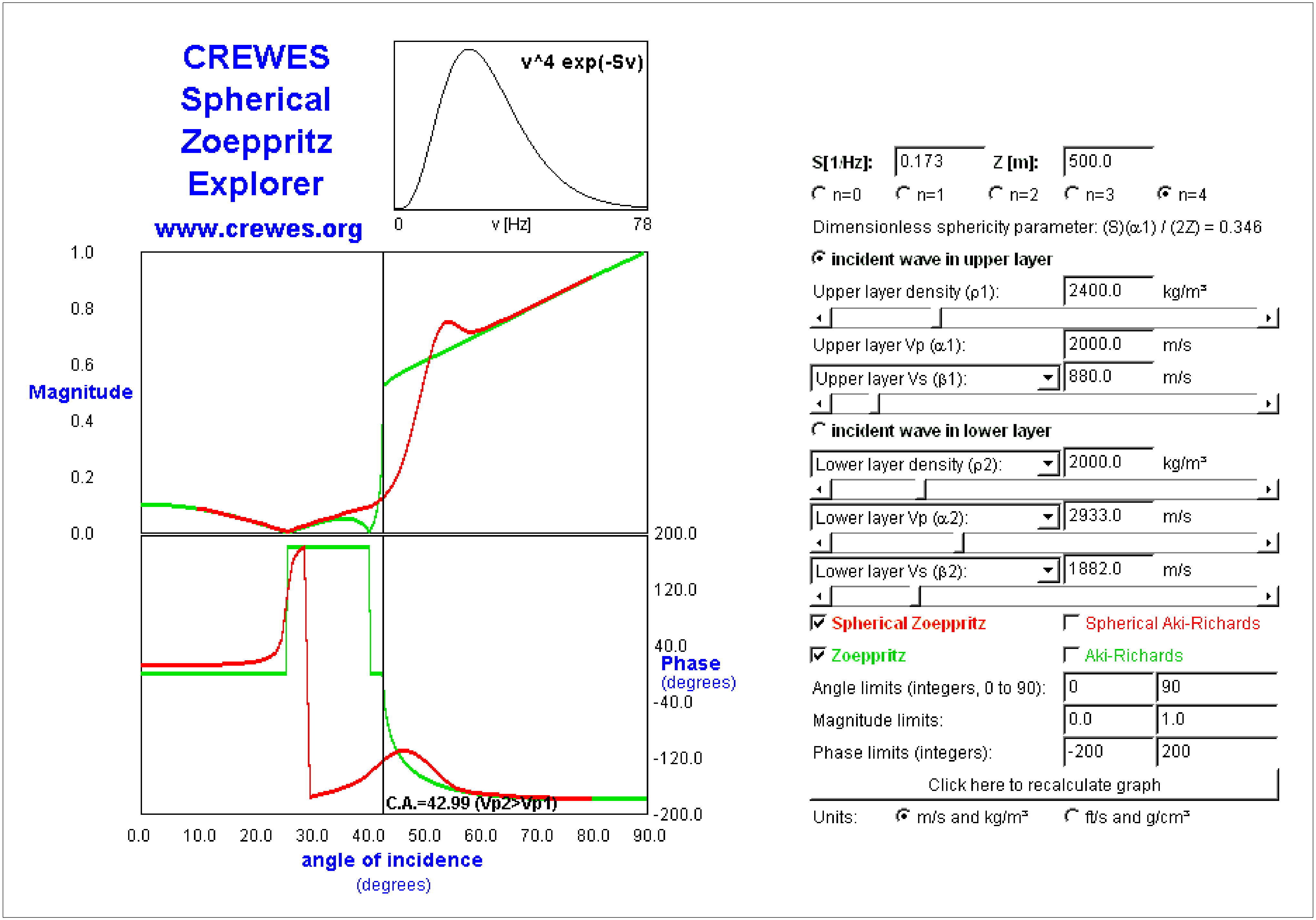
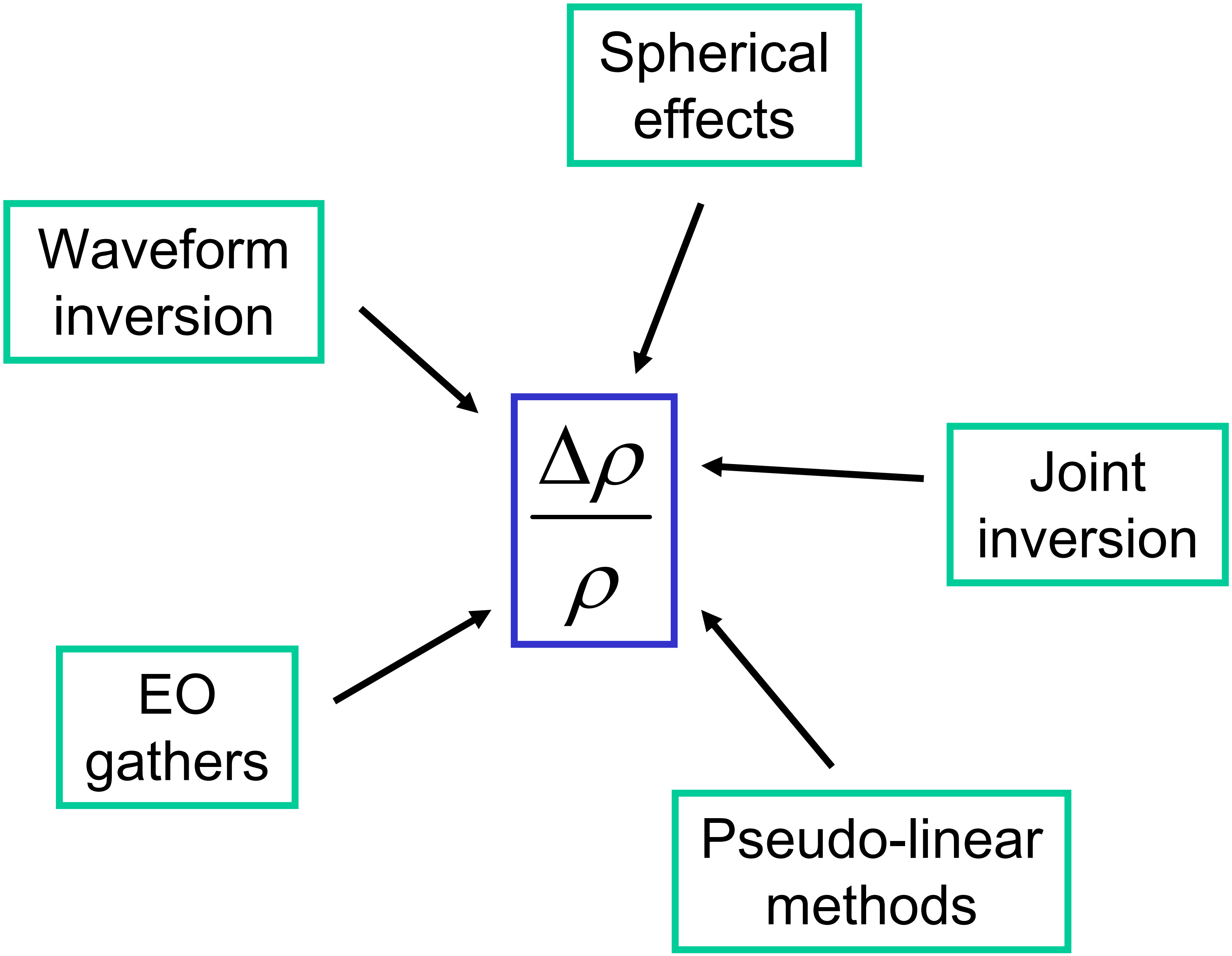

CREWES
Spherical
Zoeppritz
Explorer
www.crewes.org
v^4 exp(-Sv)
0
v [Hz]
78
1.0
0.8
0.6
Magnitude
0.4
0.2
0.0
200.0
120.0
40.0
Phase
(degrees)
-40.0
-120.0
-200.0
C.A.=42.99 (Vp2>Vp1)
0.0 10.0 20.0 30.0 40.0 50.0 60.0 70.0 80.0 90.0
angle of incidence
(degrees)
S[1/Hz]: 0.173
Z [m]: 500.0
n=0 n=1 n=2 n=3 n=4
Dimensionless sphericity parameter: (S)(α1) / (2Z) = 0.346
incident wave in upper layer
Upper layer density (ρ1): 2400.0 kg/m³
Upper layer Vp (α1): 2000.0 m/s
Upper layer Vs (β1): 880.0 m/s
incident wave in lower layer
Lower layer density (ρ2): 2000.0 kg/m³
Lower layer Vp (α2): 2933.0 m/s
Lower layer Vs (β2): 1882.0 m/s
Spherical Zoeppritz
Spherical Aki-Richards
Zoeppritz
Aki-Richards
Angle limits (integers, 0 to 90): 0 90
Magnitude limits: 0.0 1.0
Phase limits (integers): -200 200
Click here to recalculate graph
Units: m/s and kg/m³ ft/s and g/cm³


Spherical effects
Waveform inversion
$\frac{\Delta\rho}{\rho}$
Joint inversion
EO gathers
Pseudo-linear methods

Conclusions

- R_{PP}^{sph} can be calculated semi-analytically with appropriate choice of wavelet
- Spherical effects are qualitatively similar for wavelets with similar lower bounds
- New method emphasizes that R_{PP}^{sph} is a weighted integral of nearby R_{PP}^{pw}
- Calculations are efficient enough for incorporation into interactive explorer
- May help to extract density information from AVO

Possible Future Work

- Include $n > 4$
- Use multi-term wavelet: $\Sigma_n A_n \omega^n \exp(-|s_n \omega|)$
- Layered overburden
  (effective depth, non-sphericity)
- Include cylindrical wave reflection coefficients
- Extend to PS reflections

Acknowledgments

The authors wish to thank the sponsors of CREWES for financial support of this research, and Dr. E. Krebes for careful review of the manuscript.