Color correction for Gabor deconvolution and Nonstationary Phase Rotations

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Outline

• Gabor deconvolution

• Color correction methods

• Nonstationary phase rotation

• Conclusions
Gabor deconvolution

- Gabor Transform (GT)
  - Extend FT to the nonstationary realm

\[
S_G(\tau, f) = \int_{-\infty}^{\infty} s(t) g(t - \tau) e^{-i2\pi ft} dt
\]

\( g(t) \) : Gabor analysis window
\( \tau \) : window center
Gabor deconvolution

• Nonstationary model of seismic traces

\[
\hat{s}(f) = \hat{w}(f) \int_{-\infty}^{\infty} \alpha_{Q}(\tau, f) r(\tau) e^{-i2\pi f \tau} d\tau
\]

Constant-Q attenuation:

\[
\alpha_{Q}(\tau, f) = e^{\frac{-\pi f \tau}{Q} + i\text{Li}(\frac{\pi f \tau}{Q})}
\]
Gabor deconvolution

- GT of attenuated seismic trace
  \[ S_G(\tau, f) \approx \hat{w}(f)\alpha(\tau, f)R_G(\tau, f) \]
- Estimated propagating wavelet
  \[ \hat{w}(f)\alpha_Q(\tau, f) \approx \sqrt{|S_G(\tau, f)|}e^{i\varphi(\tau, f)} \]
  \[ \varphi(\tau, f) = \int_{-\infty}^{\infty} \frac{\ln(|S_G(\tau, f')|)}{f - f'} df' \]
  Assumption of white reflectivity: \[ |R_G(\tau, f)| \approx 1 \]
Gabor deconvolution

- Estimated reflectivity

\[ R_G(\tau, f)_{est} = \frac{S_G(\tau, f)}{|S_G(\tau, f)| + \mu A_{max}} e^{-i\phi(\tau, f)} \]

with assumption of white reflectivity
Color correction

• Nonwhite reflectivity in practice

\[ r_c(t) \rightarrow | R'_G(\tau, f) | \neq 1 \]
Nonwhite reflectivity

(a) Nonwhite reflectivity

(b) Amplitude spectrum
Nonwhite reflectivity
**Color correction**

- **Condition:**
  \[
  \left| R'_G(\tau, f) \right| \text{ is available from well log}
  \]

- **Estimation of nonwhite reflectivity**
  \[
  R'_G(\tau, f)_{est} = \frac{S_G(\tau, f) \left| R'_G(\tau, f) \right|}{\left| S_G(\tau, f) \right| + \mu A_{max}} e^{i\varphi_c(\tau, f)}
  \]

  \[
  \varphi_c(\tau, f) = H(\ln \frac{R'_G(\tau, f)}{S_G(\tau, f) + \mu A_{max}})
  \]
Color correction

• Effect of color correction

\[ R_G'(\tau, f)_{est} = \frac{S_G(\tau, f) R_G'(\tau, f)}{|S_G(\tau, f)| + \mu A_{max}} e^{i\phi_c(\tau, f)} \]

How much does \( |R_G'(\tau, f)| \) depart from unity?
How reliable is \( |R_G'(\tau, f)| \)?

Influential factors:

- available frequency band
- completeness of well log
An ideal case of color correction

• Complete well log

• Broad frequency band for deconvolution
An ideal case of color correction (10-150Hz)
Effect of limited frequency band (10-100Hz)
Effect of limited frequency band (10-60Hz)
Practical color correction

• Incomplete well log

  seismic trace: \( s(t) \) \( t \in (0, t_{\text{max}}) \)

  well log: \( \tilde{r}_c(t) \) \( t \in (t_1, t_2) \) \( 0 < t_1 < t_2 < t_{\text{max}} \)

\[ \tilde{r}_c(t) \rightarrow \left| R_G'(\tau, f) \right| ? \]
Practical color correction

• Creation of correction pattern

Method 1:

\[ \tilde{R}_c(t) \rightarrow \tilde{R}_c(f) \]

\[ |\tilde{R}_c(f)| \approx a_0 + a_1f + a_2f^2 \]

\[ |R'_G(\tau, f)| = a_0 + a_1f + a_2f^2 \]

Color feature is temporally stationary
Practical color correction (method 1)

(a) Incomplete nonwhite reflectivity

(b) Amplitude spectrum

true amplitude

polynomial approximation
Practical color correction (method 1)
Practical color correction

• Method 2: color feature is time-variant

\[ \tilde{R}_c(t) \rightarrow \tilde{R}_G(\tau, f) \]

\[ |\tilde{R}_G(\tau, f)| \approx a_0'(\tau) + a_1'(\tau)f + a_2'(\tau)f^2 \quad \tau \in [t_1, t_2] \]

\[ |R'_G(\tau, f)| = a_0(\tau) + a_1(\tau)f + a_2(\tau)f^2 \quad \tau \in [0, t_{\max}] \]

\[ a_i(\tau) = \begin{cases} 
\alpha_i'(t_1), & 0 \leq \tau \leq t_1 \\
\alpha_i'(t_1), & t_1 < \tau < t_2 \quad i = 1, 2, 3 \\
\alpha_i'(t_2), & t_2 \leq \tau \leq t_{\max} 
\end{cases} \]
Practical color correction (method 2)

(a) Coefficient curve: $a_2$
(b) Coefficient curve: $a_1$
(c) Coefficient curve: $a_0$

Amplitude vs. Time (sec)
Practical color correction (method 2)
Practical color correction

• Method 3: extension of method 2

\[ a_i(\tau) \] can be obtained through interpolation when multiple well logs are available.
Practical color correction (method 3)

(a) Coefficient curve: $a_2$

(b) Coefficient curve: $a_1$

(c) Coefficient curve: $a_0$

Amplitude vs. Time (sec)
Practical color correction (method 3)
Nonstationary phase rotation

• Constant phase rotation

\[ s_\theta(t) = s(t)\cos\theta + s_{\pi/2}(t)\sin\theta \]

• Nonstationary phase rotation

\[ s'_\theta(t) = s(t)\cos\theta(t) + s_{\pi/2}(t)\sin\theta(t) \]
Removal of nonstationary phase rotation

- Method 1
  
  remove the phase rotation in Gabor windows

- Method 2
  
  \[ g(t) = s'_0(t) \]

  \[ s'(t) = g(t) \cos \theta(t) - g_{\pi/2}(t) \sin \theta(t) \]
Measured T.V. phase rotation after G.D.
Measured T.V. phase rotation after phase rotation removal

![Graph showing removal of phase rotation over time for different methods.](image-url)
Conclusions

• Real reflectivity is not white and its color feature is time-variant.
• White assumption causes distortion in conventional Gabor decon.
• Color correction is the most important if frequency band is broad
• Three effective methods for practical color correction were shown
• Practical way to define and remove nonstationary phase rotation was investigated
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