P-impedance, S-impedance and density from linear AVO inversion: Application to a VSP dataset from Alberta

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Outline

- The linear AVO inversion description
- Methodology
- damped SVD method
- A synthetic surface seismic example
- AVO inversion of the Red Deer VSP data
- Conclusions
- Acknowledgments
Inversion Problem

Inverse problem: to estimate Physical properties
\((I: P\text{-impedance}, J: S\text{-impedance}, \rho: \text{density})\)

Physical property:
- Imaging subsurface structure
  - Directly detecting changes in the subsurface
  - An aid in the interpretation of seismic reflection data

Pre-stack reflection AVO data

Aki-Richards approximations

\((I, J, \rho)\)

Compressional data \(\rightarrow\) PP inversion

Converted P-Sv data \(\rightarrow\) PS inversion

Compressional data

Converted data \(\rightarrow\) Joint inversion
Methodology

Aki-Richards

\[
\begin{align*}
R_{PP} &= A \frac{\Delta I}{I} + B \frac{\Delta J}{J} + C \frac{\Delta \rho}{\rho} \\
R_{PS} &= E \frac{\Delta J}{J} + D \frac{\Delta \rho}{\rho}
\end{align*}
\]

Sample depth

\[
\begin{bmatrix}
A_1 & B_1 & C_1 \\
\vdots & \vdots & \vdots \\
A_1 & B_1 & C_1 \\
0 & E_1 & D_1 \\
\vdots & \vdots & \vdots \\
0 & E_m & D_m
\end{bmatrix}_{2 m \times 3} =
\begin{bmatrix}
\Delta I / I \\
\Delta J / J \\
\Delta \rho / \rho
\end{bmatrix}
\]

\[Gm = d\]
Workflow: Joint inversion

PP data in time

Conversion to depth

PS data in time

Scaling

PP data in depth

PS data in depth

damped SVD

Gm=d

Background Vp, Vs, \( \rho \) in depth

Raytracing

\[
\begin{bmatrix}
\Delta I / I \\
\Delta J / J \\
\Delta \rho / \rho 
\end{bmatrix}
\]

+ Low-freq trend
Inverse problem

- **Least-squares** method minimizing the prediction error \( Gm = d \)
- **Ill-posed**: a small change in data will cause the large change in solution.
- **3-parameter linear inversion**: ill-posed problem
  - Nonlinearity of the problem
  - Limited data acquisition aperture
- **Singular Value Decomposition, SVD**

\[
m^{est} = G_g^{-1} d
\]

**Generalized inverse**

\[
G_g^{-1} = V \Lambda^{-1} U^T
\]

(Lay, 1996)

\[
\Lambda = \begin{bmatrix}
\sigma_1 & 0 & 0 \\
0 & \sigma_2 & 0 \\
0 & 0 & \sigma_3
\end{bmatrix}
\]

\[
\sigma_i = \text{non-zero singular value of } G
\]

\[
\sigma_1 \geq \sigma_2 \geq \sigma_3 > 0
\]

\[
+\sqrt{\text{eigenvalues of } G^T G}
\]
SVD analysis

**Condition number** = \( \frac{\text{Largest singular value}}{\text{Smallest singular value}} \)

A matrix is **well-posed** when its condition number is not far from 1 (Jin et al., 2000).

**Damped SVD**

\[
G_g^{-1} = V \Lambda (\Lambda^2 + \epsilon^2 I)^{-1} U^T
\]

\( \epsilon^2 = \text{Damping factor} \)

Damping factor = \( \epsilon \) percent of the largest singular value

Small damping factor is desired, with large \( \epsilon \) the model parameter will not be resolved correctly

**Model parameter resolution matrix:**

\[
R = G_g^{-1} G
\]

\[
m^{est} = G_g^{-1} d = (G_g^{-1} G) m
\]

**Perfect resolution:**

\[
G_g^{-1} G = I
\]
AVO inversion testing

SYNGRAM, CREWES software

Real log from Blackfoot field, owned and operated by Encana, south-eastern Alberta, Canada
Condition number

The ratio is important
Joint inversion, 10% damping

Joint inversion with damping factor = %10

Depth (meters)

I

J

ρ

0.6 0.8 1 1.2 1.4
km/s

4 6
km/s

2000 2500 3000
kg

x 10^7

x 10^6

True
%10-damp joint inversion
no-damped joint inversion
Joint inversion vs. PP inversion

Joint inversion vs. PP inversion, %10 damping factor

Depth (meters)

Joint inversion

PP inversion

ρ
Joint inversion vs. PS inversion

Joint inversion vs. PS inversion, %10 damping factor

Depth (meters)

Joint inversion vs. PS inversion, %10 damping factor

Depth (meters)

Joint inversion vs. PS inversion, %10 damping factor

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Joint inversion vs. PS inversion, %10 damping factor

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Depth (meters)

Joint inversion vs. PS inversion, %10 damping factor

Depth (meters)

Joint inversion vs. PS inversion, %10 damping factor

Depth (meters)
Study area: Cygnet 9-34 in Red Deer, Alberta

(Courtesy of Richardson, 2003)
Survey geometry

(Courtesy of Richardson, 2003)

Source: Compressional vibroseis, with sweep 8-250 HZ

AVO inversion input: deconvolved upgoing wavefield

Ardley coal seams are laterally continues with no lateral variation (Beaton, 2003)
CCP gather = Common shot gather
Joint inversion, offset 3

\( \rho \): Density

- \( \varepsilon = 0 \)
- \( \varepsilon = 0.1\% \)
- \( \varepsilon = 1\% \)
- \( \varepsilon = 2\% \)
- \( \varepsilon = 3\% \)
- \( \varepsilon = 4\% \)
- \( \varepsilon = 5\% \)
- \( \varepsilon = 9\% \)

Depth (meters)

Kg
Resolution matrix

Row 1
- P-impedance
- S-impedance
- \( \varepsilon = 0 \)

Row 2
- P-impedance
- S-impedance
- \( \varepsilon = 1 \%1 \)

Row 3
- P-impedance
- S-impedance
- \( \varepsilon = 2 \%2 \)

Row 4
- P-impedance
- S-impedance
- \( \varepsilon = 3 \%3 \)

Row 5
- P-impedance
- S-impedance
- \( \varepsilon = 4 \%4 \)

Row 6
- P-impedance
- S-impedance
- \( \varepsilon = 9 \%9 \)
Density relation for Red Deer area

Resolution

\[
\begin{bmatrix}
\frac{\Delta I_{est}}{I} \\
\frac{\Delta J_{est}}{J} \\
\frac{\Delta \rho_{est}}{\rho}
\end{bmatrix}
= \begin{bmatrix}
0.98 & 0.016 & 0.003 \\
0.016 & 0.86 & 0.27 \\
0.003 & 0.27 & 0.17
\end{bmatrix}
\begin{bmatrix}
\frac{\Delta I_{true}}{I} \\
\frac{\Delta J_{true}}{J} \\
\frac{\Delta \rho_{true}}{\rho}
\end{bmatrix}
\]

\[
\frac{\Delta \rho_{est}}{\rho} = 0.25 \frac{\Delta J_{true}}{J}
\]
I estimate
J estimate

J: S-impedance from the AVO inversion of walkaway VSP data

- Offset 1
- Offset 2
- Offset 3
- Offset 4

Depth (meters)

Horizontal distance to the well (meters)
Density estimate from the AVO inversion of walkaway VSP data

- **Density estimate**

- **Horizontal distance to the well (meters):**
  - Offset 1: 50
  - Offset 2: 75
  - Offset 3: 95
  - Offset 4: 122

- **Depth (meters):**
  - 210
  - 220
  - 230
  - 240
  - 250
  - 260
  - 270
  - 280
  - 290
  - 300

- **Legend:**
  - True value
  - PP inversion
  - PS inversion
  - Joint inversion
Conclusions

- A favorable density estimate is obtained from linear AVO inversion.
- A good compressional impedance estimate can be obtained from the AVO inversion of PP data.
- Good shear impedance and density can be obtained from the AVO inversion of converted data.
- Converted waves data provides information on density not obtainable from compressional data.

Red Deer case study

- The shear wave velocity contributes more to improving the density estimate.
- Possible discontinuity in coal properties at the lateral distance between 95-125 m to the well.
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