Microseismic focal mechanisms: A tutorial
...beyond dots in a box

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McGillivray, 2005
Outline

• Magnitude scales
• Earthquake spectra
• Demystifying beach balls
• Moment tensors
• Stress transfer
General form:

\[ M = \log \left( \frac{A}{T^n} \right) + Q(h, \Delta) \]

where

- \( A \) is signal amplitude
- \( T \) is dominant period
- \( h \) is focal depth
- \( \Delta \) is distance

Bolt, 1993
Richter magnitude

Richter scale (California):

\[ M_L = \log A + 2.56 \log \Delta - 1.67 \]

where

- \( A \) is maximum amplitude in mm measured on a Wood-Anderson seismograph
- \( \Delta \) is epicentral distance in km

Bolt, 1993
Magnitude versus Energy

- Each unit increase in magnitude corresponds to a 30-fold increase in Energy.

Typical range for micro-seismic events.
\[ M_0 = \mu DA \]

where

- \( \mu \) is shear modulus (rigidity)
- \( D \) is average slip
- \( A \) is rupture area

**Moment magnitude:**

\[ M_w = \log M_0 / 1.5 - 10.73 \]

- 1906 San Francisco (7.8)
- 1964 Alaska (9.1)
- 1960 Chile (9.5)
- 2004 Sumatra (9.0 – 9.3)
Spectral Characteristics

Dislocation on a small circular crack (Brune source model)

Far-field spectra

\[ \tilde{a}(\omega) = \frac{M_0 \omega^2}{1 + \left( \frac{\omega}{\omega_c} \right)^2} \]

\[ \tilde{d}(\omega) = \frac{M_0}{1 + \left( \frac{\omega}{\omega_c} \right)^2} \]

\[ \omega_c = 1/\tau \text{ is the corner frequency} \]

\[ d(t) = D \left[ 1 - (1 + t/\tau) e^{-t/\tau} \right] \]
Spectral Characteristics

Example: small earthquake in Georgian Bay, Ontario

\[ \omega_c \sim 25 \, \text{s}^{-1} \]

\[ \lim_{\omega \to 0} \tilde{d} = M_0 \]

\[ \lim_{\omega \to \infty} \tilde{a} = M_0 \omega_c \]

Dineva et al., 2006
Spectral Characteristics

Source radius

\[ R \approx 2.34 \frac{V_s}{\omega_c} = 280m \]

Stress drop

\[ \omega_c \approx 2.34 \times 2V_s \left( \frac{\Delta \sigma}{M_0} \right)^{1/3} \]

\[ \Delta \sigma \sim 20 \text{ bars} \]
Seismic Moment Tensor

\[ \mathbf{M} = \mathbf{M}_0 \begin{bmatrix} M_{xx} & M_{xy} & M_{xz} \\ M_{yx} & M_{yy} & M_{yz} \\ M_{zx} & M_{zy} & M_{zz} \end{bmatrix} \]

- A more general representation of an earthquake source
- Each tensor component represents a force couple
- Since \( \mathbf{M} \) is symmetric (zero net torque), 6 are independent

Aki and Richards, 1980
Most earthquakes can be approximated by a **double-couple**

As with other forms, eigenvectors of $\mathbf{M}$ yield principal stress axes ($P$, $T$)

\[
\begin{bmatrix}
0 & M_0 & 0 \\
M_0 & 0 & 0 \\
0 & 0 & 0 \\
\end{bmatrix}
\]
An explosive source is represented by an isotropic moment tensor

\[
\begin{pmatrix}
M_0 & 0 & 0 \\
0 & M_0 & 0 \\
0 & 0 & M_0
\end{pmatrix}
\]
A crack opening under tension (fluid injection) can be represented by the sum of an isotropic moment tensor and a compensated linear vector dipole (CVLD).

\[
\begin{bmatrix}
M_0 & 0 & 0 \\
0 & -2M_0 & 0 \\
0 & 0 & M_0
\end{bmatrix}
\]

http://www.iwb.uni-stuttgart.de/grosse/aet/mti.htm
Moment Tensor Inversion

- Waveform inversion for source mechanism
- Requires good velocity model

Ma and Eaton, 2008
Stress transfer due to an earthquake can be modelled using the so-called Coulomb failure function (Stein, 1999)

\[ \Delta \sigma_f = \Delta \tau + \mu(\Delta \sigma_n + \Delta P) \]

Where

- \( \Delta \tau \) is the change in shear stress
- \( \mu \) is the coefficient of friction
- \( \Delta \sigma_n \) is the normal stress
- \( \Delta P \) is change in pore pressure

King et al., 1994
Although far-field stress changes are small (a few bars or less), earthquake aftershocks are more probable in regions of increased $\Delta \sigma_f$ and less probable in regions of decreased $\Delta \sigma_f$ (Stein, 1999)

→ Potential application to induced microseismicity from hydraulic fracturing?

King et al., 1994
Key points

• Various methods are used to describe earthquakes (magnitude, seismic moment, focal mechanism) and are applicable, in principle, to microseismic monitoring studies.

• Application of these methods may yield useful information about stress state and failure mechanisms.

• Recent models for stress transfer may also have applicability to modelling and understanding induced seismicity.