Gabor deconvolution, Hilbert transform and Phase corrections

Carlos A. Montaña and Gary F. Margrave

CREWES
Overview

- Gabor deconvolution is a nonstationary extension of Wiener deconvolution.
- Minimum phase and Hilbert transform are fundamental concepts in the Gabor method.
- The estimation of the phase through the digital Hilbert transform can be improved by
  - Adding up a correction term to the digital Hilbert
  - Downsampling.
Outline

- Introduction: Gabor deconvolution
  - Constant Q theory
  - Nonstationary convolutional model
  - Gabor transform
- Minimum phase and Hilbert transform
- Phase correction in Gabor deconvolution
- Conclusions
Constant Q theory for attenuation

\[ B(f) = \exp \left[ -\frac{\pi fx}{V(f_0)\mathcal{Q}} \right] \exp \left[ -\frac{2\pi fx}{V(f)} \right] \]

\[ V(f) = V(f_0) \left[ 1 + \frac{1}{\pi \mathcal{Q}} \log \left( \frac{f}{f_0} \right) \right] \]

(With dispersion
Without dispersion)

(Aki and Richards, 2002)
Nonstationary convolutional model

\[ \hat{s}(f) = \hat{w}(f) \int_0^\infty \alpha_Q(f, \tau) r(\tau) e^{-2\pi if\tau} d\tau \]

\[ \alpha_Q(\omega, \tau) = \text{Fourier transform of the pseudo differential operator} \]
Gabor transform

\[ G[s](\tau, f) = \int_{-\infty}^{\infty} s(t) g(t - \tau) e^{-2\pi if t} \, dt \]

\[ s(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G[s](\tau, f) g(t - \tau) e^{2\pi if t} \, df \, d\tau \]
Gabor deconvolution

- Factorization of the nonstationary convolutional model

\[ \hat{s}(f) = \hat{w}(f) \int_{-\infty}^{\infty} \alpha_{Q}(f, \tau) r(\tau) e^{-2\pi if\tau} d\tau \]

\[ G[s](\tau, f) \approx \hat{w}(f) \alpha_{Q}(\tau, f) G[r](\tau, f) \]

- Fourier of the source wavelet
- Gabor of the trace
- Approximated factorization
- Attenuation function
- Gabor of the reflectivity

\[ \approx \]
Gabor deconvolution

Gabor transform of the seismic trace

Deconvolutional operator:
- smoothing Gabor of trace
- phase through Hilbert transform

Estimate of Gabor of reflectivity:
Gabor transform of the trace deconvolutional operator

Expected Gabor of the trace

Source wavelet removal and compensation for attenuation are simultaneous
Minimum phase, linearity, causality
and Hilbert transform.

- **Minimum phase wavelet:**
  - The wavelet with the minimum phase delay of all possible causal, invertible wavelets with the same amplitude spectrum
  - A wavelet whose Fourier phase spectrum is the Hilbert transform of the logarithm of its amplitude spectrum

Futterman (1962) showed that wave attenuation in a causal, linear theory is always minimum phase.
Analog Hilbert transform

$$\varphi_A(f) = H[\alpha(f)] = \frac{f}{V(f)} - \frac{f}{V_\infty}$$

$$V(f) = V_\infty \left[ 1 + \frac{1}{\pi Q} \log \left( \frac{f}{f_\infty} \right) \right]$$

- Depends explicitly on the attenuation parameter $Q$

and the velocity $V_\infty$ at a reference frequency $f_\infty$
Digital Hilbert transform

\[ \varphi_D(f) = \int_{-\infty}^{\infty} \frac{\ln|\sigma(f')|}{f-f'} \, df' \approx \sum_{f'=-Nyq}^{Nyq} \frac{\ln|\sigma(f')|}{f-f'} \Delta f \]

- Q does not appear explicitly in this formula
- Phase estimated from this expression is data-driven
Reference traces, $Q=100$
After standard Gabor
Phase computed using digital Hilbert transform

\[ \varphi(f) \approx \sum_{f'=-N_{yq}}^{N_{yq}} \frac{\ln|\sigma(f')|}{f - f'} \Delta f' \]
After Gabor + phase correction

\[ \varphi_c(t, f) = H\left(\ln|\sigma(\tau, f)|\right) + \frac{t}{Q}(a + bf + cf^2) \]
Analog vs. digital phase correction

\[ \varphi_2(t, f) = H \left( \ln |\sigma(t, f)| \right) + \frac{t}{Q} \left( a + b f + cf^2 \right) \]

\[ \varphi(f) = \frac{f}{V(f)} - \frac{f}{V_o} \]

Accurate Q
Analog vs. digital phase correction

\( \varphi(t, f) = H \left( \ln |\sigma(\tau, f)| \right) + \frac{t}{Q} \left( a + b f + c f^2 \right) \)

\( \varphi(f) = \frac{f}{V(f)} - \frac{f}{V_0} \)

Inaccurate

Q = 150
Phase through digital Hilbert + downsampling

$$\phi(f) \approx \sum_{f'=-Nyq}^{Nyq} \frac{\ln|\sigma(f')|}{f-f'} \Delta f'$$
Conclusions

- Gabor deconvolution is based on minimum phase assumptions, therefore it applies the Hilbert transform for computing the phase spectrum.

- A residual phase error remains after Gabor deconvolution is applied due to the inaccuracy of the digital implementation of the Hilbert transform.
Conclusions

- The error in the phase can be corrected by adding a correction term, linear in time and quadratic in frequency to the digital Hilbert transform.
Work in progress

- Phase correction without explicit knowledge of Q
- Consideration of nonwhite reflectivity
- Surface consistent Gabor deconvolution
Acknowledgements

- CREWES sponsors
- POTS1 sponsors
- Geology and Geophysics Department
- MITACS
- NSERC
- CSEG
Acknowledgements

- Mike Perz (Geo-X)
Questions?
Analog vs. digital Hilbert transform

\[ \Phi(f) = \frac{f}{V(f)} - \frac{f}{V_\infty} \]

\[ \Phi(f) = \sum_{\omega=\omega_0}^{\omega_n} \ln \left| \frac{\sigma(f')}{f' - f''} \Delta f' \right| \]

- **dt=2 ms, fnyq=250 Hz**
- **dt=0.08 ms, fniq=6000 Hz**
- **dt=0.04 ms, fniq=12500 Hz**
- **dt=0.025 ms, fniq=20000 Hz**