High-fidelity time stepping for reverse-time migration

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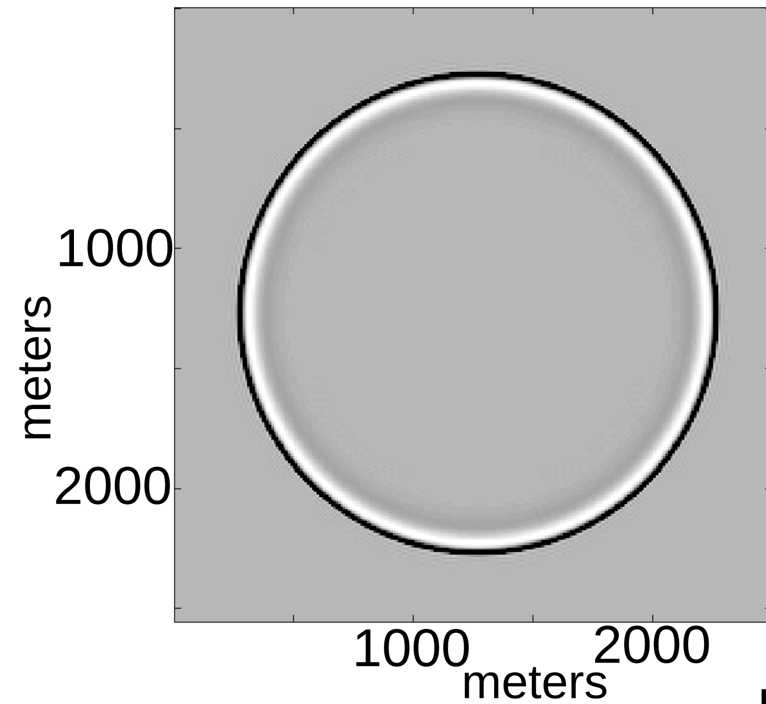
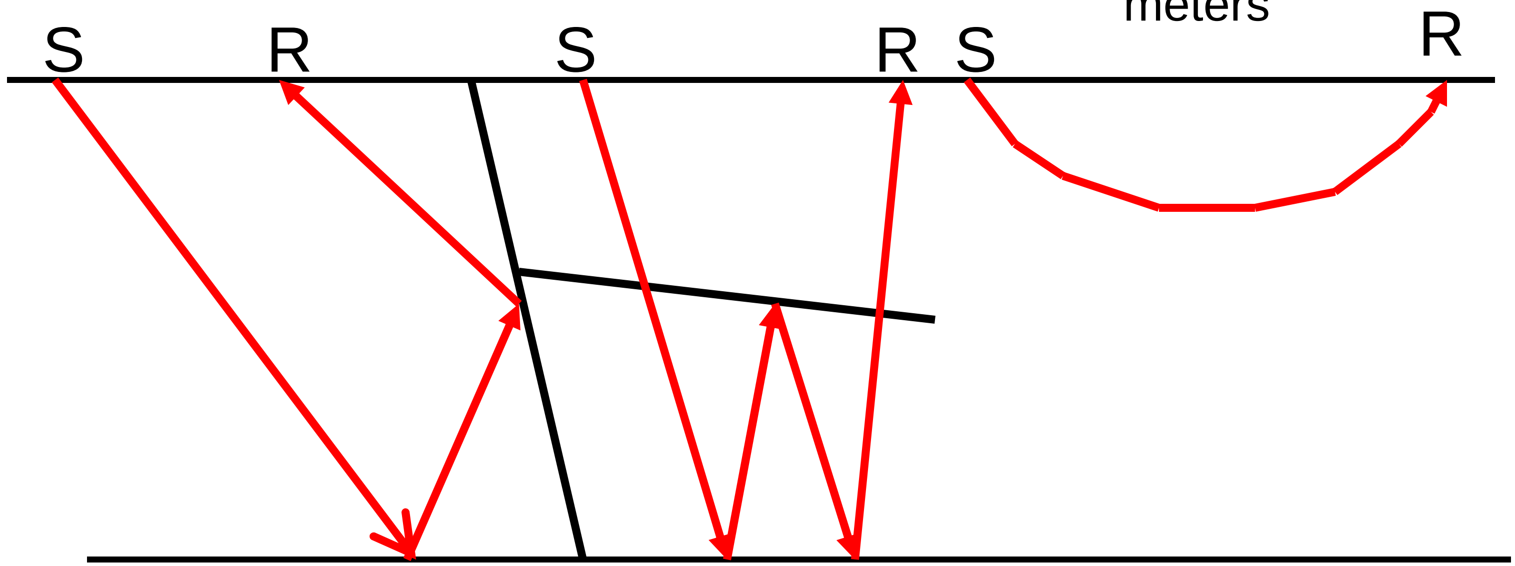
Advantages of RTM





Disadvantages

- Computation
- Memory
- Cross-correlation imaging condition artifacts (prestack)
- Precise velocity model required
- 3-D images are low frequency

Phase-Shift Time Stepping

- Fourier transformation of constant velocity wave equation over space coordinates

$$\hat{U}_{tt} = -c^2({k_x}^2 + {k_z}^2)\hat{U}$$

- If $\hat{U}(t=0,\vec{k}) = \hat{f}(\vec{k})$ and $\dfrac{\partial \hat{U}}{\partial t}(t=0,\vec{k}) = \hat{g}(\vec{k})$

then the D'Alembert solution is

$$2\hat{U}(\Delta t) = \left(\hat{f} + \frac{\hat{g}}{i\omega}\right)e^{i\omega\Delta t} + \left(\hat{f} - \frac{\hat{g}}{i\omega}\right)e^{-i\omega\Delta t}$$

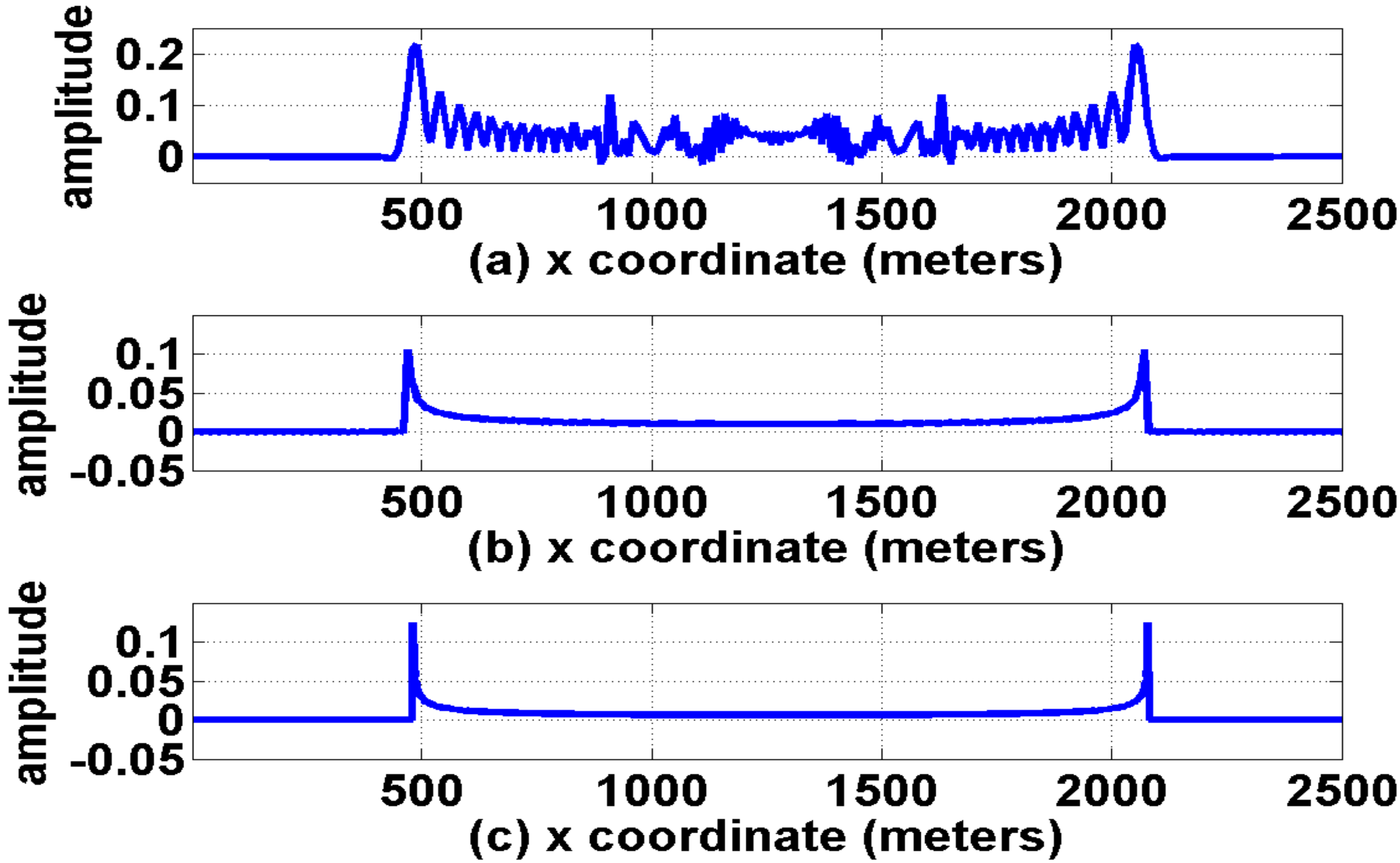
$$2\hat{U}(-\Delta t) = \left(\hat{f} + \frac{\hat{g}}{i\omega} \right) e^{-i\omega\Delta t} + \left(\hat{f} - \frac{\hat{g}}{i\omega} \right) e^{i\omega\Delta t}$$

$$\hat{U}(\Delta t) + \hat{U}(-\Delta t) = \hat{f}\left(e^{-i\omega\Delta t} + e^{i\omega\Delta t} \right)$$

$$\hat{U}(\Delta t) + \hat{U}(-\Delta t) = 2\hat{f}\cos(\omega\Delta t)$$

Exact solution of homogenous wave equation

$$U(\Delta t, \vec{x}) = -U(-\Delta t, \vec{x}) + 2\mathcal{F}^{-1}\{\cos(\omega\Delta t)\mathcal{F}\{U(0, \vec{x})\}\}$$



(a) Finite-difference impulse response, (b) Phase-shift impulse response

(c) Theoretical Green's function.

Aliasing and the time-stepping equation

$$\vec{k} = \left(\pm \frac{1}{2 \Delta x}, \pm \frac{1}{2 \Delta x} \right)$$ Nyquist Numbers

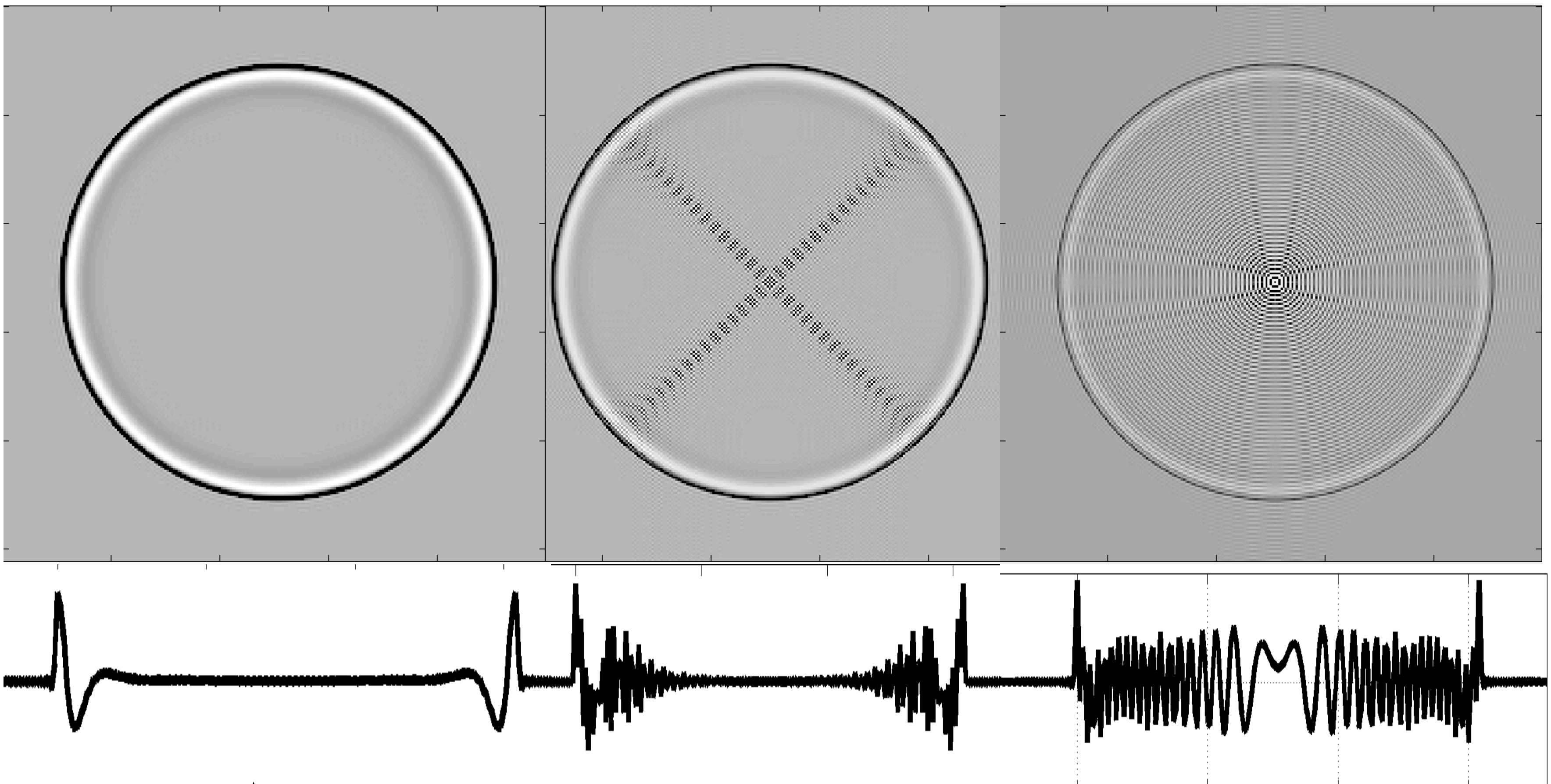
From the dispersion relation $$f(\vec{k}) = c \| \vec{k} \|$$

generate frequencies $$f = \frac{c}{\sqrt{2}\Delta x}$$

but time variable has a Nyquist number $$f_{nyq} = \frac{1}{2\Delta t}$$

Therefore, $$\frac{c\Delta t}{\Delta x} < \frac{1}{\sqrt{2}}$$

Propagation Near Stability



$$0.6 = \frac{c\Delta t}{\Delta x} < 0.71$$

$$0.8 = \frac{c\Delta t}{\Delta x} > 0.71$$

$$1.0 = \frac{c\Delta t}{\Delta x} > 0.71$$

Variable Velocity Model

- Use localized Fourier Tranforms i.e Gabor transform

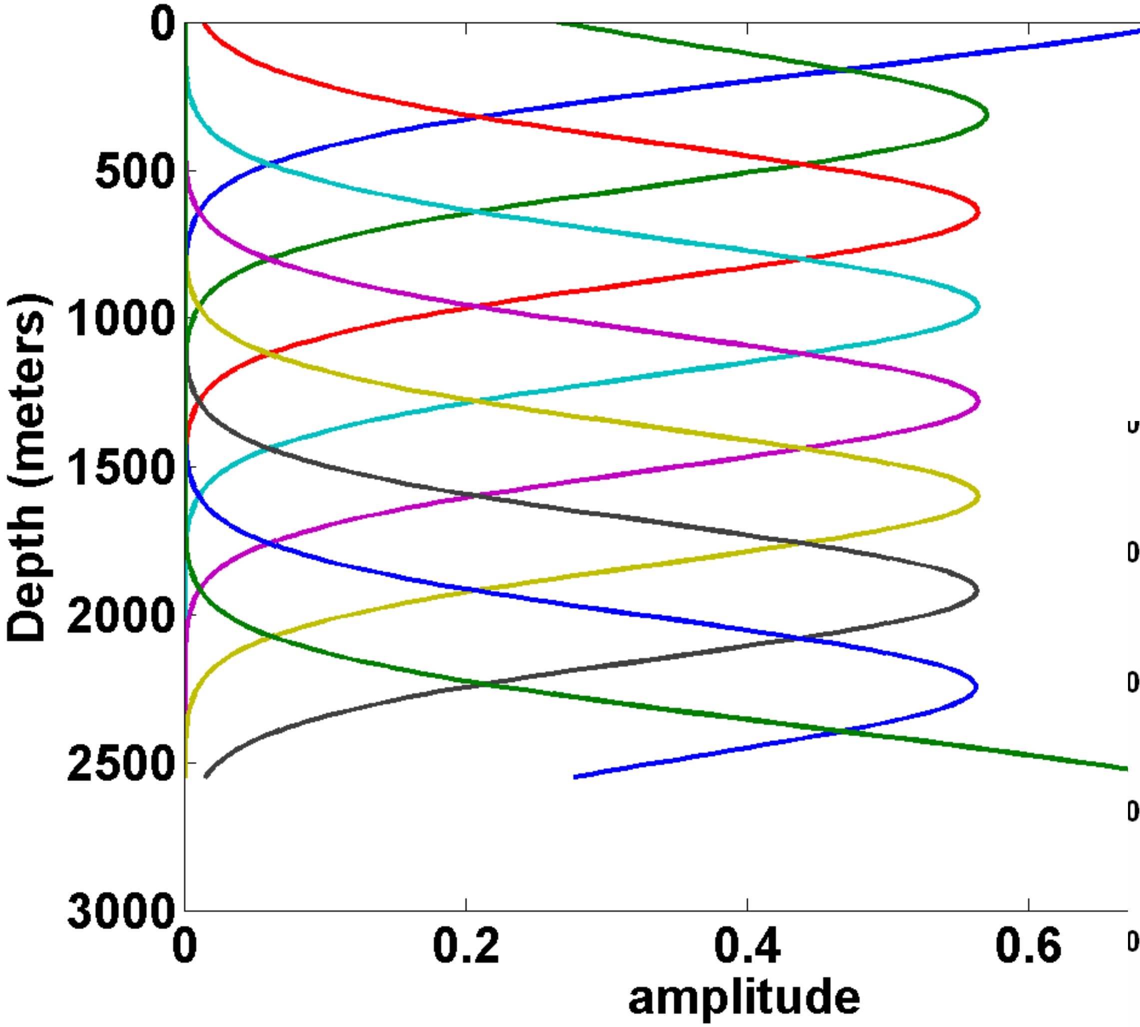
$$\sum_i \Omega_i(\vec{x}) = 1$$

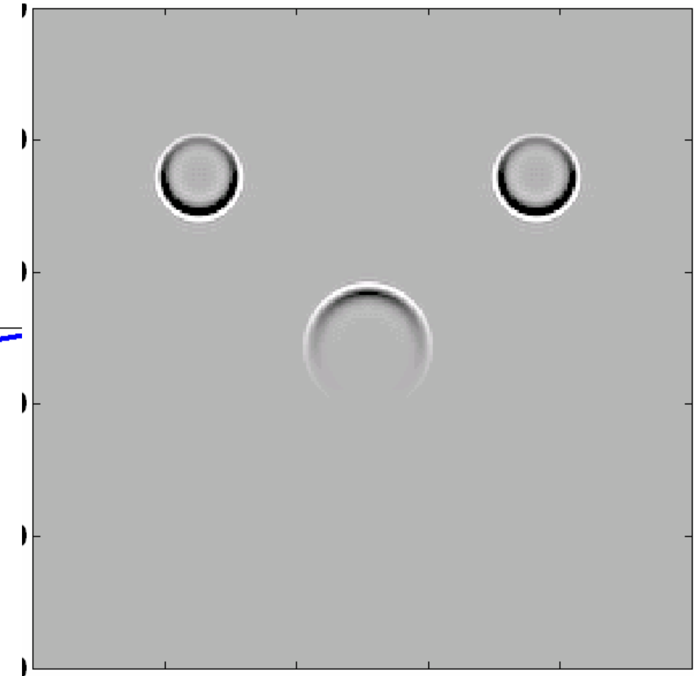
$$U(\Delta t, \vec{x}) = -U(-\Delta t, \vec{x})$$

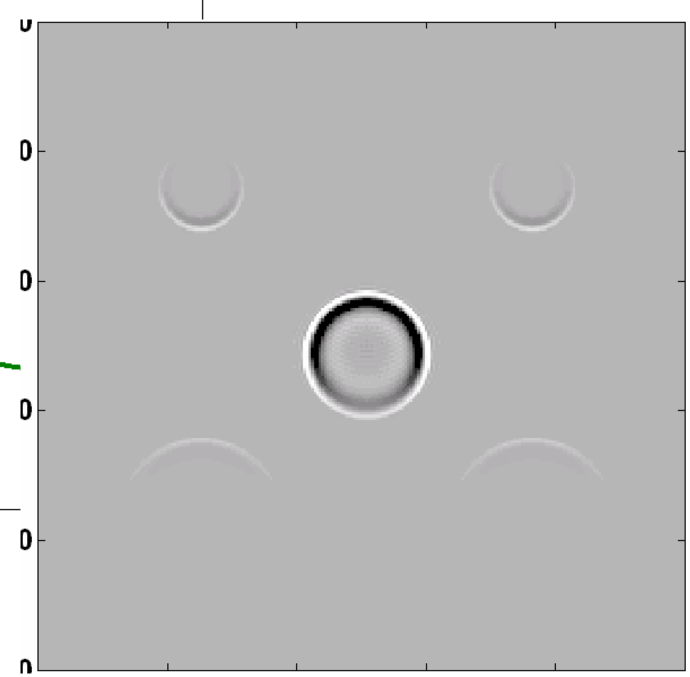
$$+\sum_i \mathcal{F}^{-1}\{2\cos(\omega_i \Delta t)\mathcal{F}\{\Omega_i(\vec{x})U(0,\vec{x})\}\}$$

$$\omega_i = v_i \,\|\vec{k}\,\|$$

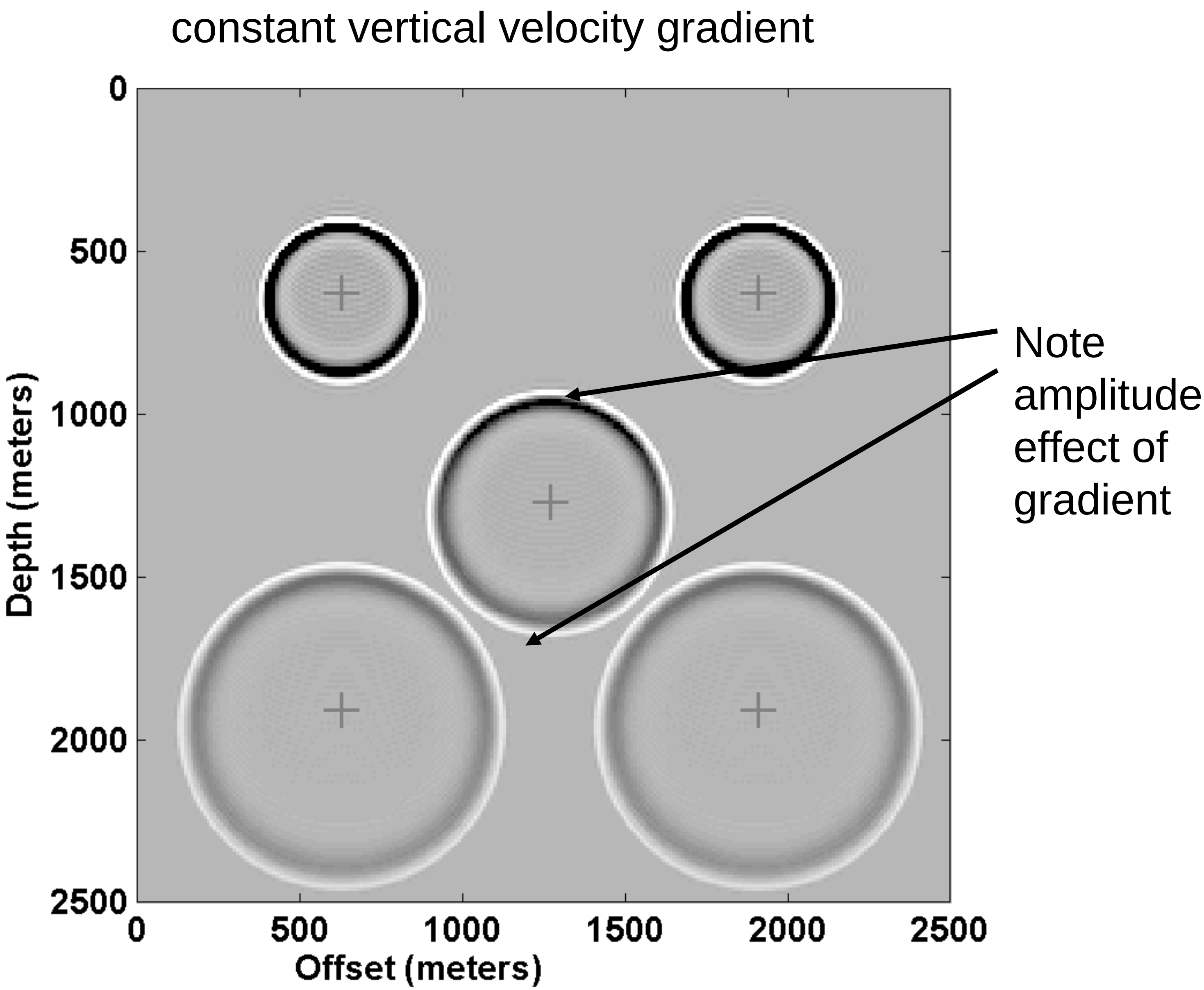
Windows in Depth







Response to minimum phase wavelet



Post-Stack RTM

- Recorded wavefield used as a time-dependent boundary condition.
- Imaging condition: Stacked seismic record is back propagated to time t=0.

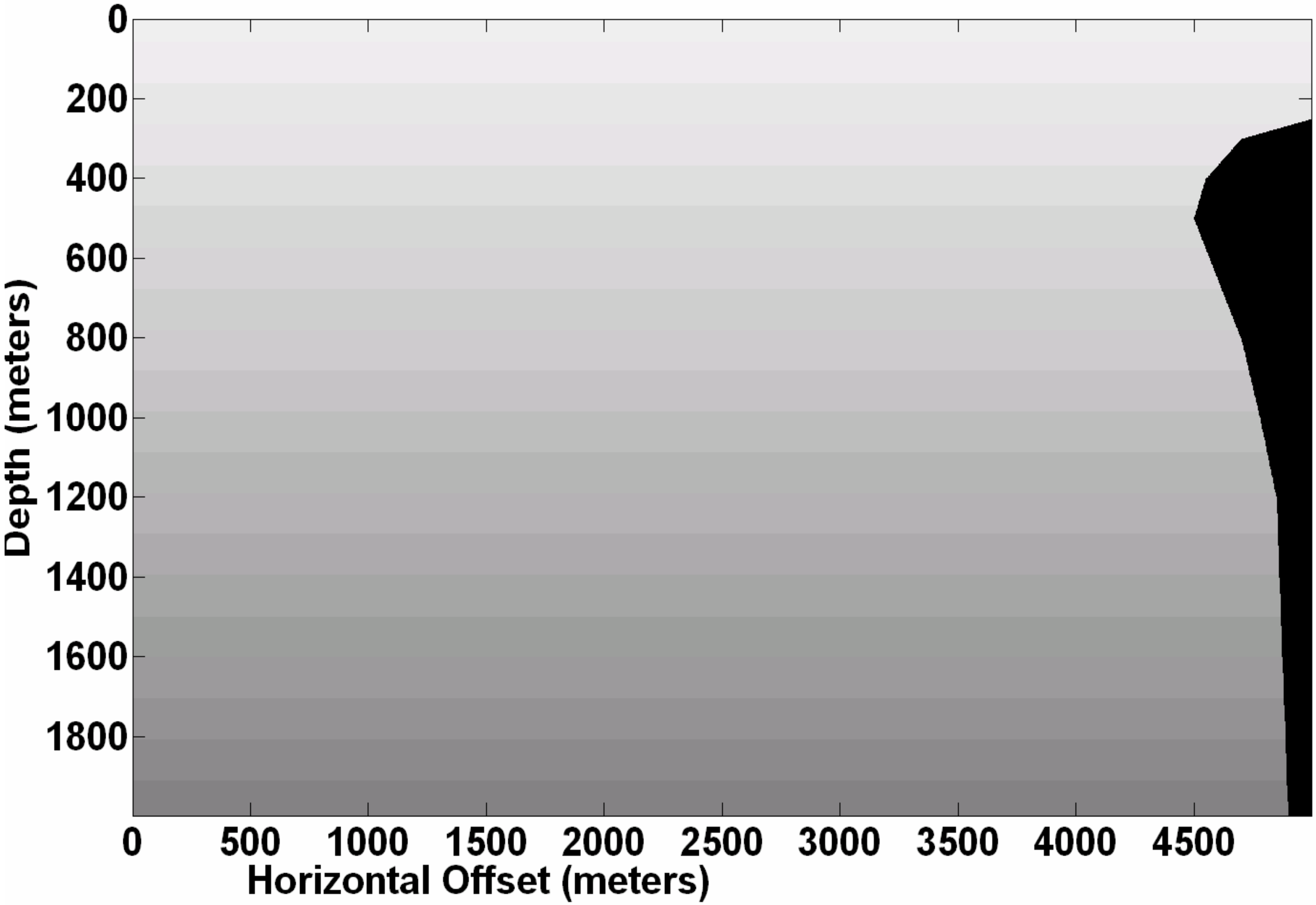
2nd order time Finite-Difference Time Stepping

$$U(t, z, x) = U(k\Delta t, z, x)$$

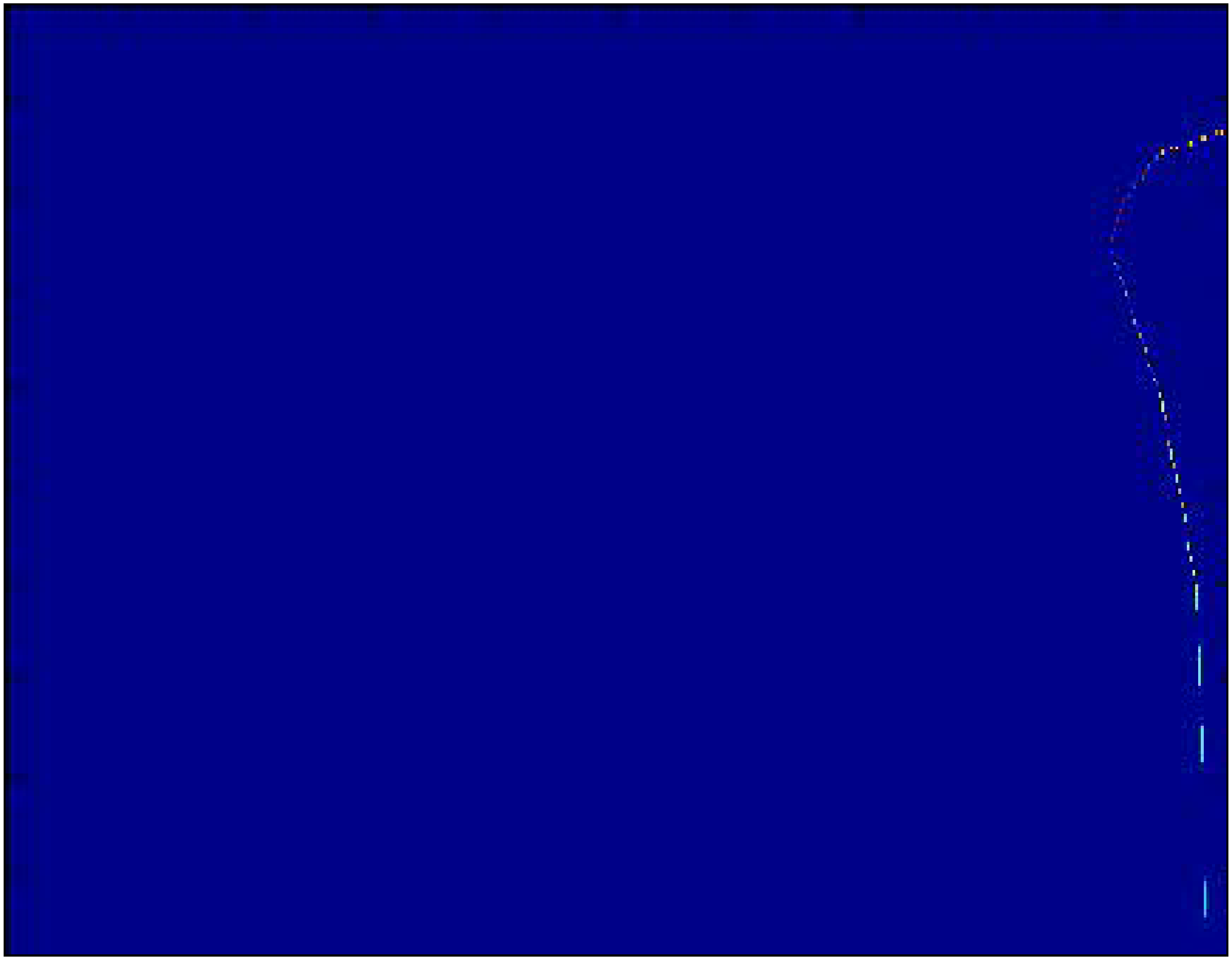
$$U_{k+1} = 2U \;\; - U_{k-1} + \frac{(\Delta t)^2}{c^2} \Delta U$$

- Requires expensive oversampling in time and space to control numerical dispersive

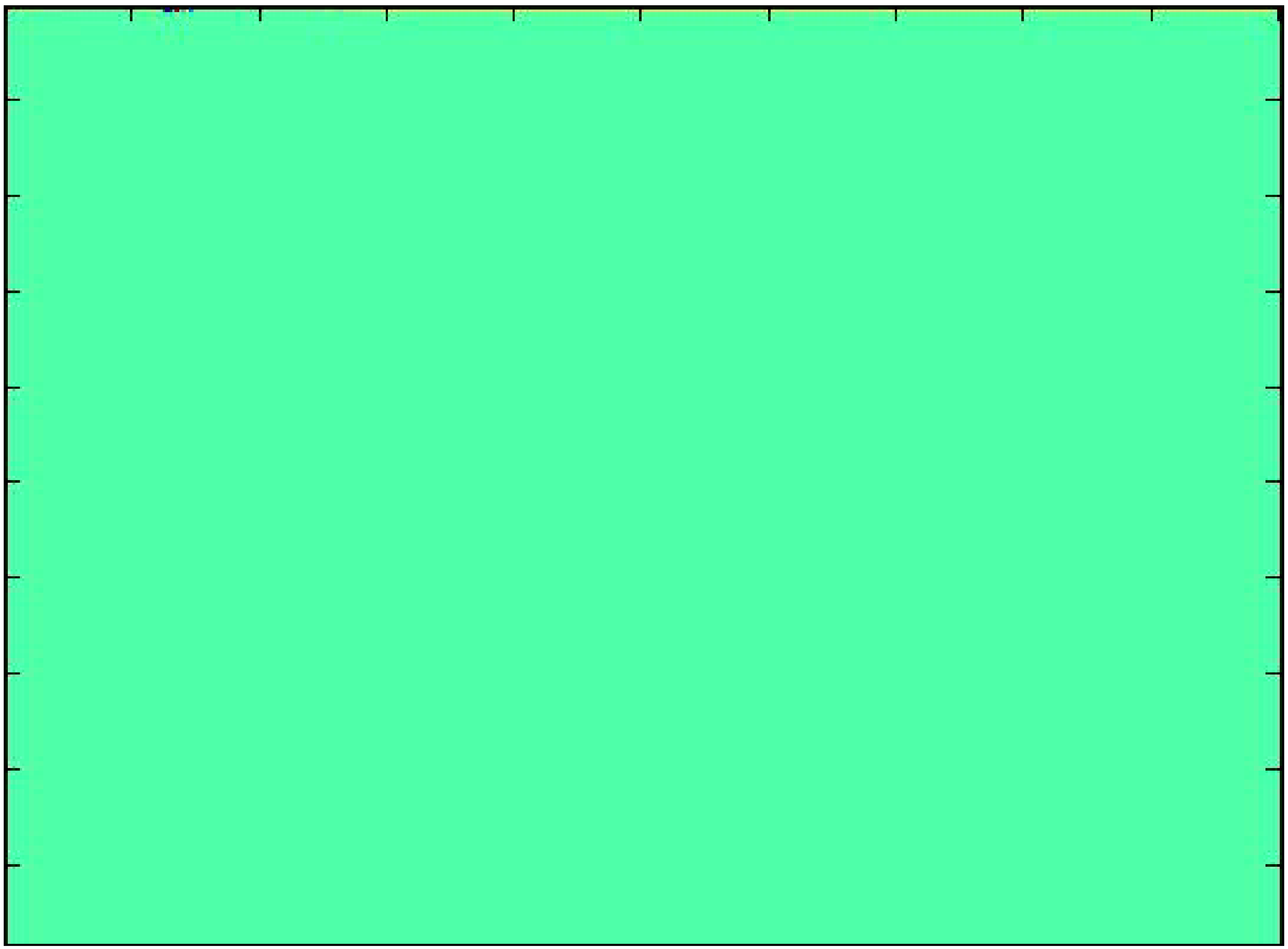
Velocity Model



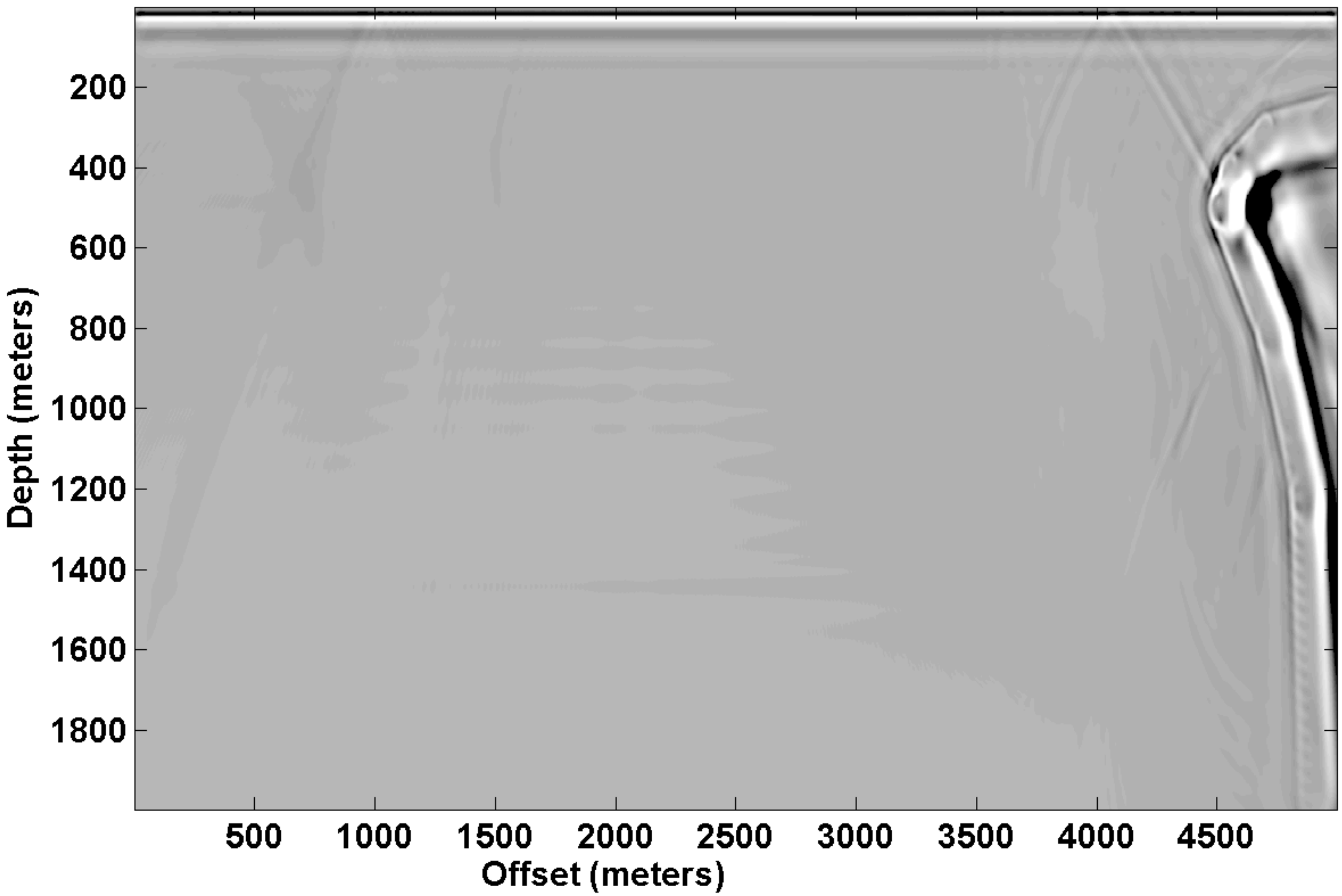
Exploding Reflector Propagation



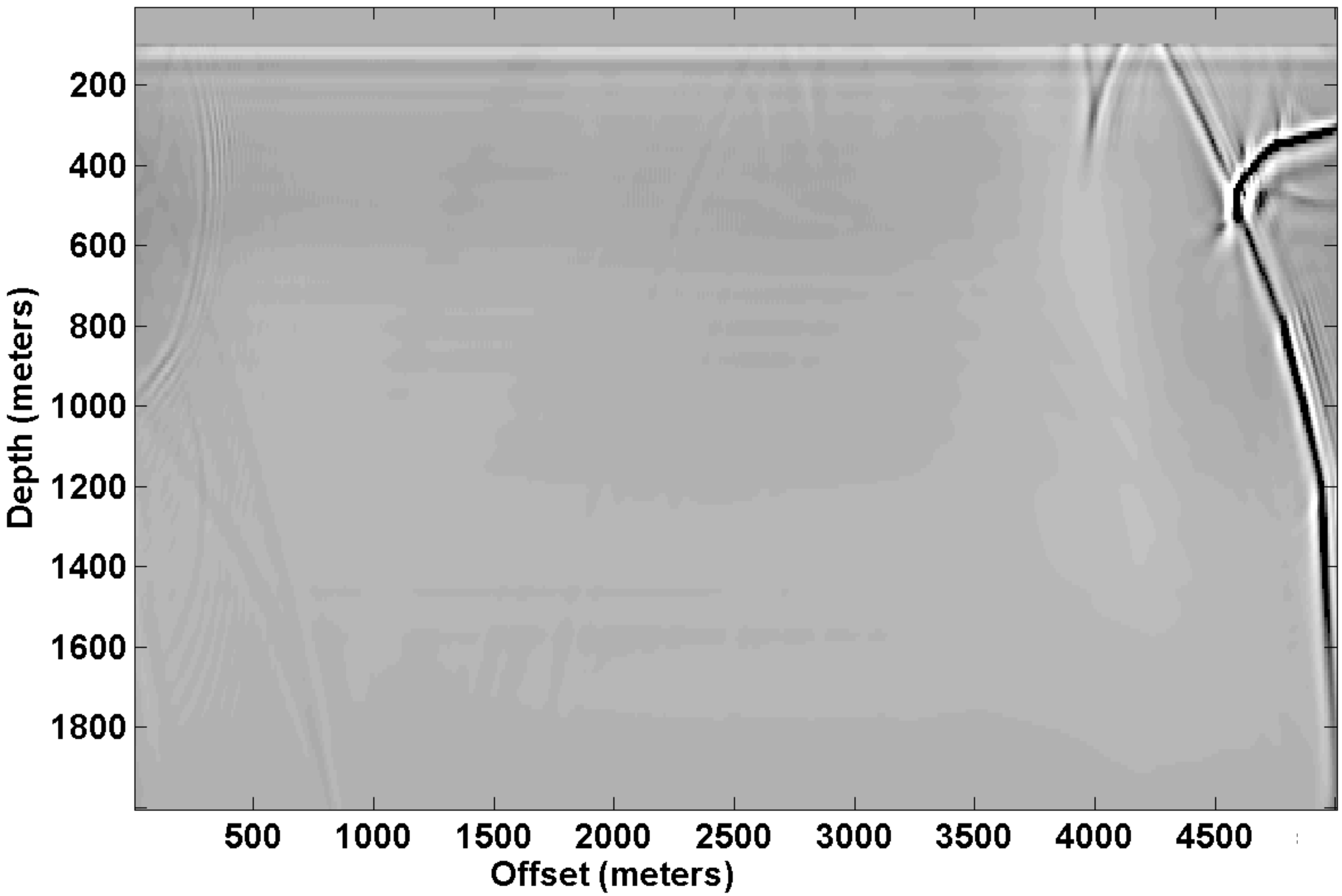
Phase-Shift time stepping for RTM of Saltdome



RTM of a Exploding Reflector model



Phase-Shift RTM



Conclusions

- The phase shift time stepper used for RTM
  - an exact solution to the homogenous wave equation
  - non dispersive
  - no dip limitation
  - Much larger time step than finite difference
  - Not presently capable of reflecting energy (no multiples)
  - highly accurate impulse response
  - Adapted to variable velocity by windowing the spatial wavefield and propagating with a constant velocity time stepper

Future Work

- Prestack Migration
- Extend method to V(x,z) media
- Construct a time stepper that can reflect energy
- Experiment with different windows/partitions of unity.