Locating microseismic events and traveltime mapping using locally spherical wavefronts

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Motivation

Traveltime grids
Motivation

Locate the center of curvature

Knowing three traveltimes

Traveltimes on raypaths to grids
Motivation

Well fracing

CO₂ injection
Motivation

Monitoring hazardous geological sites
The main point

• Given three receiver locations (2D)
• Traveltimes of an event at those receiver locations

1. Locate the source and time of the event
2. Compute another traveltime at a nearby location

• Later, four receivers for 3D
Vidale’s method (still plane waves)

\[
\left(\frac{dt}{dx}\right)^2 + \left(\frac{dt}{dx}\right)^2 = \frac{1}{v^2}
\]

\[
t_4 = t_1 + \sqrt{\frac{2h^2}{v^2} - (t_3 - t_2)^2}
\]

\[
(t_4 - t_1)^2 + (t_3 - t_2)^2 = \frac{2h^2}{v^2}
\]

Fast, OK accuracy, industry standard
Curved wavefront assumption

1. Locally constant velocity
2. Estimate center of curvature
3. Apparent source location \((x_0, z_0)\)
4. Source time \(t_0\)
5. From apparent source
   - get time \(t_4\)
Difference between travel times \((t_2 - t_1)\) and \((t_3 - t_1)\)

Loran C navigation system
Error in estimating $t_4$

Vidale’s solution

Iterative solution
Now, three arbitrarily located points

Clock times $t_0$, $t_1$, $t_2$, and $t_3$
Traveltime circles for $t_1$, $t_2$, and $t_3$ with $t_0 \neq 0$

Radius of clock times $t_1$, $t_2$, and $t_3$

They don’t help find the source location, yet
Three receivers

Delta times $t_{01}$, $t_{02}$, and $t_{03}$

Traveltime circles for $t_1$, $t_2$, and $t_3$ with $t_0 = 0$

Circles of $\Delta$ times

But we don’t know these $\Delta$ times
Traveltime circles for t1, t2, and t3 with t0 ≠ 0

Back to clock time \( t_3 = t_0 + t_{03} \)

Normal to the circle
Traveltime circles for $t_1$, $t_2$, and $t_3$ with $t_0 \neq 0$

These three arrows:

1. Have the same length $v t_0$
2. Are normal to the circles
3. Have the same origin $(x_0, z_0)$

A circle at $(x_0, z_0)$:
Will be tangent to the circles
1. This circle is tangent to the other three circles.
2. Its center is the source location \((x_0, z_0)\).
3. Its radius is \(vt_0\)

Our problem now becomes one of finding this circle that is tangent to three other circles.
Go to the internet and look up…

• A circle tangent to three other circles

• 1,610,000 responses
Go to the internet and look up...

- A circle tangent to three other circles
- 1,610,000 responses
Apollonius 262 BC to 190 BC

The Great Geometer

Introduced terms:
parabola
ellipse
hyperbola
The three-circle problem was solved by Viète (Boyer 1968), and the solutions are called Apollonius circles. There are eight total solutions. The simplest solution is obtained by solving the three simultaneous quadratic equations

\[(x - x_1)^2 + (y - y_1)^2 - (r \pm r_1)^2 = 0\]  
\[(x - x_2)^2 + (y - y_2)^2 - (r \pm r_2)^2 = 0\]  
\[(x - x_3)^2 + (y - y_3)^2 - (r \pm r_3)^2 = 0\]  

in the three unknowns $x, y, r$ for the eight triplets of signs (Courant and Robbins 1996). Expanding the equations gives

\[(x^2 + y^2 - r^2) - 2x x_i - 2y y_i \mp 2rr_i + (x_i^2 + y_i^2 - r_i^2) = 0\]  

for $i = 1, 2, 3$. Since the first term is the same for each equation, taking (2) – (1) and (3) – (1) gives

\[\alpha x + \beta y + \gamma r = d\]  
\[\alpha' x + \beta' y + \gamma' r = d',\]  

where

\[\alpha = 2(x_1 - x_2)\]  
\[\beta = 2(y_1 - y_2)\]  
\[\gamma = \pm 2(r_1 - r_2)\]  
\[d = (x_1^2 + y_1^2 - r_1^2) - (x_2^2 + y_2^2 - r_2^2)\]  

and similarly for $\alpha', \beta', \gamma'$ and $d'$ (where the 2 subscripts are replaced by 3s). Solving these two simultaneous linear equations gives

\[x = \frac{\beta' d - \beta d' - \beta' c r + \beta c' r}{\alpha \beta' - \beta \alpha'}\]  
\[y = \frac{-\alpha' d + \alpha d' + \alpha' c r - \alpha c' r}{\alpha \beta' - \alpha' \beta}\]  

which can then be plugged back into the quadratic equation (1) and solved using the quadratic formula.
It turns out...

• In general there are 8 possible solutions

• But our case is limited by the problem to only 2 solutions.
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• In general there are 8 possible solutions.

• But our case is limited by the problem to only 2 solutions.
3D solution

- Use the identical form as for the 2D solution
- Requires four receivers and their clock times
- Find one sphere tangent to four spheres
- Only two solutions
Comments on the clock time

• The clock times can be very large relative to the $\Delta$ times. (Difference of large numbers)

• Subtract the minimum clock time from all other clock times:
  – simpler solutions
  – 2D: two circles and one point
  – 3D: three spheres and one point
  – $t_0$ will be negative
Final words

- Receivers at any location
- Circular or spherical wavefronts
- Source time and location is computed
- Assumes locally constant velocities
- Many types of solutions for Apollonius’ problem
- Illustrated with 2D data, solved for 3D data
- Applications for:
  - Microseismic events, fraccing, CO₂ sequestration
  - Monitoring of hazardous geological sites (Frank slide), volcanoes, …
  - Computing gridded traveltimes
  - Converting ray traveltimes to gridded traveltimes
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The End