Long offset moveout correction and spherical traveltimes

John C. Bancroft and Xiang Du

CREWES/University of Calgary

2007

CREWES

Consortium for **R**esearch in

Elastic **W**ave **E**xploration **S**eismology

NSERC

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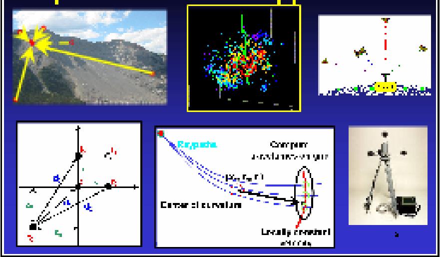
Spherical traveltime applications

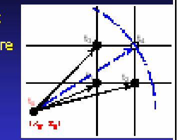


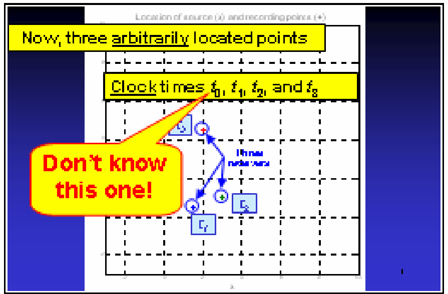
The main point

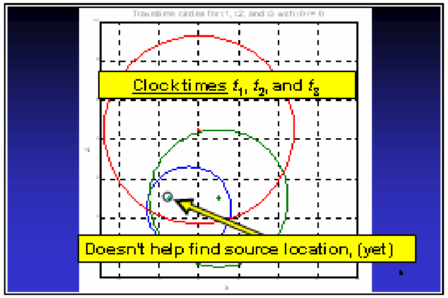
- Given:
  - four receiver locations for 3D
  - clock-times of an event recorded at receiver locations
- Compute the source location and time of the event
- Compute another clock-time at a nearby location on a grid (depth migration)

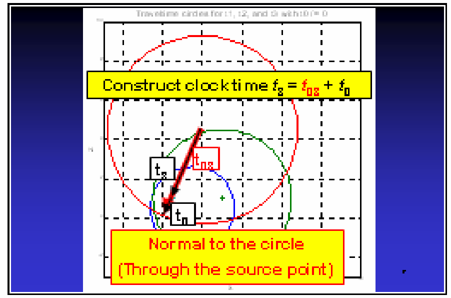
Curved wavefront assumption (2D)

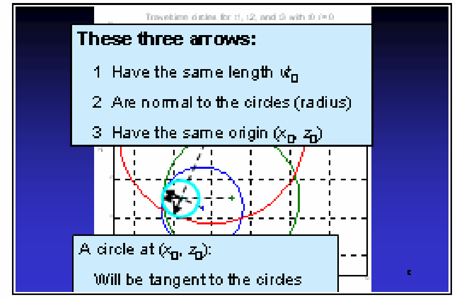
1. Assume velocity locally constant
2. Source point at center of curvature (x_0, z_0)
3. Source time t_0
4. Estimate (x_0, z_0) and t_0
5. Hyperbolic intersections
6. Get time t_g
7. Mentioned in Vidale's paper

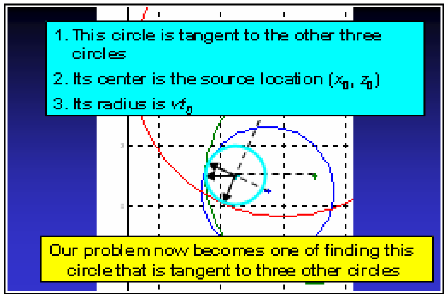


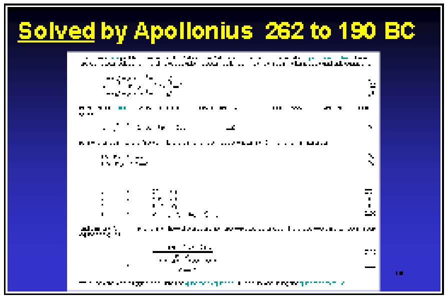


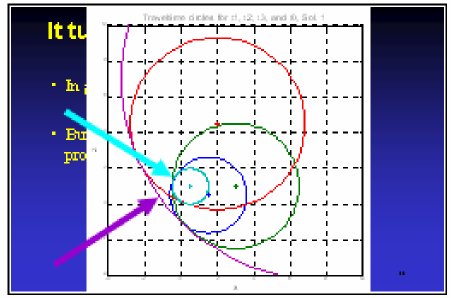


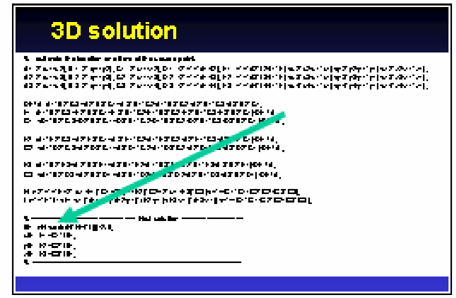


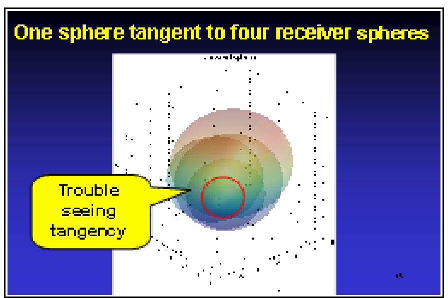


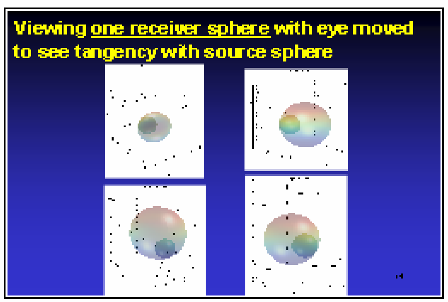








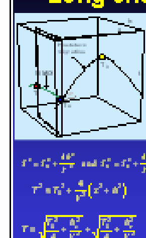




Comments on the velocities

- Micro seismic events:
  - Assumes velocity constant in the monitoring area
- Gridded traveltimes:
  - Velocity of the square or cube extended without error to the apparent source
- Mapping rays to gridded traveltimes:
  - Constant velocity in the neighborhood of the raytimes extended without error to the apparent source

Long offset moveout correction



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- Equivalent offset migration
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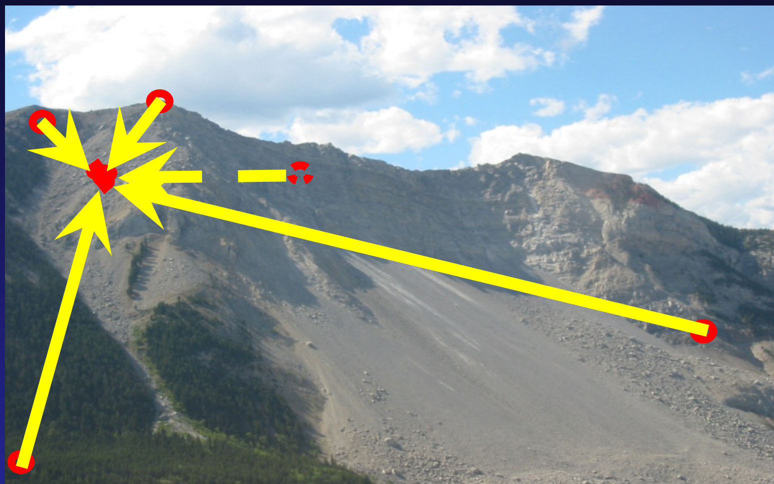
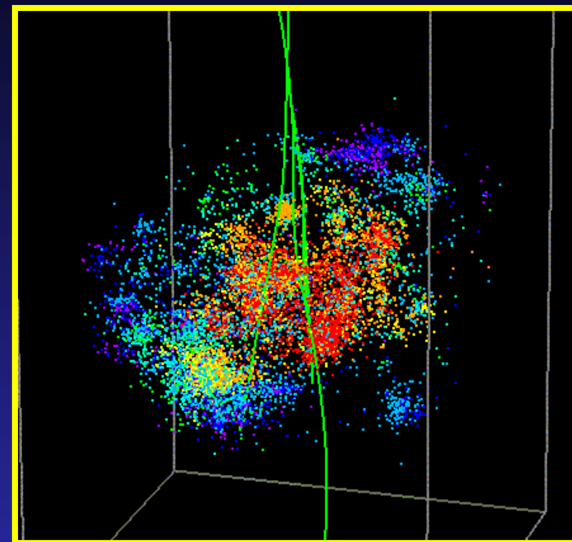
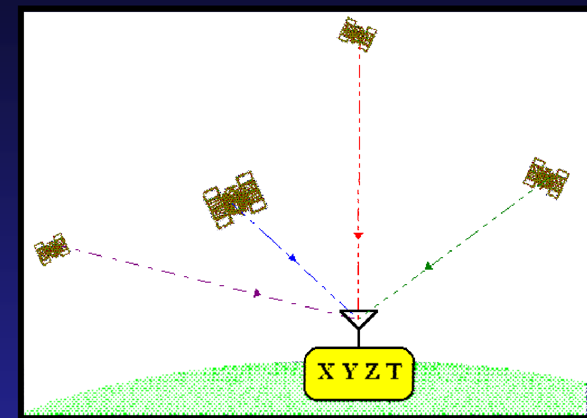
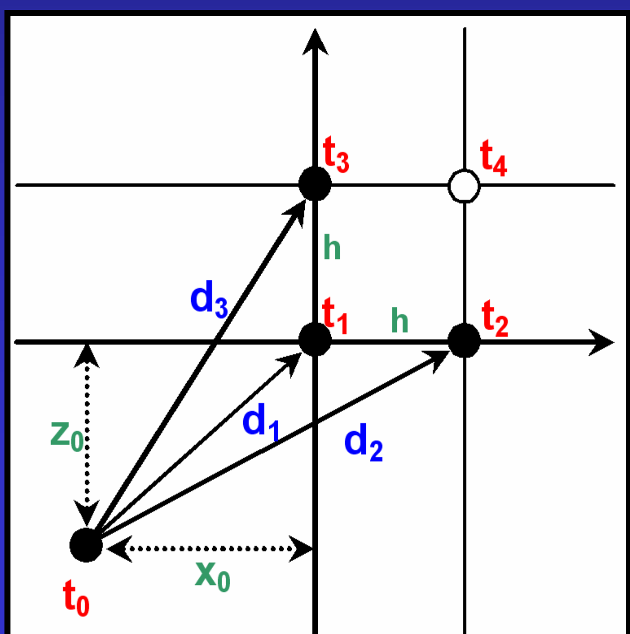
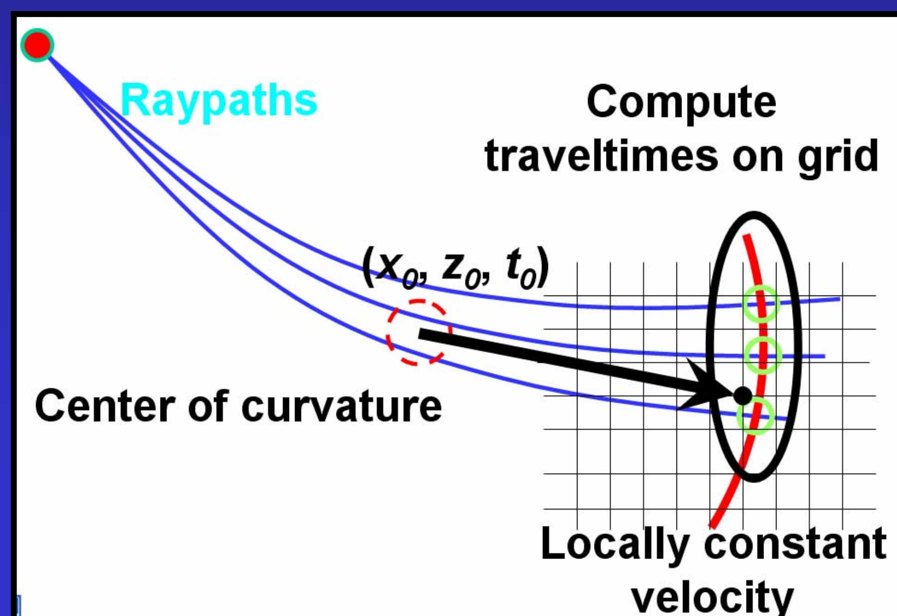
Spherical traveltime applications











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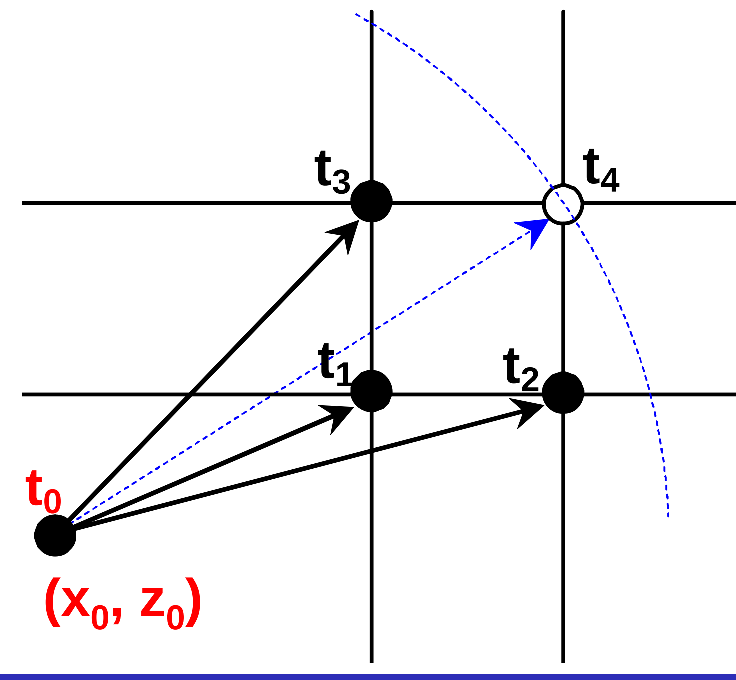
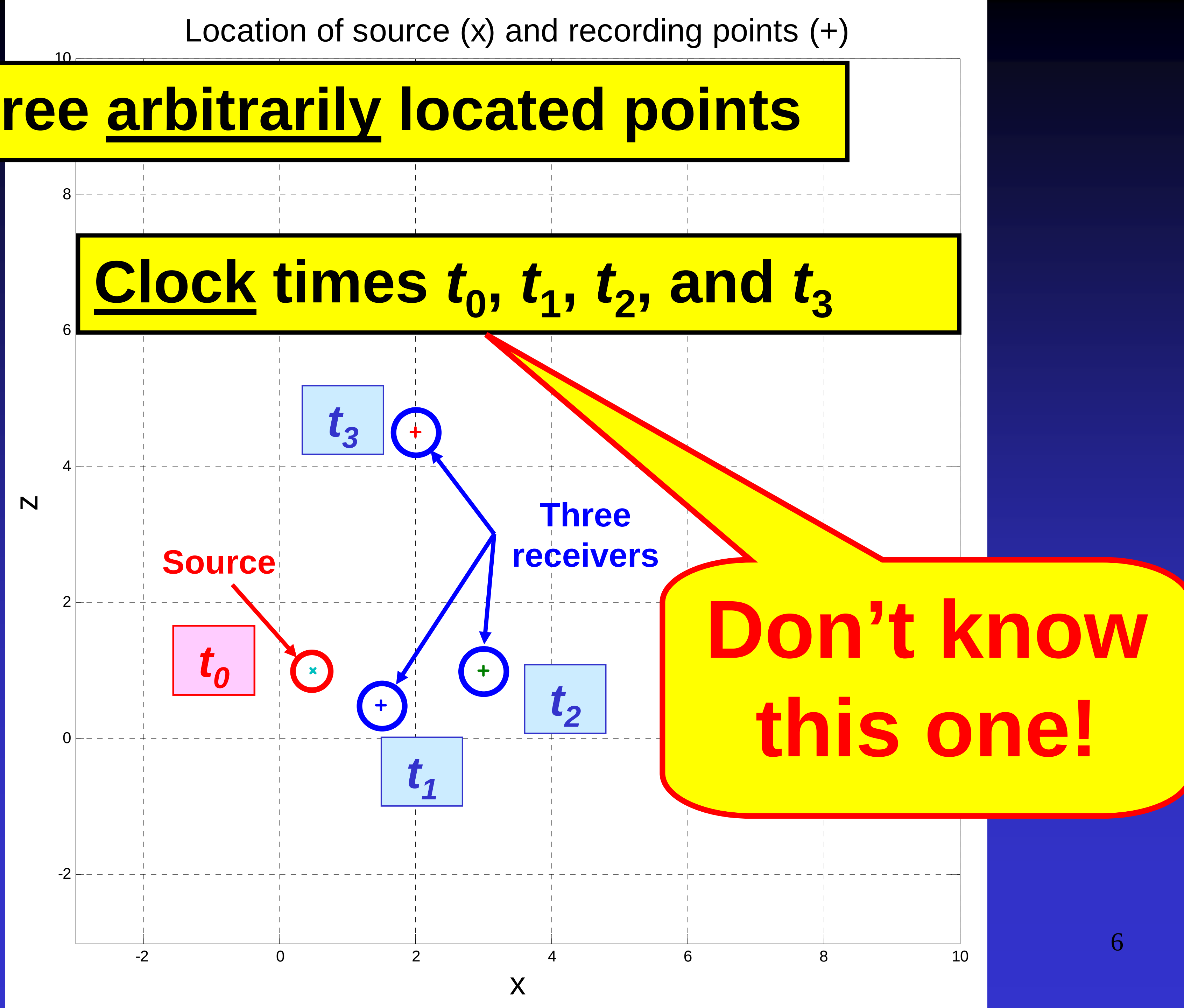
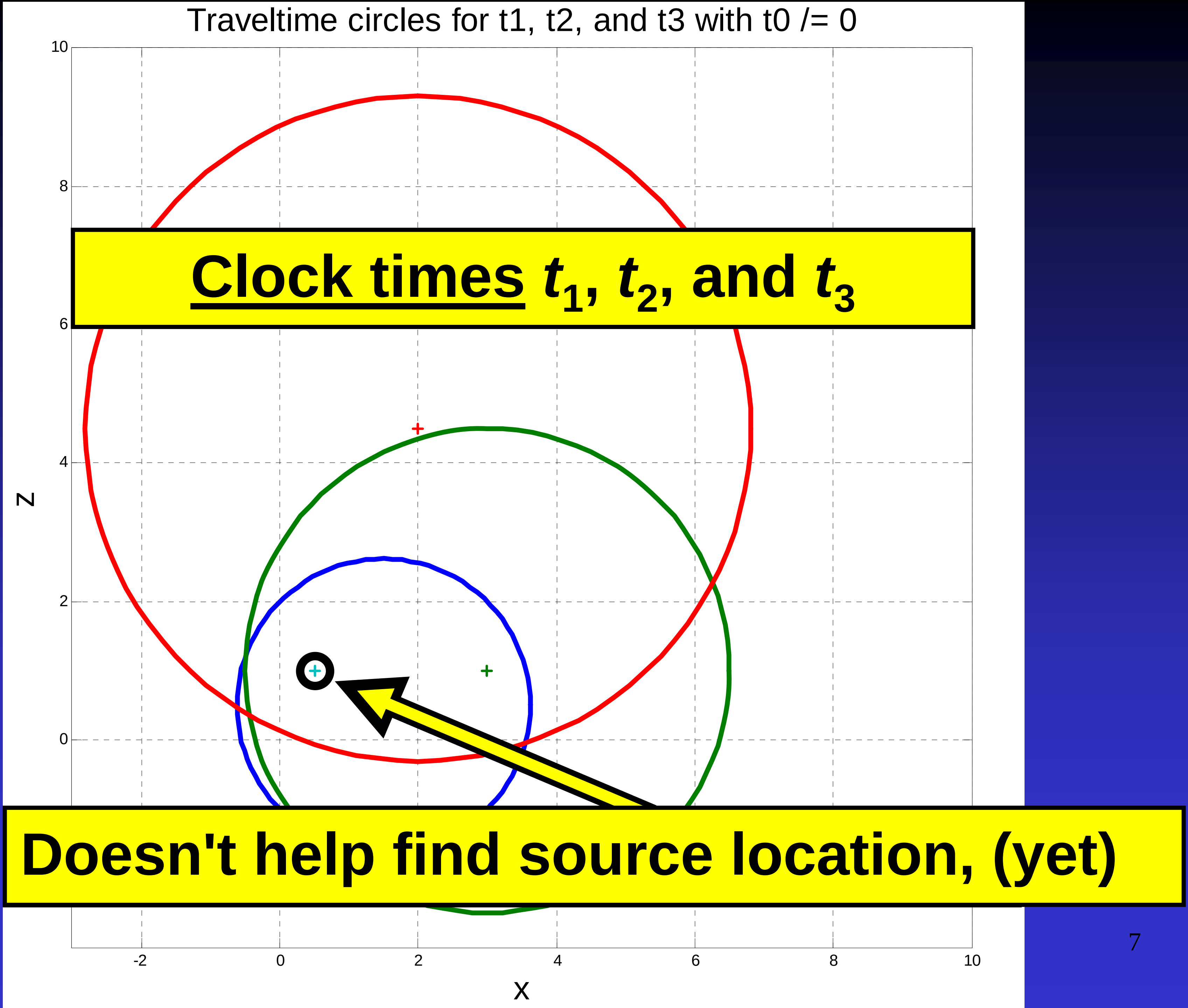
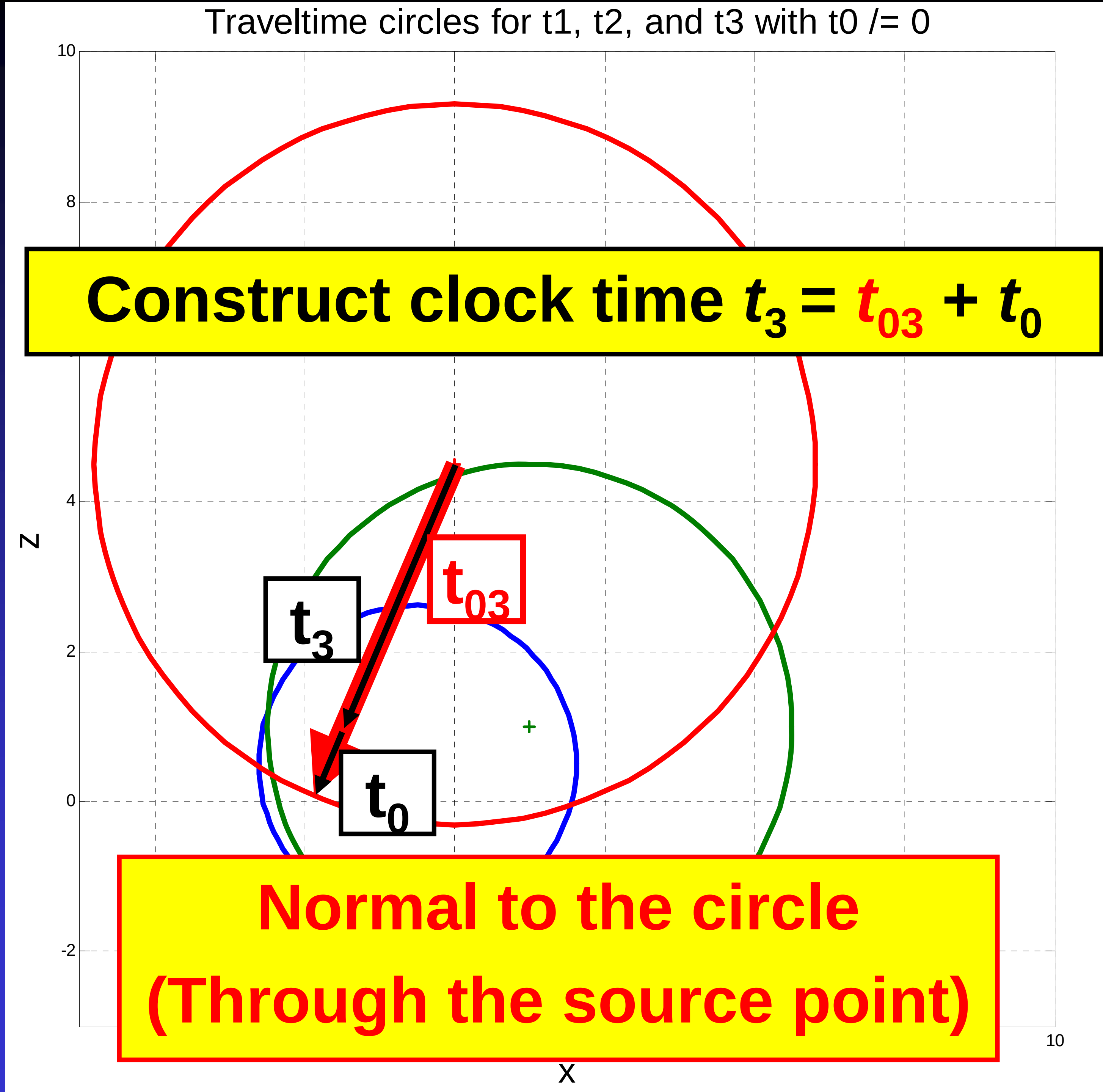
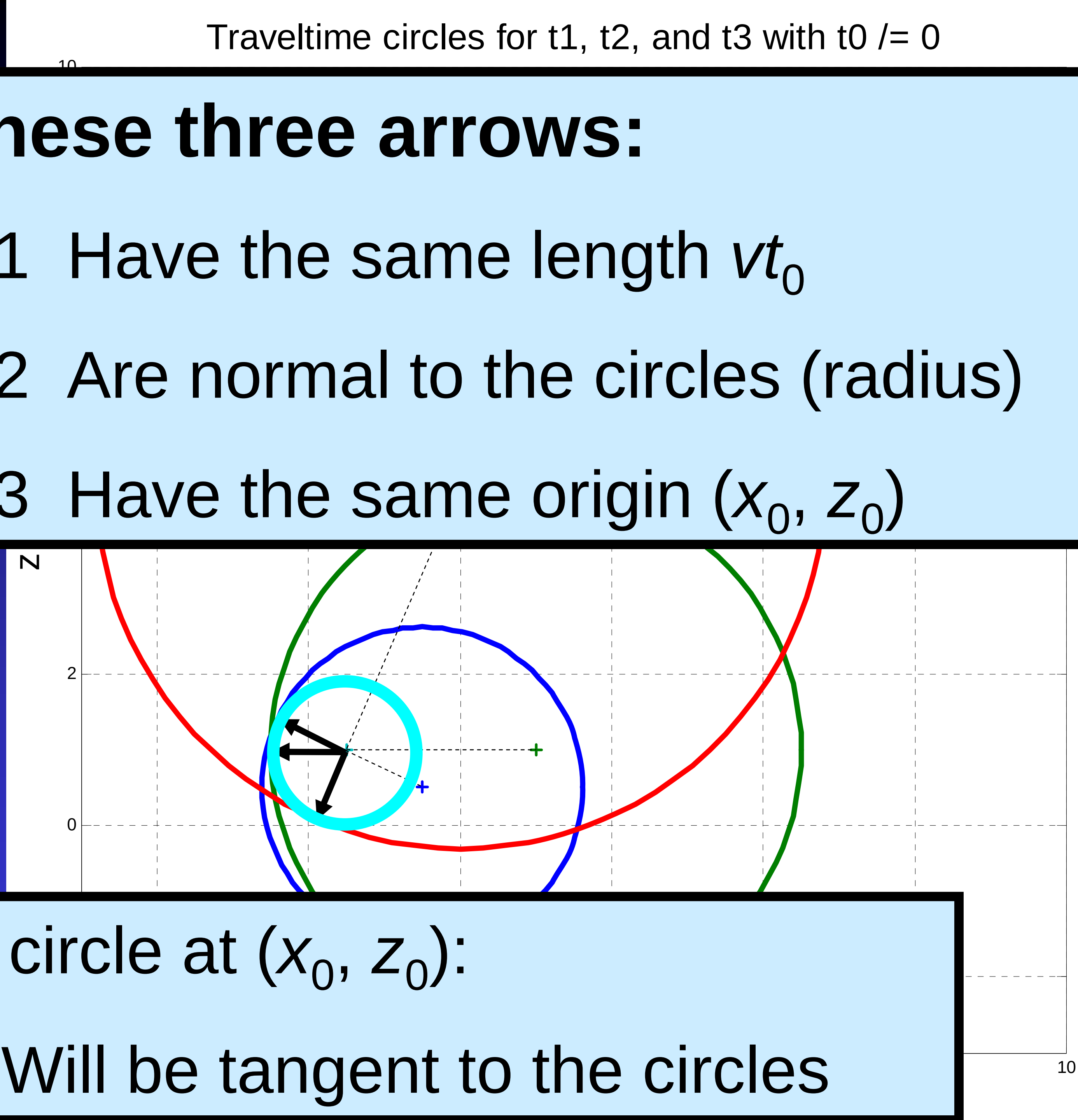


Now, three <u>arbitrarily</u> located points




Traveltime circles for t1, t2, and t3 with t0 /= 0
Clock times t_1, t_2, and t_3
Doesn't help find source location, (yet)
X
Z
-2
0
2
4
6
8
10


Traveltime circles for t1, t2, and t3 with t0 /= 0
Construct clock time $t_3 = t_{03} + t_0$
t_3
t_{03}
t_0
Normal to the circle
(Through the source point)
10
8
4
2
0
-2
10
Z
X



These three arrows:

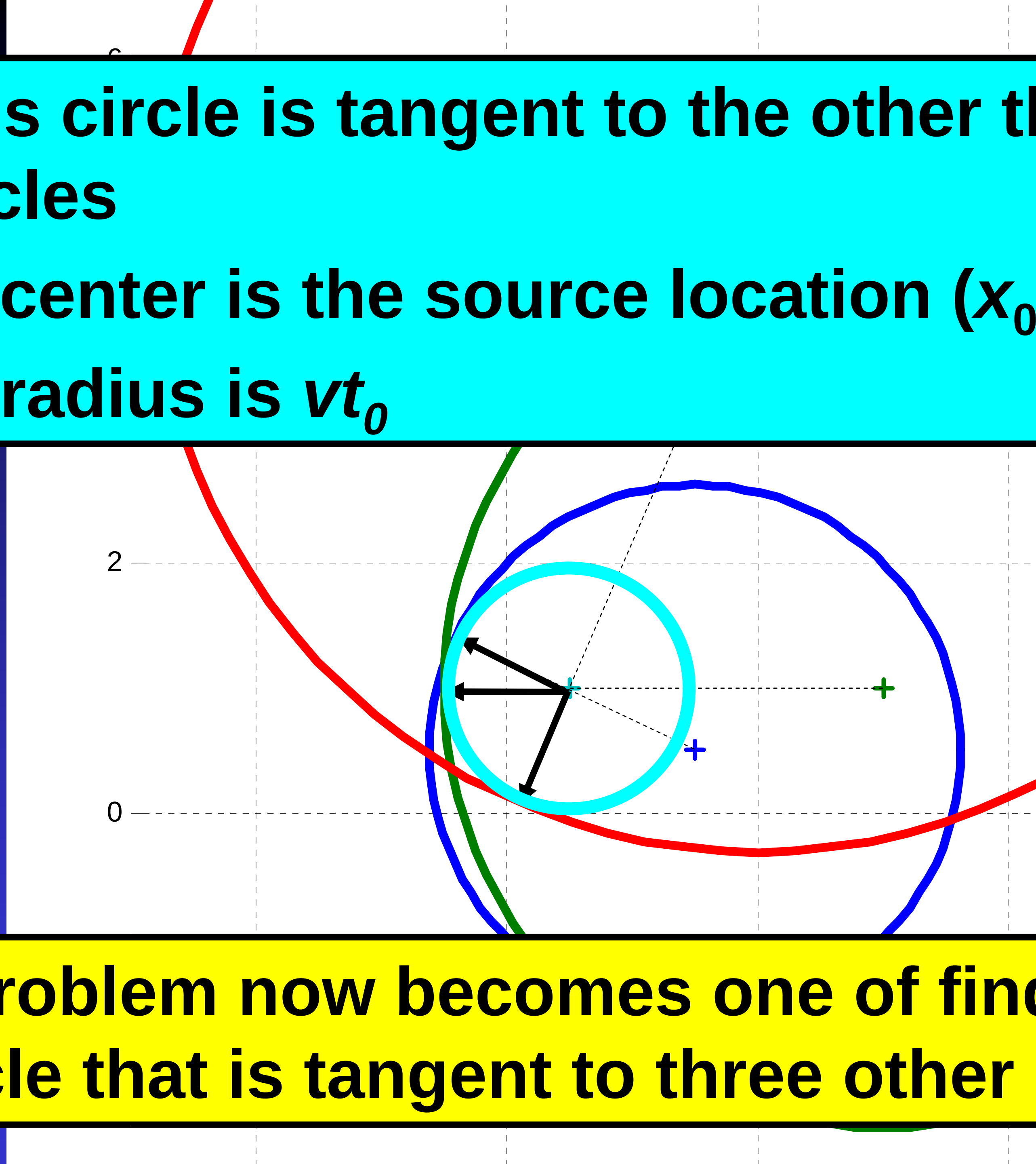
1 Have the same length vt_0

2 Are normal to the circles (radius)

3 Have the same origin (x_0, z_0)

A circle at (x_0, z_0):

Will be tangent to the circles



1. This circle is tangent to the other three circles
2. Its center is the source location (x_0, z_0)
3. Its radius is vt_0

Our problem now becomes one of finding this circle that is tangent to three other circles

Solved by Apollonius 262 to 190 BC

The three-circle problem was solved by Viète (Boyer 1968), and the solutions are called Apollonius circles. There are eight total solutions. The simplest solution is obtained by solving the three simultaneous quadratic equations

$$(x-x_1)^2+(y-y_1)^2-(r\pm r_1)^2=0 \quad (1)$$
$$(x-x_2)^2+(y-y_2)^2-(r\pm r_2)^2=0 \quad (2)$$
$$(x-x_3)^2+(y-y_3)^2-(r\pm r_3)^2=0 \quad (3)$$

in the three unknowns x, y, r for the eight triplets of signs (Courant and Robbins 1996). Expanding the equations gives

$$(x^2+y^2-r^2)-2\,x\,x_i-2\,y\,y_i\mp 2\,r\,r_i+(x_i^2+y_i^2-r_i^2)=0 \quad (4)$$

for $i=1, 2, 3$. Since the first term is the same for each equation, taking (2) – (1) and (3) – (1) gives

$$a\,x+b\,y+c\,r=d \quad (5)$$
$$a'\,x+b'\,y+c'\,r=d', \quad (6)$$

where

$$a = 2\,(x_1-x_2) \quad (7)$$
$$b = 2\,(y_1-y_2) \quad (8)$$
$$c = \pm 2\,(r_1-r_2) \quad (9)$$
$$d = (x_1^2+y_1^2-r_1^2)-(x_2^2+y_2^2-r_2^2) \quad (10)$$

and similarly for a', b', c' and d' (where the 2 subscripts are replaced by 3s). Solving these two simultaneous linear equations gives

$$x = \frac{b'\,d-b\,d'-b'\,c\,r+b\,c'\,r}{a\,b'-b\,a'} \quad (11)$$
$$y = \frac{-a'\,d+a\,d'+a'\,c\,r-a\,c'\,r}{a\,b'-a'\,b}, \quad (12)$$

which can then be plugged back into the quadratic equation (1) and solved using the quadratic formula.

It turns out…

- In general there are 8 possible solutions
- But our case is limited by the geometry of the problem to only 2 solutions.

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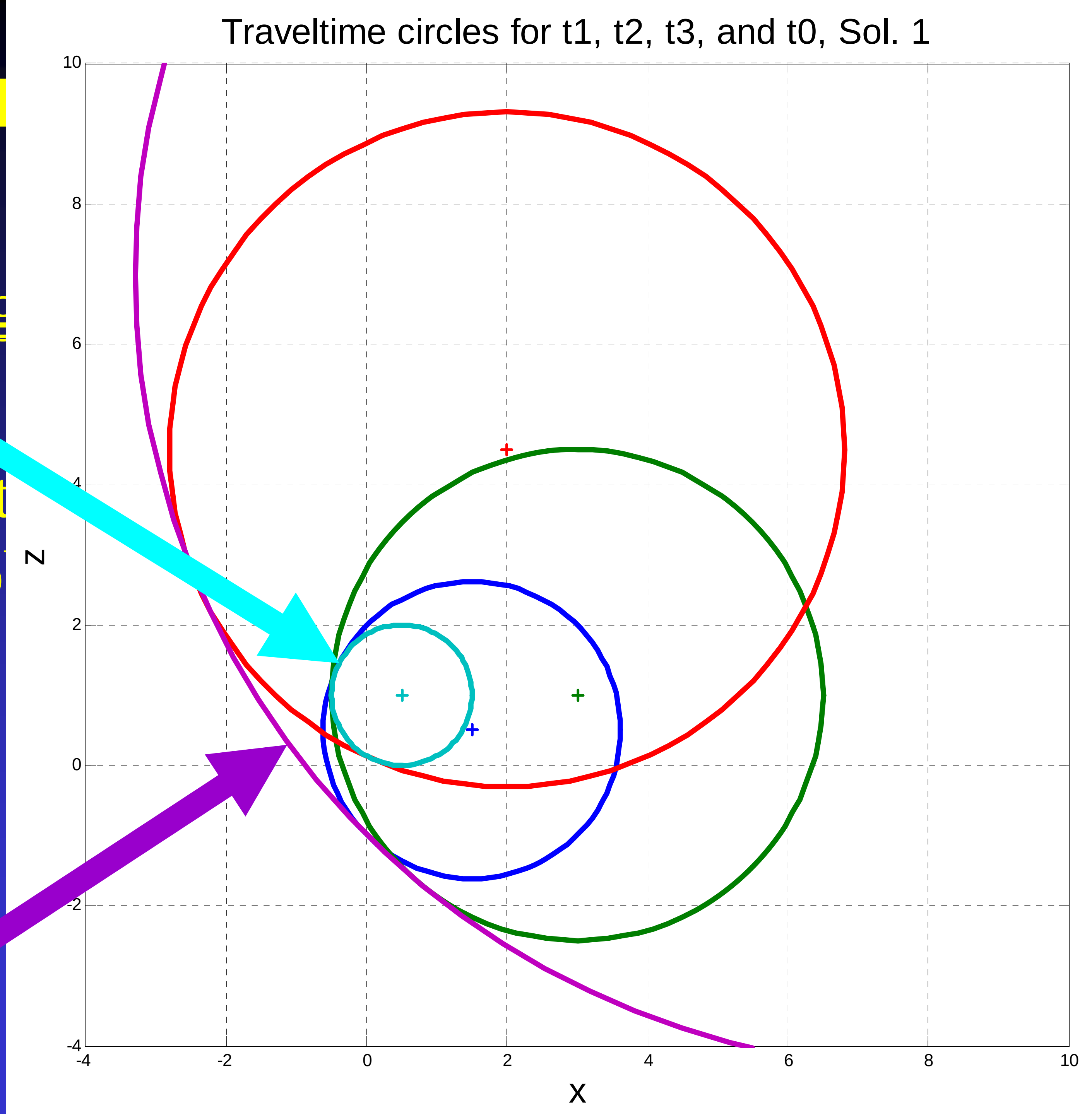
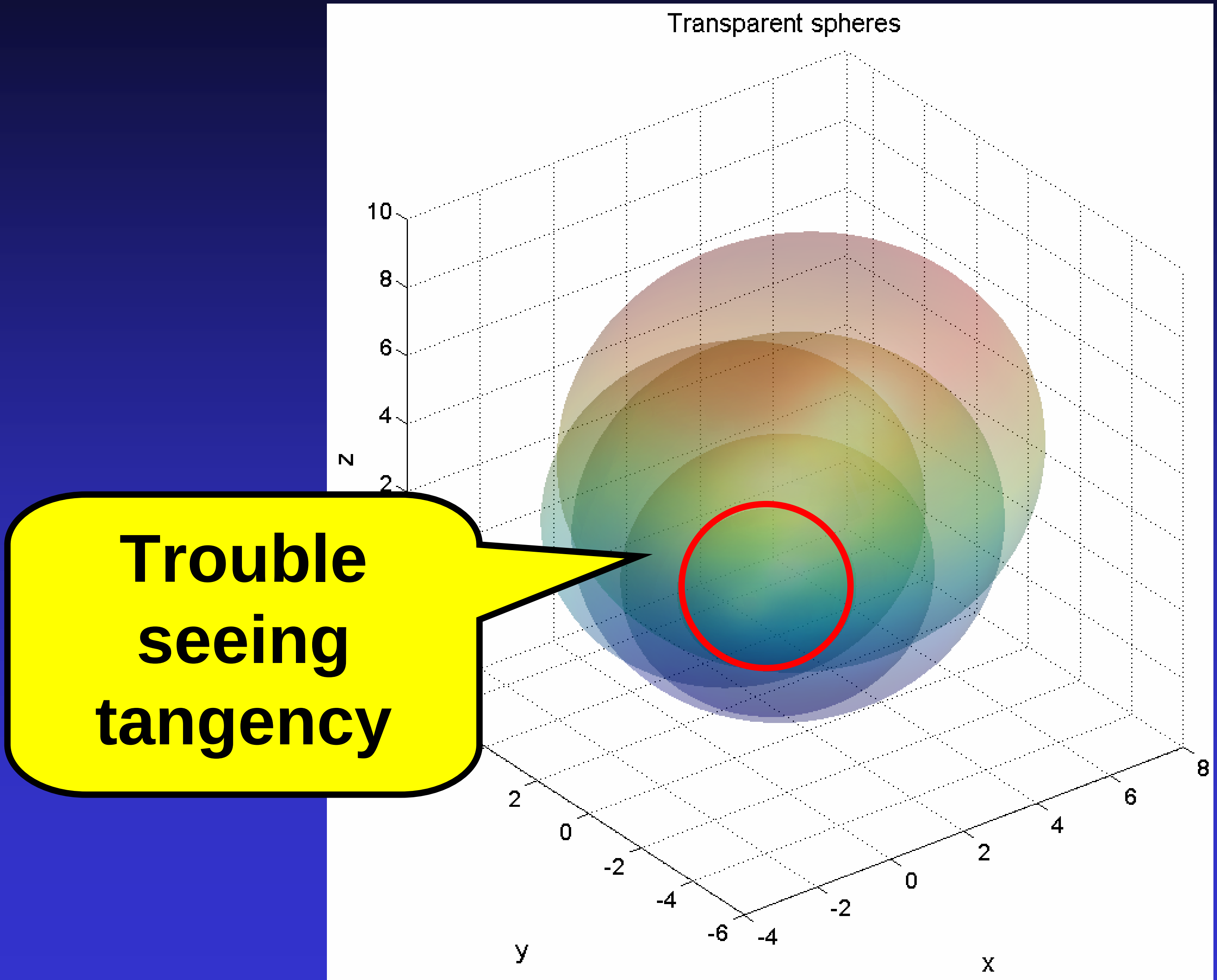


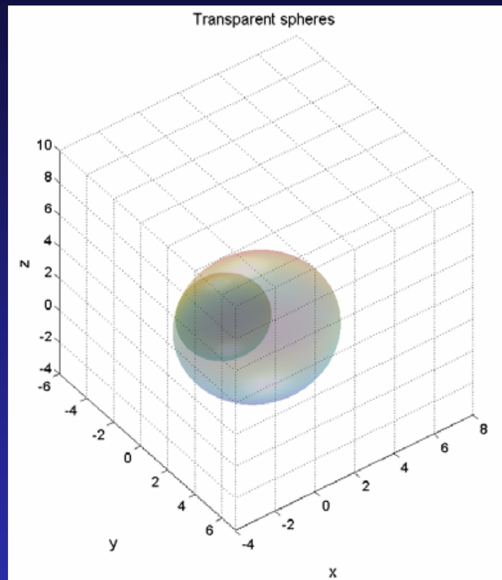
3D solution

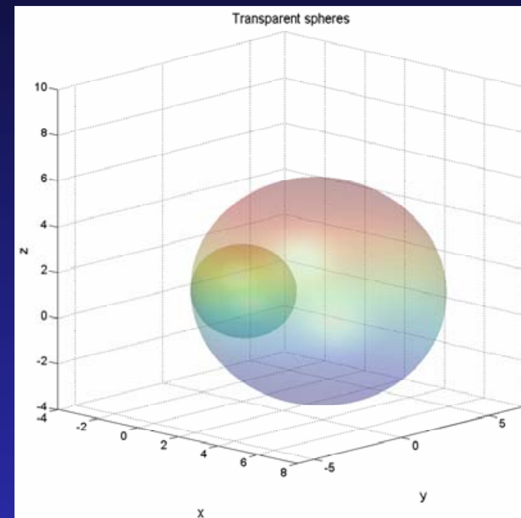
```
%   estimate the location and time of the source point.
A1=2*(x1-x2); B1=2*(y1-y2); C1=2*(z1-z2); D1=-2*v*v*(t1-t2); E1=v*v*(t2*t2-t1*t1)-(x2*x2-x1*x1)-(y2*y2-y1*y1)-(z2*z2-z1*z1);
A2=2*(x1-x3); B2=2*(y1-y3); C2=2*(z1-z3); D2=-2*v*v*(t1-t3); E2=v*v*(t3*t3-t1*t1)-(x3*x3-x1*x1)-(y3*y3-y1*y1)-(z3*z3-z1*z1);
A3=2*(x1-x4); B3=2*(y1-y4); C3=2*(z1-z4); D3=-2*v*v*(t1-t4); E3=v*v*(t4*t4-t1*t1)-(x4*x4-x1*x1)-(y4*y4-y1*y1)-(z4*z4-z1*z1);

DETA=A1*B2*C3+A2*B3*C1+A3*B1*C2-A1*B3*C2-A2*B1*C3-A3*B2*C1;
F1=(E1*B2*C3+E2*B3*C1+E3*B1*C2-E1*B3*C2-E2*B1*C3-E3*B2*C1)/DETA;
G1=-(D1*B2*C3+D2*B3*C1+D3*B1*C2-D1*B3*C2-D2*B1*C3-D3*B2*C1)/DETA;

F2=(A1*E2*C3+A2*E3*C1+A3*E1*C2-A1*E3*C2-A2*E1*C3-A3*E2*C1)/DETA;
G2=-(A1*D2*C3+A2*D3*C1+A3*D1*C2-A1*D3*C2-A2*D1*C3-A3*D2*C1)/DETA;

F3=(A1*B2*E3+A2*B3*E1+A3*B1*E2-A1*B3*E2-A2*B1*E3-A3*B2*E1)/DETA;
G3=-(A1*B2*D3+A2*B3*D1+A3*B1*D2-A1*B3*D2-A2*B1*D3-A3*B2*D1)/DETA;

H=(-2*v*v*t1+2*(x1-F1)*G1+2*(y1-F2)*G2+2*(z1-F3)*G3)/(v*v-G1*G1-G2*G2-G3*G3);
I=(v*v*t1*t1-(F1-x1)*(F1-x1)-(F2-y1)*(F2-y1)-(F3-z1)*(F3-z1))/(v*v-G1*G1-G2*G2-G3*G3);

%vvvvvvvvvvvvvvvvvvvvvvvvvvvvvvv First solution vvvvvvvvvvvvvvvvv
t01=(-H-sqrt((H*H-4*I)))/2.0;
x01=F1+G1*t01;
y01=F2+G2*t01;
z01=F3+G3*t01;
% ^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^
```

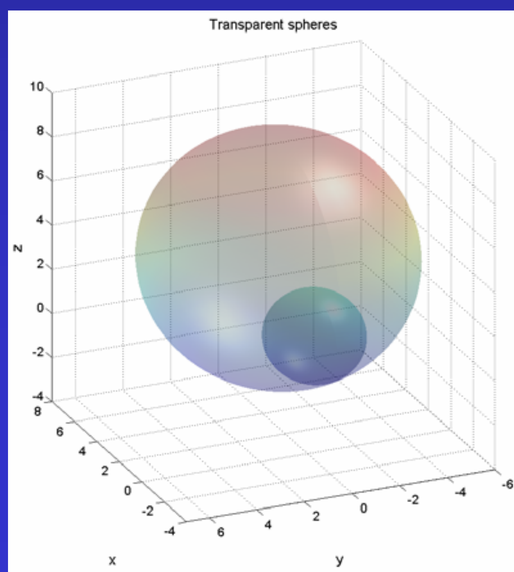
One sphere tangent to four receiver spheres

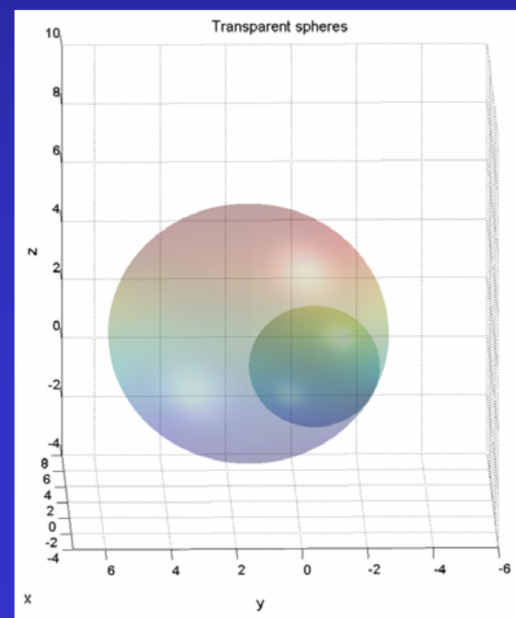


Viewing one receiver sphere with eye moved to see tangency with source sphere









Comments on the velocities

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- Gridded traveltimes:
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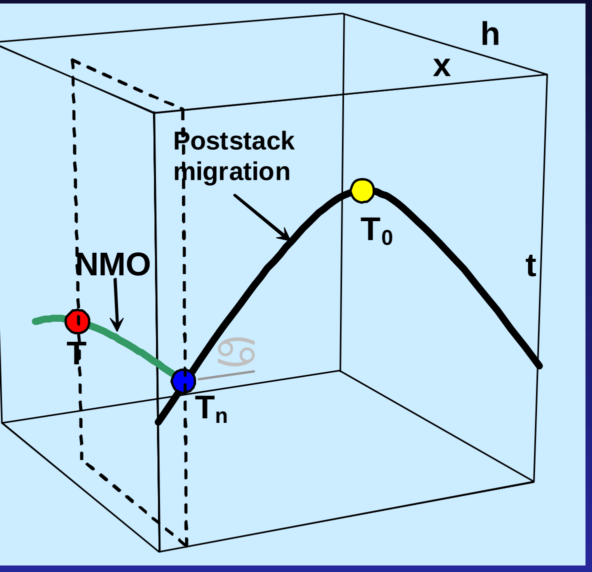
  - Constant velocity in the neighborhood of the ray times extended without error to the apparent source

Long offset moveout correction



$$T^2 = T_N^2 + \frac{4h^2}{V^2} \quad \text{and} \quad T_N^2 = T_0^2 + \frac{4x^2}{V^2}$$

$$T^2 = T_0^2 + \frac{4}{V^2}\left(x^2 + h^2\right)$$

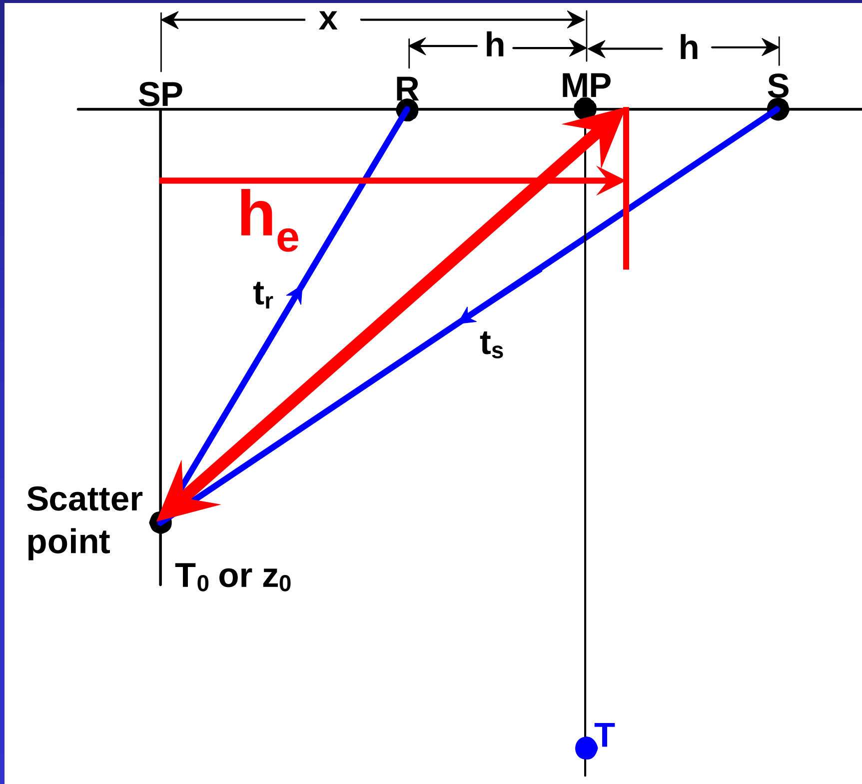
$$T = \sqrt{\frac{T_0^2}{4} + \frac{h_s^2}{V^2}} + \sqrt{\frac{T_0^2}{4} + \frac{h_r^2}{V^2}}$$

- Moveout correction is part of the migration process
- Kirchhoff migration
- Equivalent offset migration
- Anisotropy to hyperbolic trajectory
- Converted wave
- AVO and AVA

Equivalent offset

$$\sqrt{\frac{T_0^2}{4}+\frac{(x+h)^2}{V_{RMS}^2}}+\sqrt{\frac{T_0^2}{4}+\frac{(x-h)^2}{V_{RMS}^2}}=2\sqrt{\frac{T_0^2}{4}+\frac{h_e^2}{V_{RMS}^2}}$$

$$h_e^2 = x^2 + h^2 - \frac{4x^2h^2}{T^2V_{RMS}^2}$$



Converted wave Eq. Offset

$$\sqrt{\frac{T_{0P}^2}{4}+\frac{(x+h)^2}{V_P^2}}+\sqrt{\frac{T_{0S}^2}{4}+\frac{(x-h)^2}{V_S^2}}=\sqrt{\frac{T_{0P}^2}{4}+\frac{h_e^2}{V_P^2}}+\sqrt{\frac{T_{0S}^2}{4}+\frac{h_e^2}{V_S^2}}$$

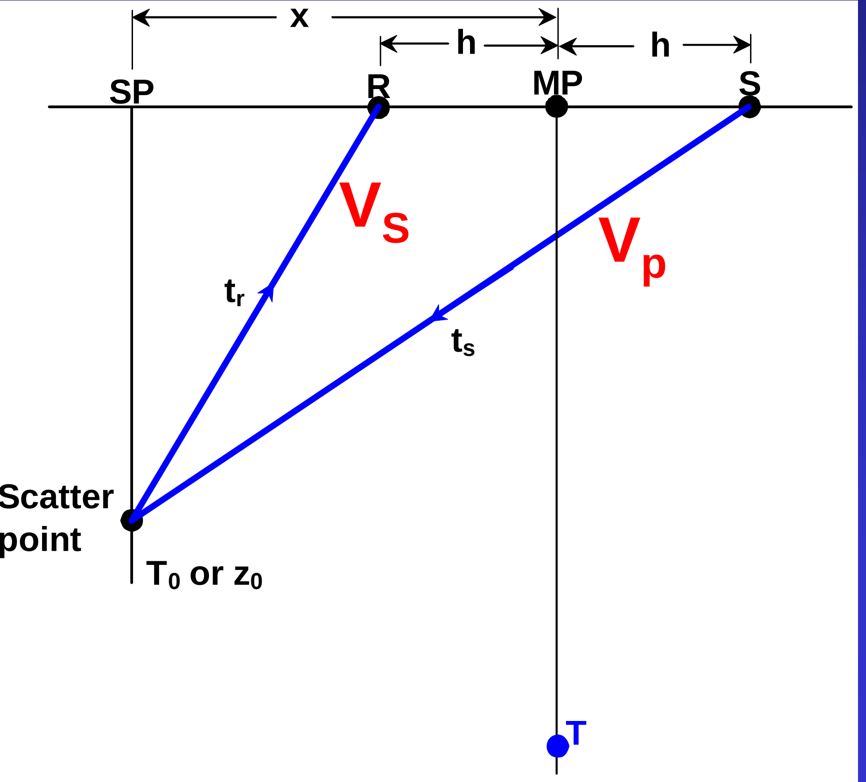
$$=\frac{1}{V_P}\sqrt{\frac{z_0^2V_P^2}{V_{Pave}^2}+h_e^2}+\frac{1}{V_s}\sqrt{\frac{z_0^2V_S^2}{V_{Save}^2}+h_e^2}$$

$$\approx\frac{1}{V_P}\sqrt{\tilde{z}^2+h_e^2}+\frac{1}{V_s}\sqrt{\tilde{z}^2+h_e^2}$$

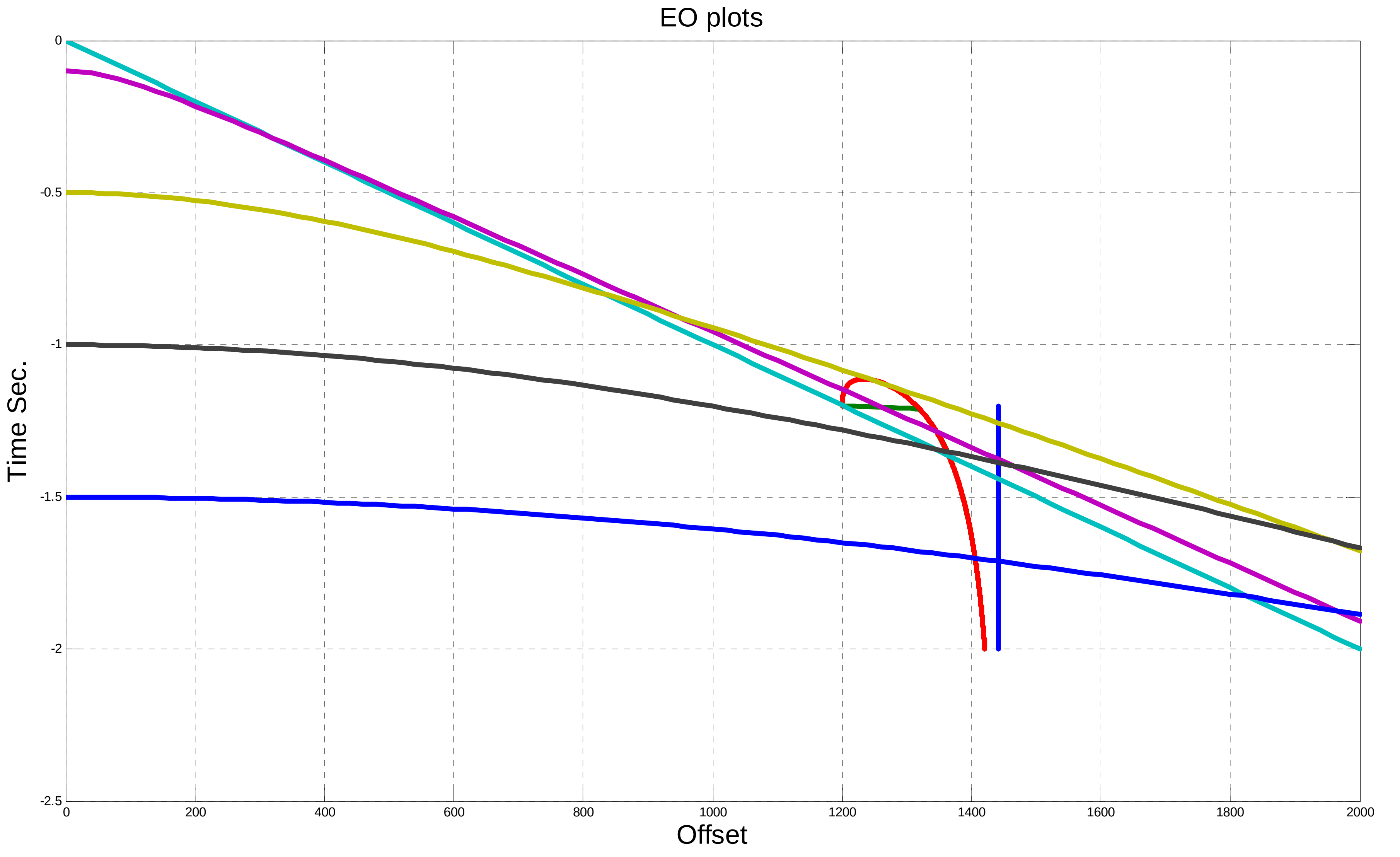
$$=\left(\frac{1}{V_P}+\frac{1}{V_s}\right)\sqrt{\tilde{z}^2+h_e^2}$$

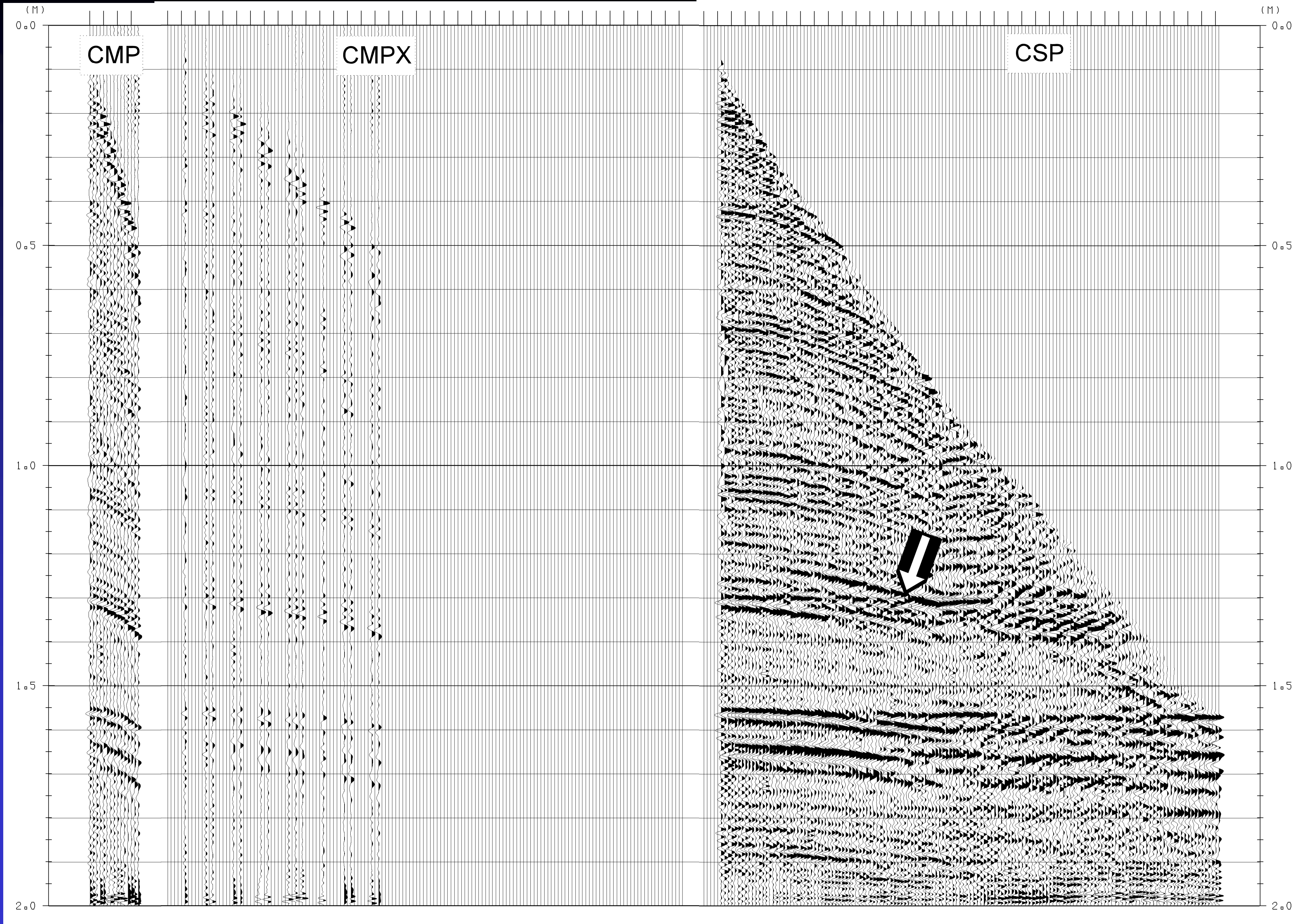
$$=\frac{1+\gamma}{V_P}\sqrt{\tilde{z}^2+h_e^2}$$

$$=2\sqrt{\frac{T_{0PS}^2}{4}+\frac{h_e^2}{V_{PS}^2}}\qquad V_{PS}=\frac{V_S}{1+\gamma}$$



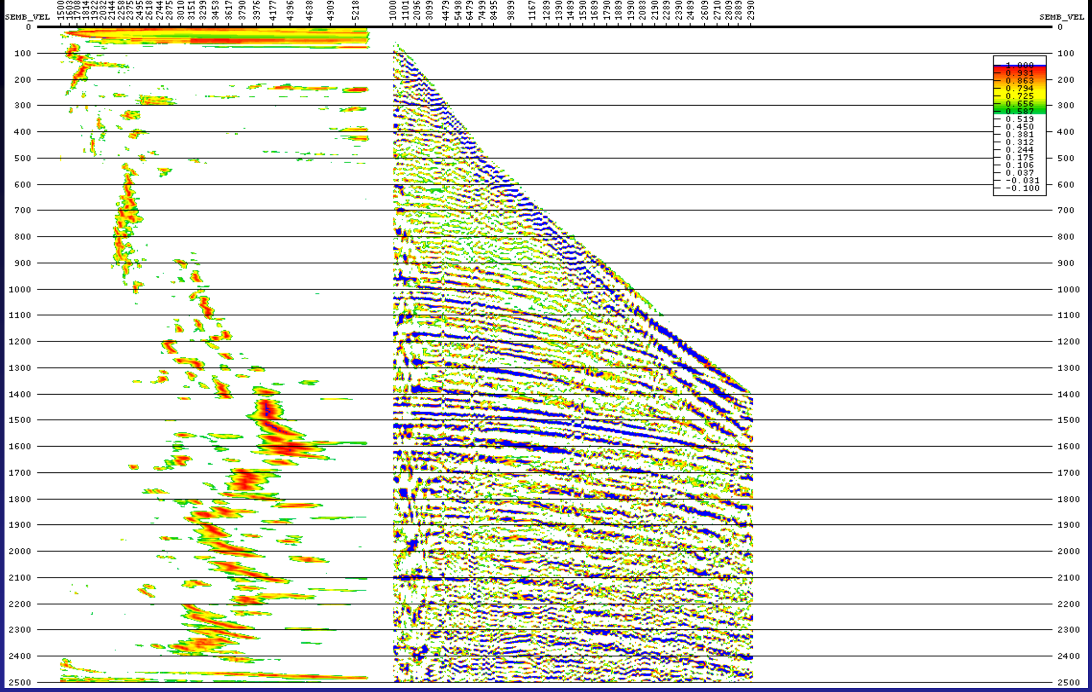
EOM gather


EO plots
Time Sec.
Offset
0
-0.5
-1
-1.5
-2
-2.5
0
200
400
600
800
1000
1200
1400
1600
1800
2000

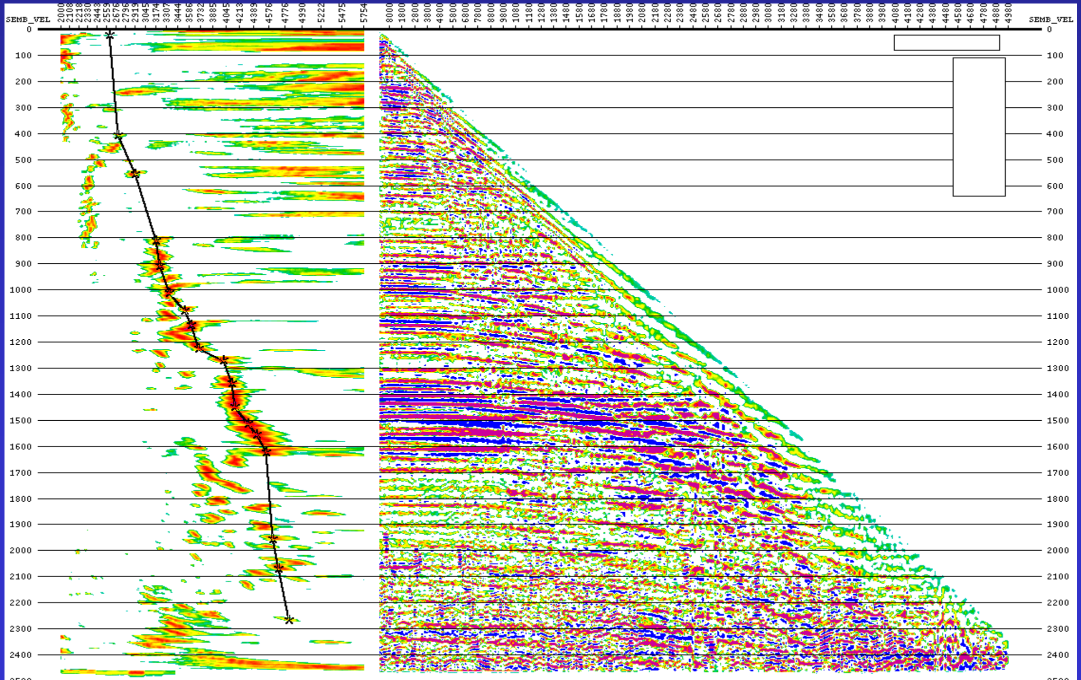


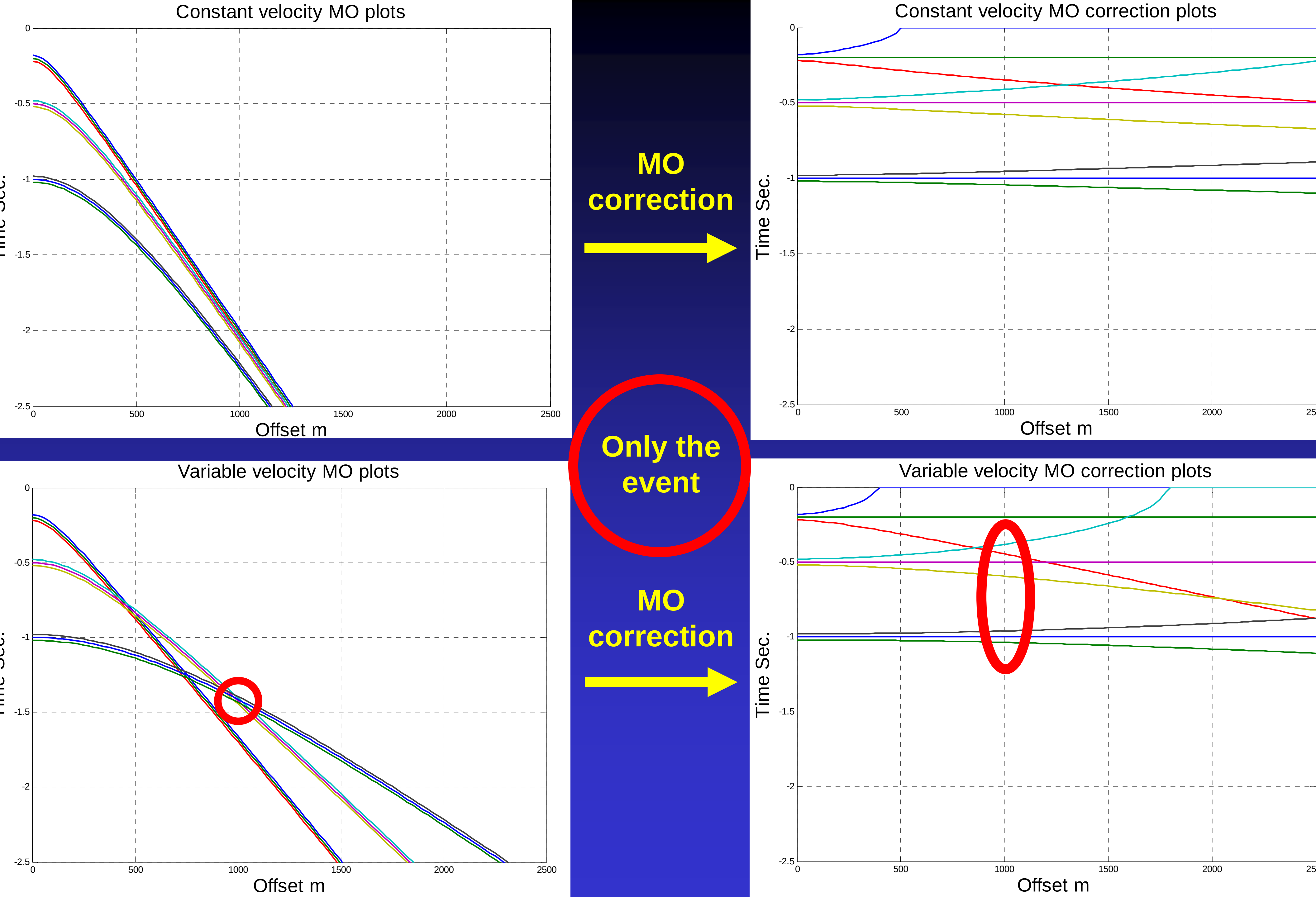
(M)
0.0
0.5
1.0
1.5
2.0
CMP
CMPX
CSP

CMP gather



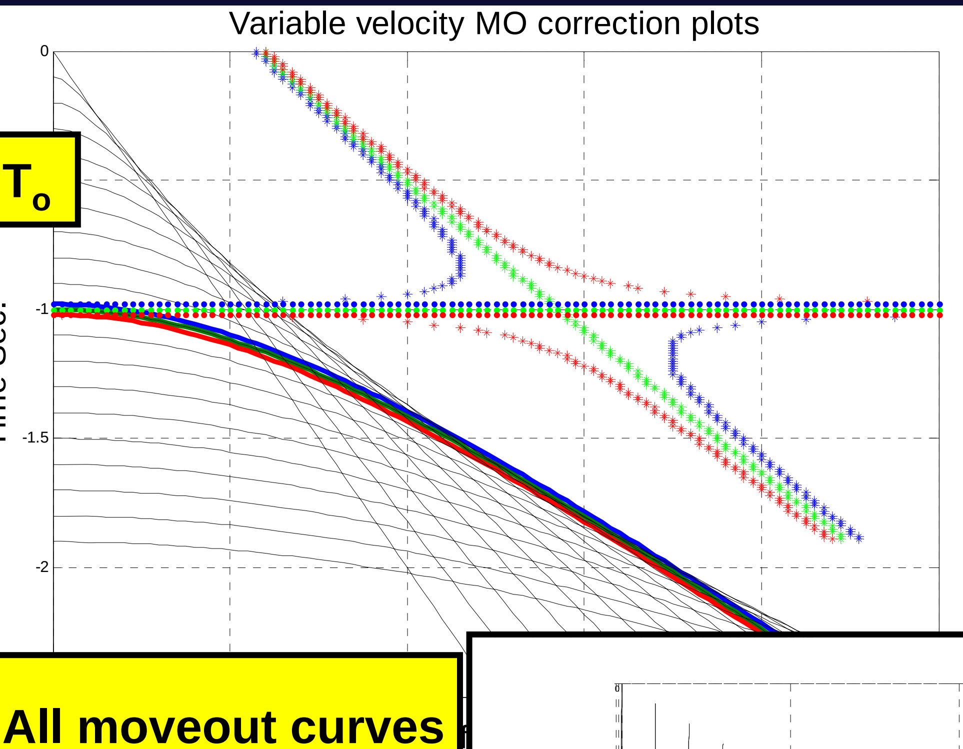
EOM gather

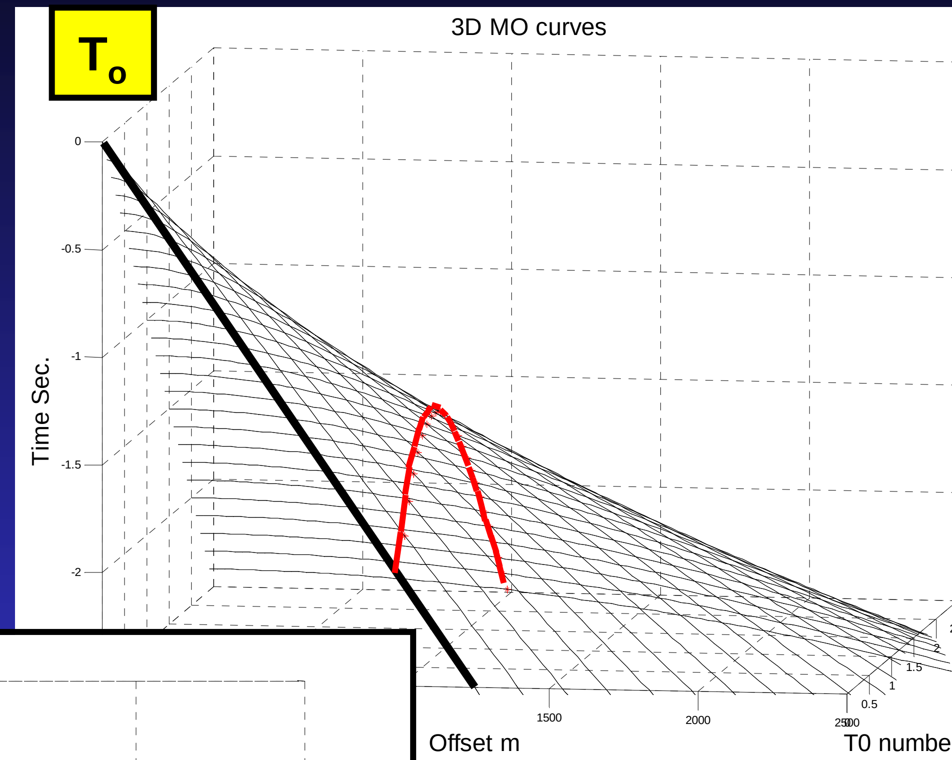




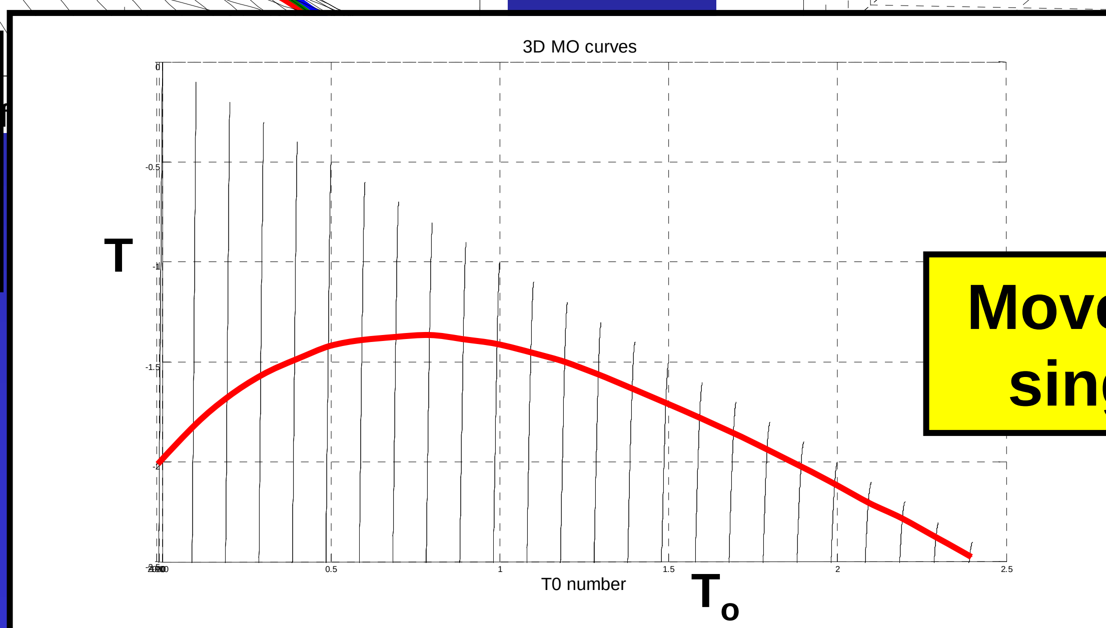
Constant velocity MO plots
Time Sec.
Offset m
MO correction
Constant velocity MO correction plots
Time Sec.
Offset m
Only the event
Variable velocity MO plots
Time Sec.
Offset m
MO correction
Variable velocity MO correction plots
Time Sec.
Offset m

Suite of moveout correction curves (T, h, T_0)





All moveout curves cross the event (multivalued)



Moveout curves single valued

Moveout objectives

- Single valued moveout correction
- Removal of excessive stretch
- Reduce or eliminate aliased condition (???)
- Stretchless moveout correction (???)

Thanks for your attention