Seismic data interpolation using a fast generalized Fourier transform

Mostafa Naghizadeh and Kris Innanen

CREWES
University of Calgary

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Outlines:

- Introduction
- Fourier analysis of nonstationary signals
- Fast Generalized Fourier Transform
- Fast Generalized Fourier Interpolation
  - 1D Synthetic chirp examples
  - Synthetic and real seismic data
- Conclusion

Introduction I:

- **Fourier transform**
  - **Stationary signals:** constant frequency/wavenumber content at all time/space samples.
- **Time-Frequency analysis**
  - **Nonstationary signals:** Dynamic frequency/wavenumber content.
  - Methods for analysis of nonstationary signals:
    - **Short-Time Fourier Transform** (for example Gabor transform)
    - **Wavelet Transform** (based on progressive resolution concept)
    - **S-Transform** (frequency dependent windowing)
    - **Curvelet Transform** (local and directional decomposition)
  - All of the methods for nonstationary signal analysis use Fourier analysis as their core building block.

Introduction II:

- **Nonstationary signals in seismic data processing**
  - **Time axis:**
    - **Dispersive and attenuated seismic traces**
  - **Spatial axes:**
    - **Hyperbolic and parabolic events**
    - **Dispersive events**
    - **Discontinuities**
- **Interpolation of spatially nonstationary seismic data**
  - **Windowing before interpolation**
  - **Adaptive prediction filters (Naghizadeh and Sacchi, 2009)**

Fourier analysis of nonstationary signals

The source

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A General Description of Linear Time-Frequency Transforms and Formulation of a Fast, Invertible Transform That Samples the Continuous S-Transform Spectrum Nonredundantly

Robert A. Brown, M. Louis Lauzon, and Richard Frayne

***Abstract*—Examining the frequency content of signals is critical in many applications, from neuroscience to astronomy. Many techniques have been proposed to accomplish this. One of these, the S-transform, provides simultaneous time and frequency information similar to the wavelet transform, but uses sinusoidal basis functions to produce frequency and globally referenced phase**

dimensions into frequency or frequency-analogue spaces have proliferated [1]–[4].

While the original Fourier transform (FT) is an extremely important signal and image analysis tool, it assumes that a signal is stationary, i.e., that the frequency content is constant at all

S-Transform (Stockwell et al., 1996)

S-Transform (Time lag, Frequency)

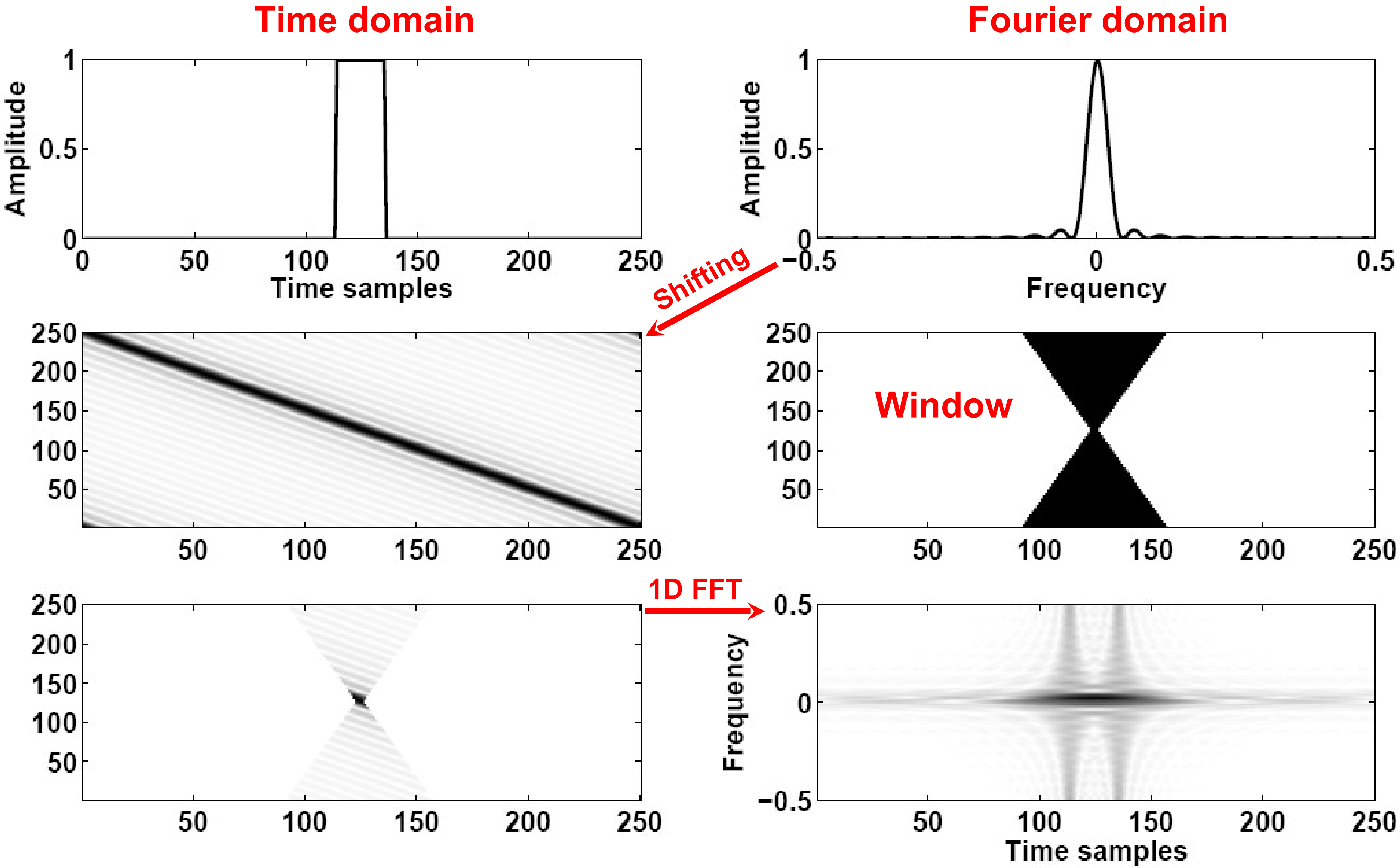
$$S(\tau,\omega) = \int_{-\infty}^{\infty} g(t) \frac{|\omega|}{\sqrt{2\pi}} e^{-\frac{(\tau-t)^2\omega^2}{2}} e^{-i2\pi\omega t} dt$$

Signal

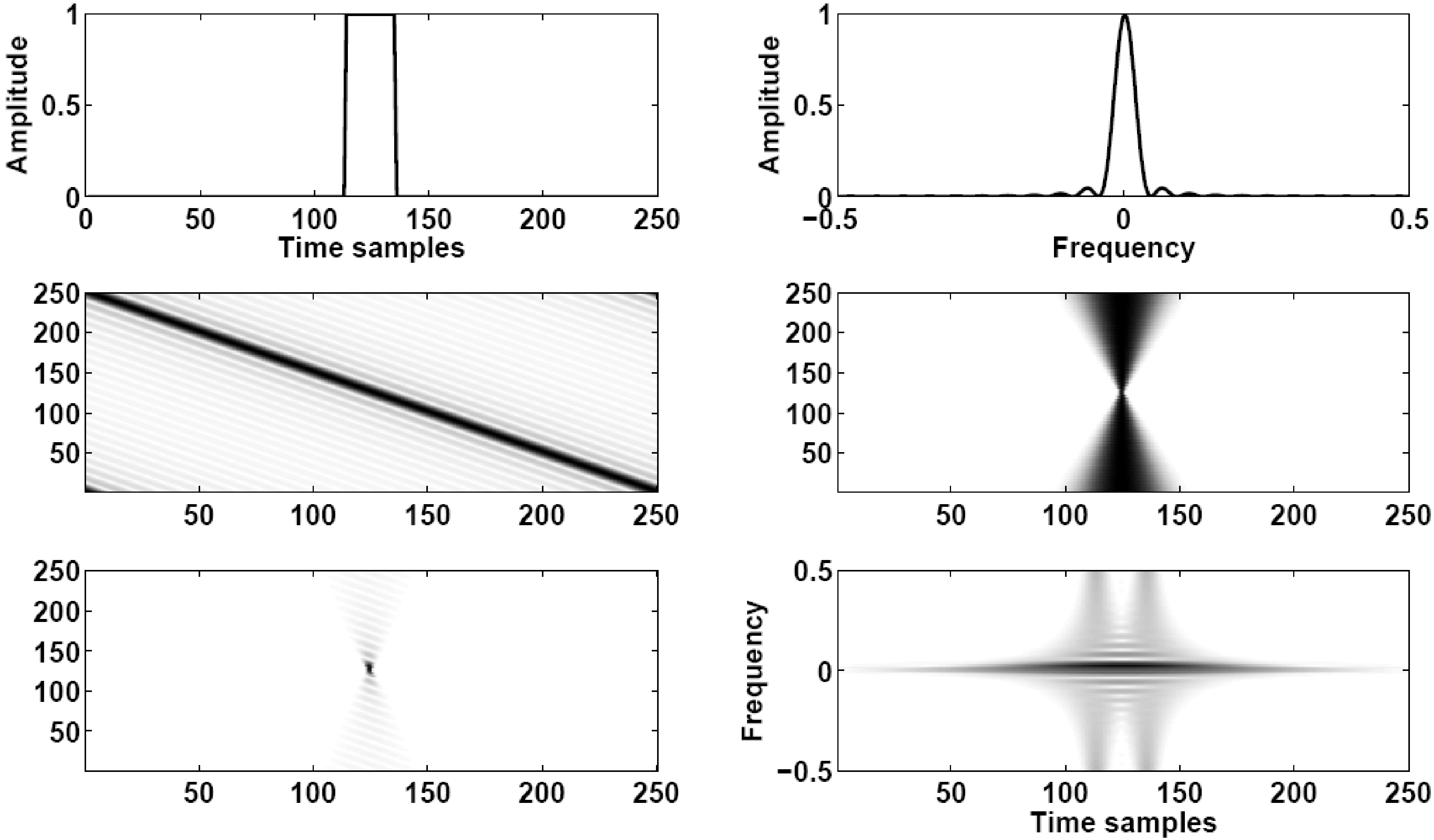
Frequency-dependent time window

Fourier Kernel

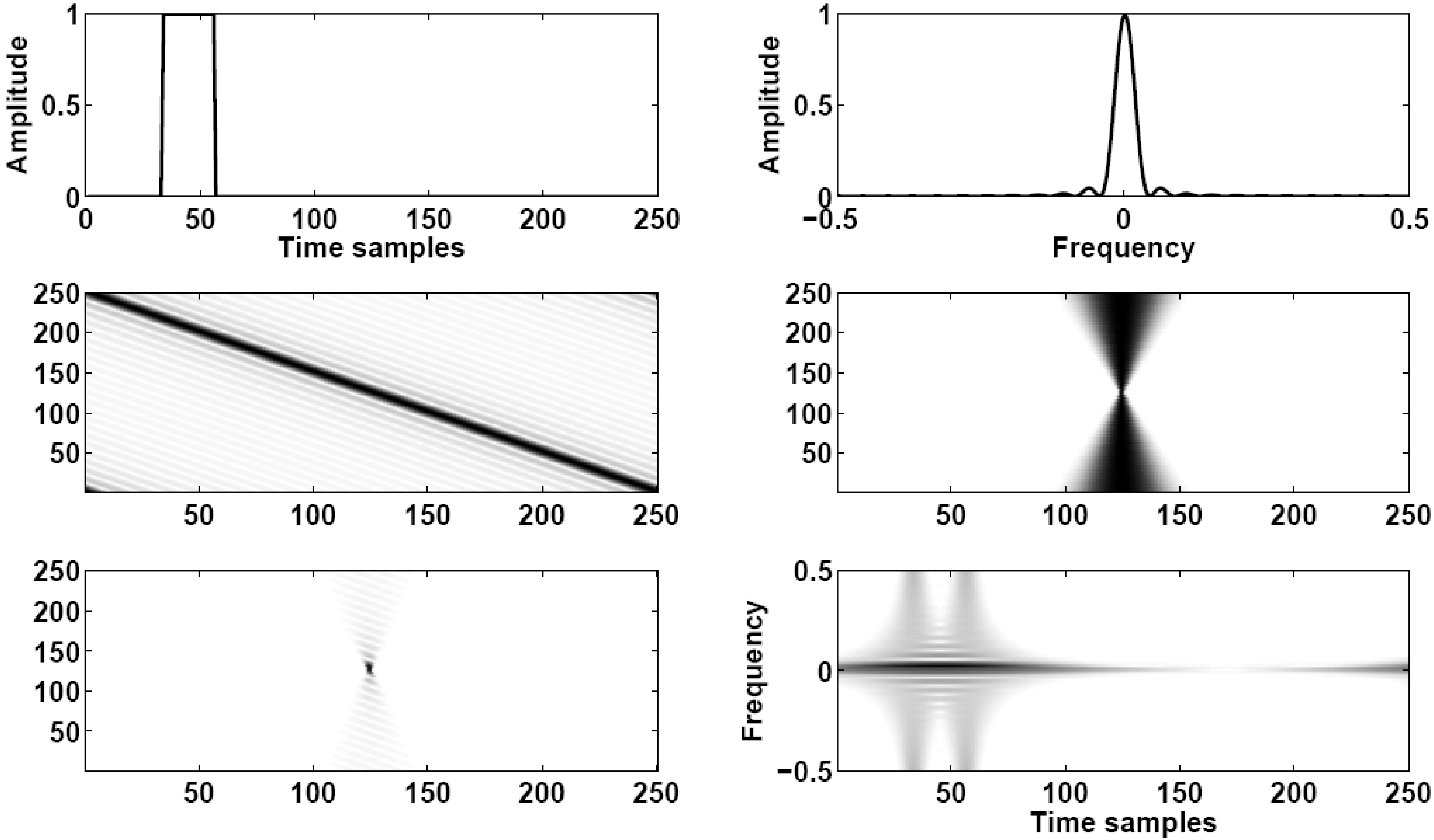
S-Transform in frequency domain
Box function (1)



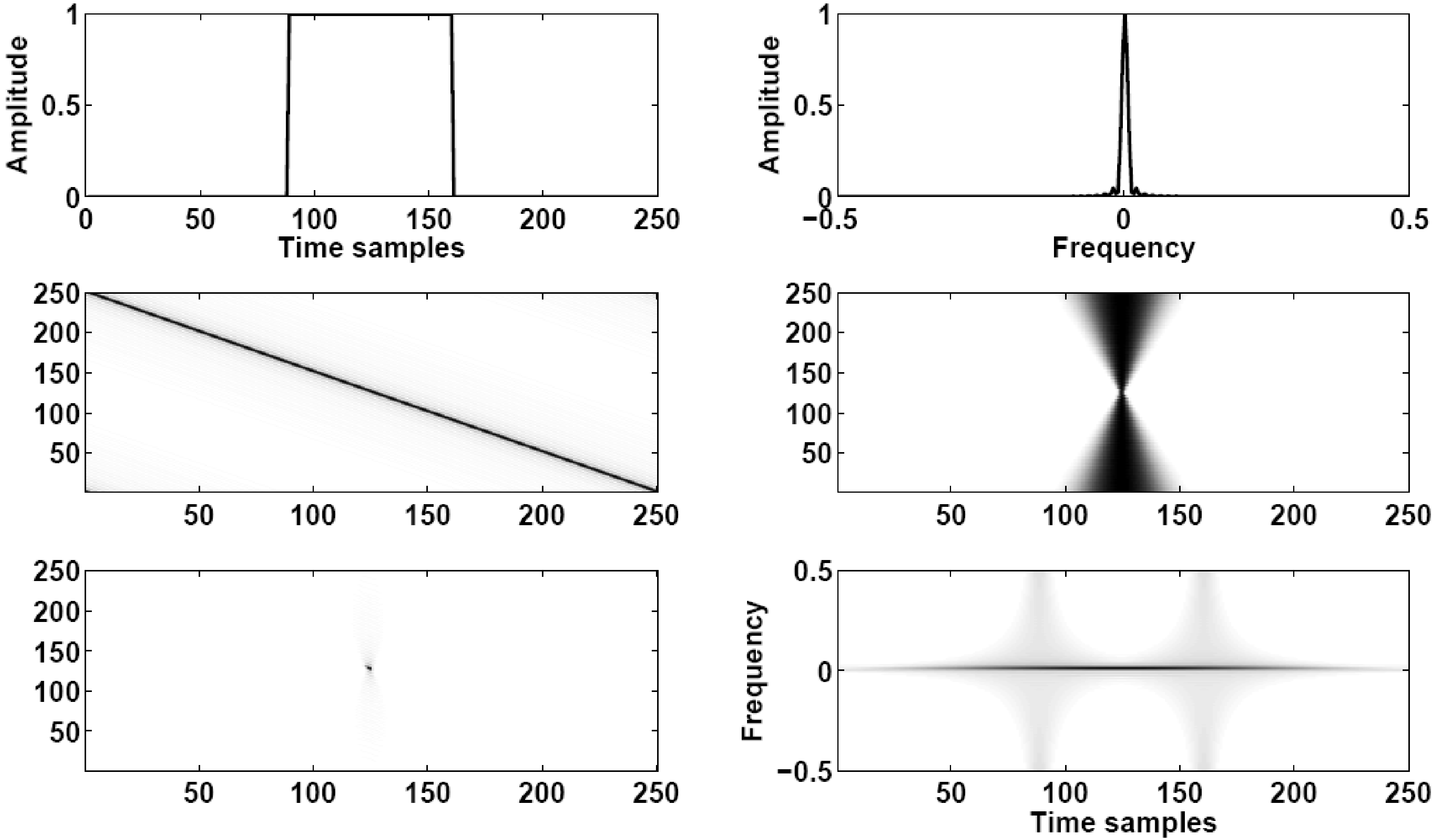
Box function (2)



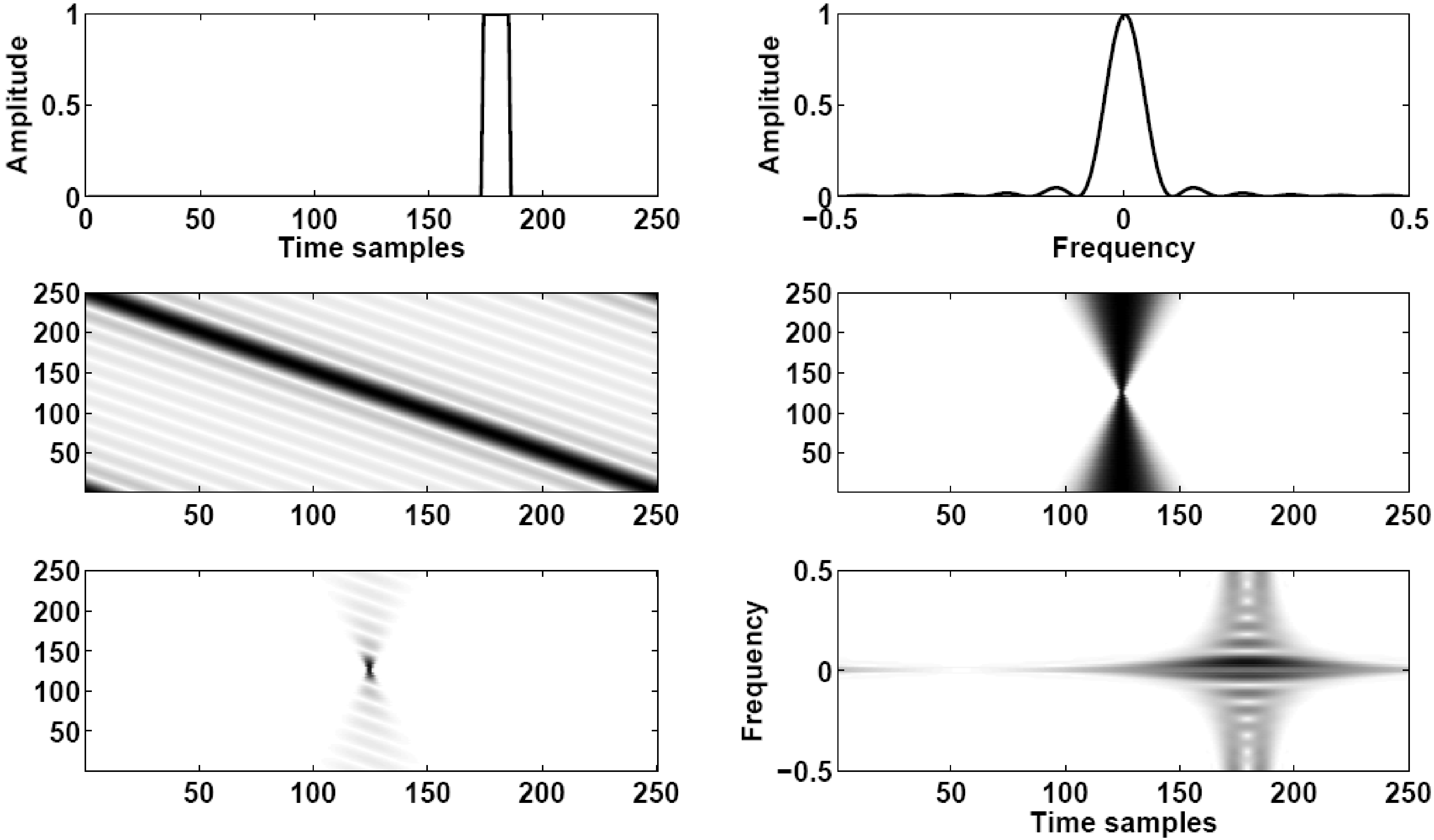
Box function (3)



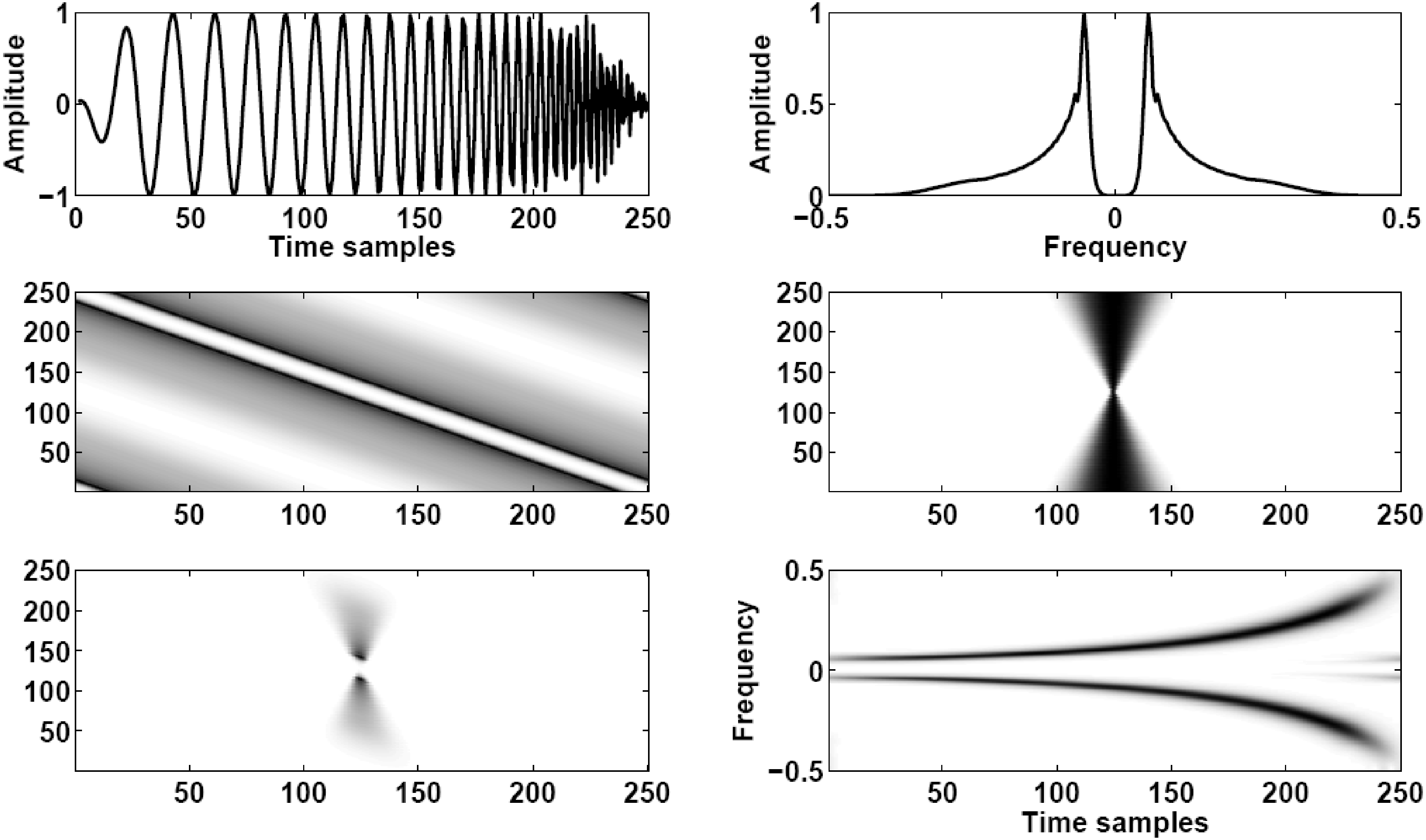
Box function (4)



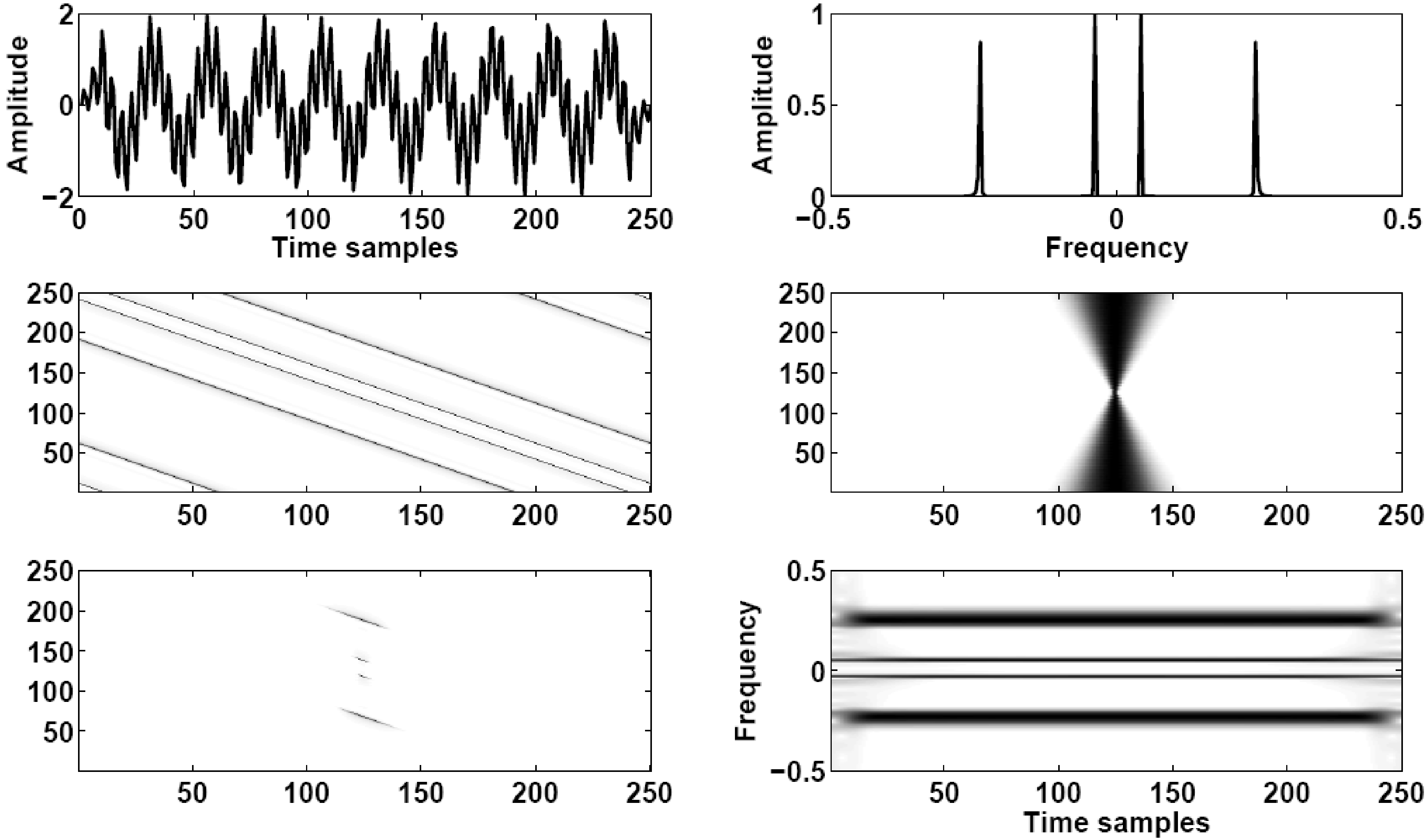
Box function (5)



Chirp function

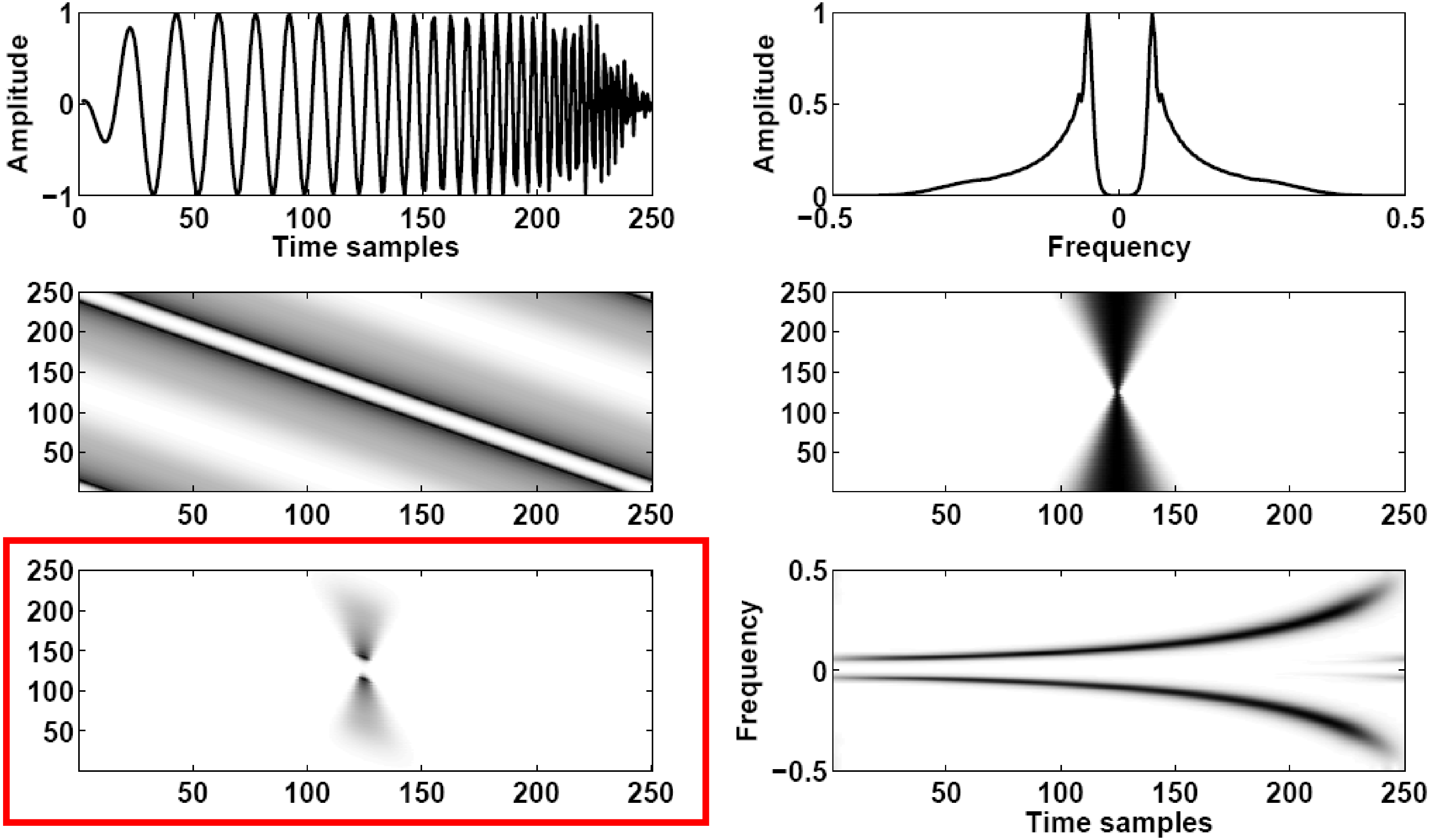


Sine function

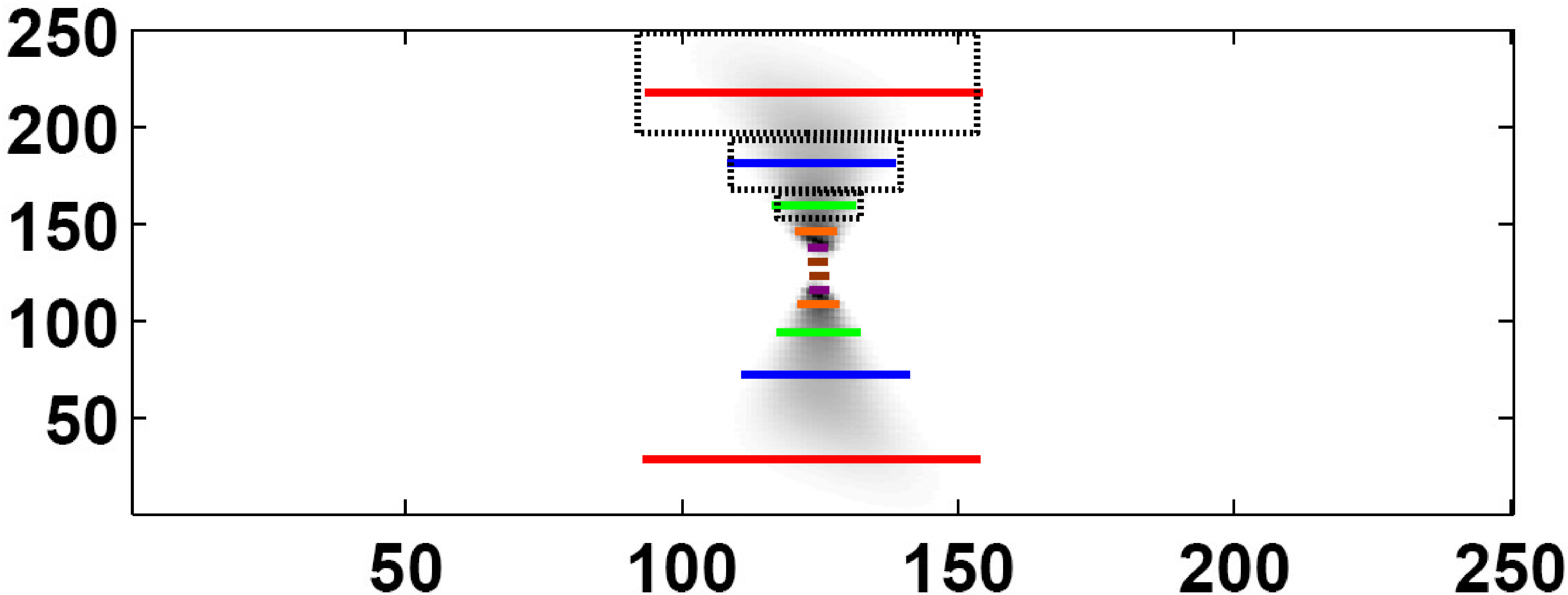


Fast Generalized Fourier Transform

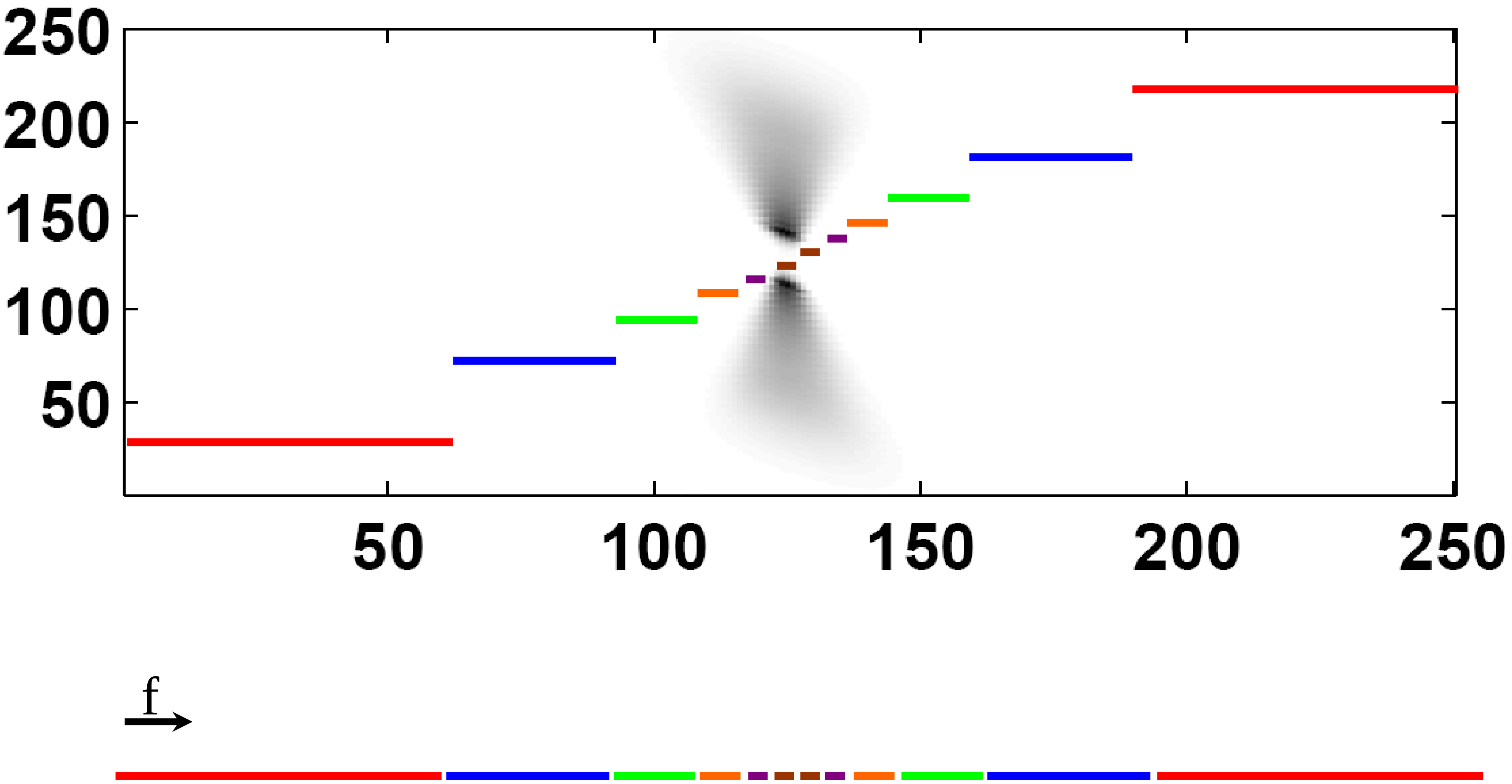
Chirp function



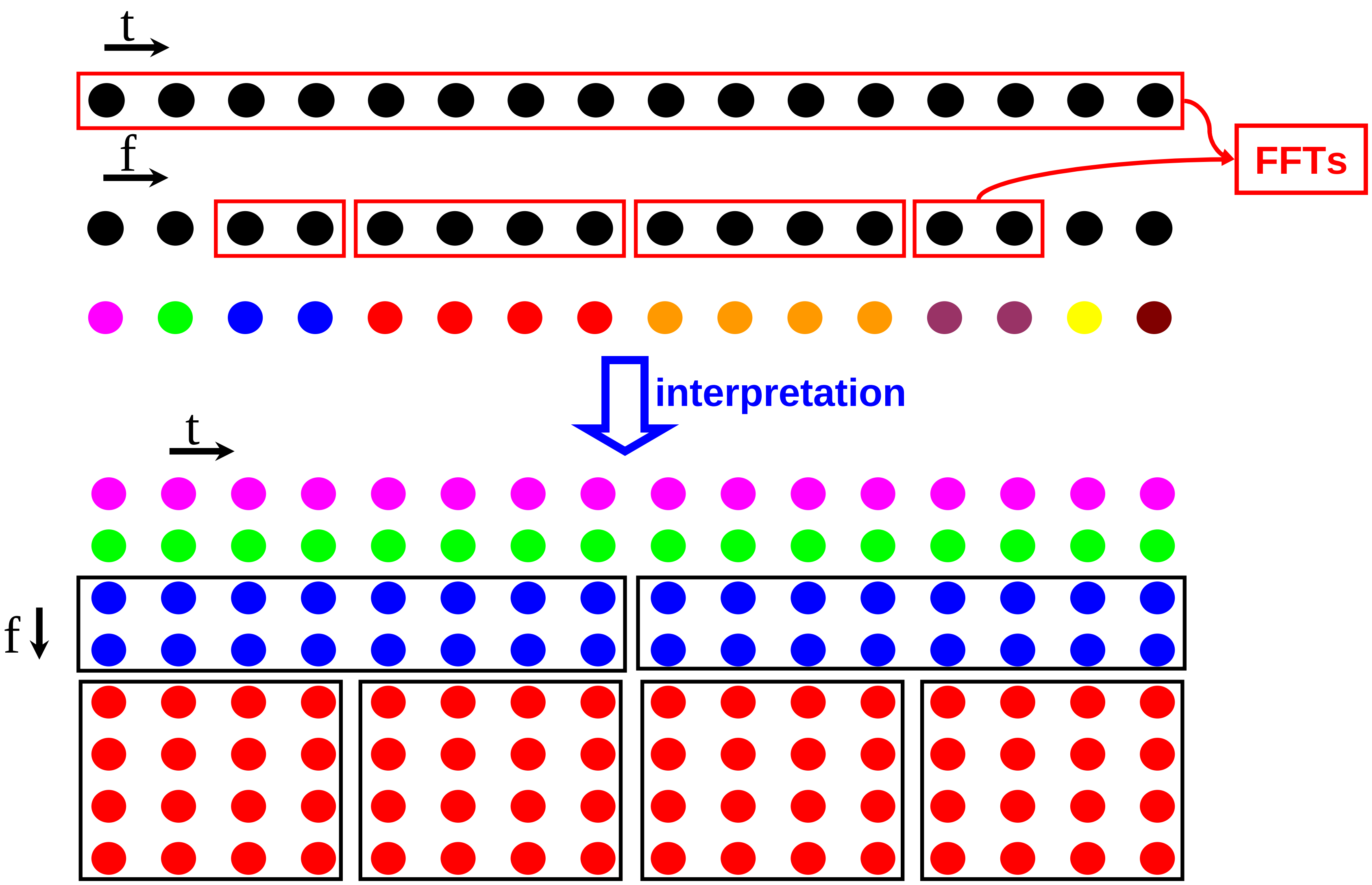
Dyadic sampling of Alpha-domain



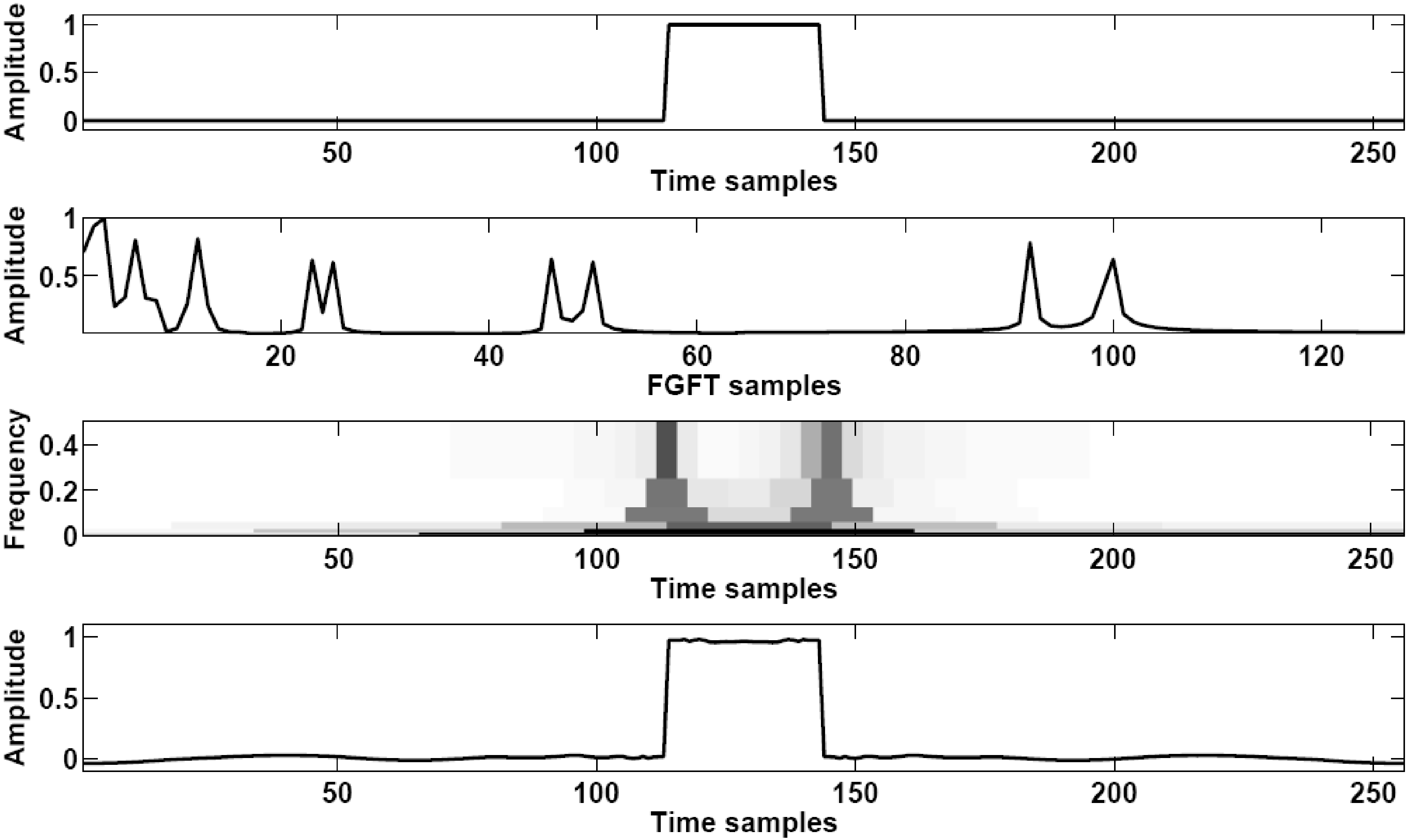
In-place non-redundant S-Transform



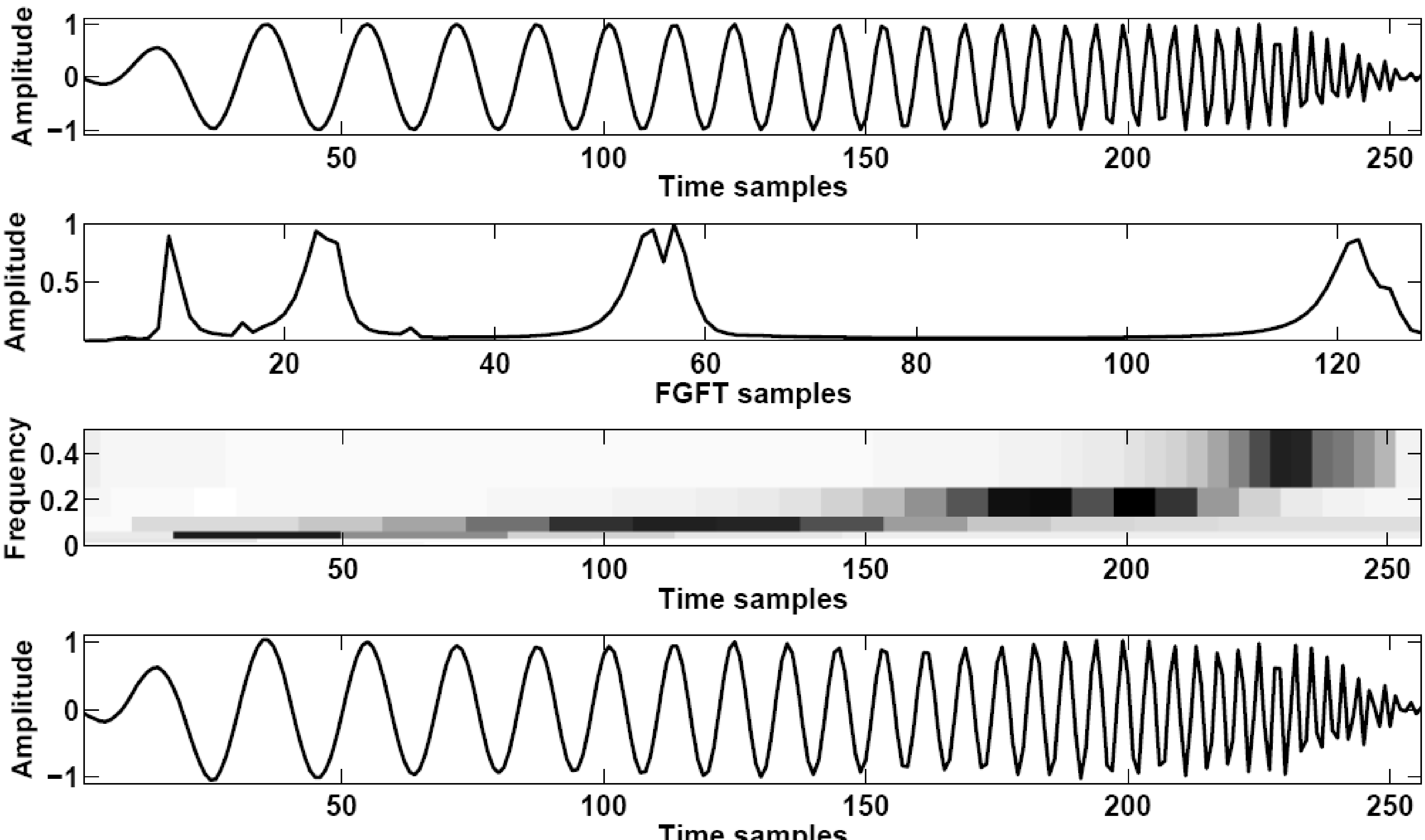
In-place non-redundant S-Transform
(Fast Generalized Fourier Transform (FGFT))



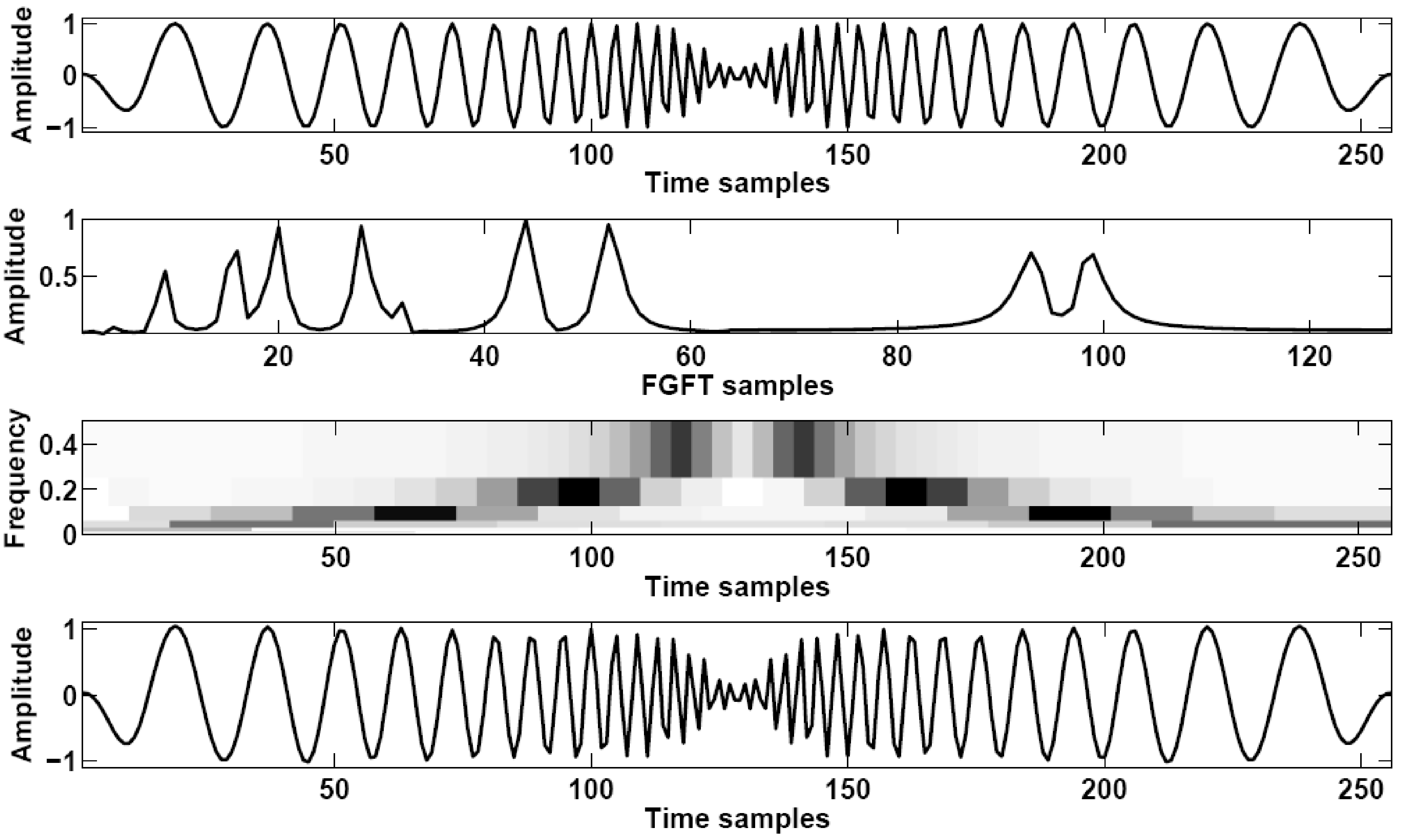
Box function



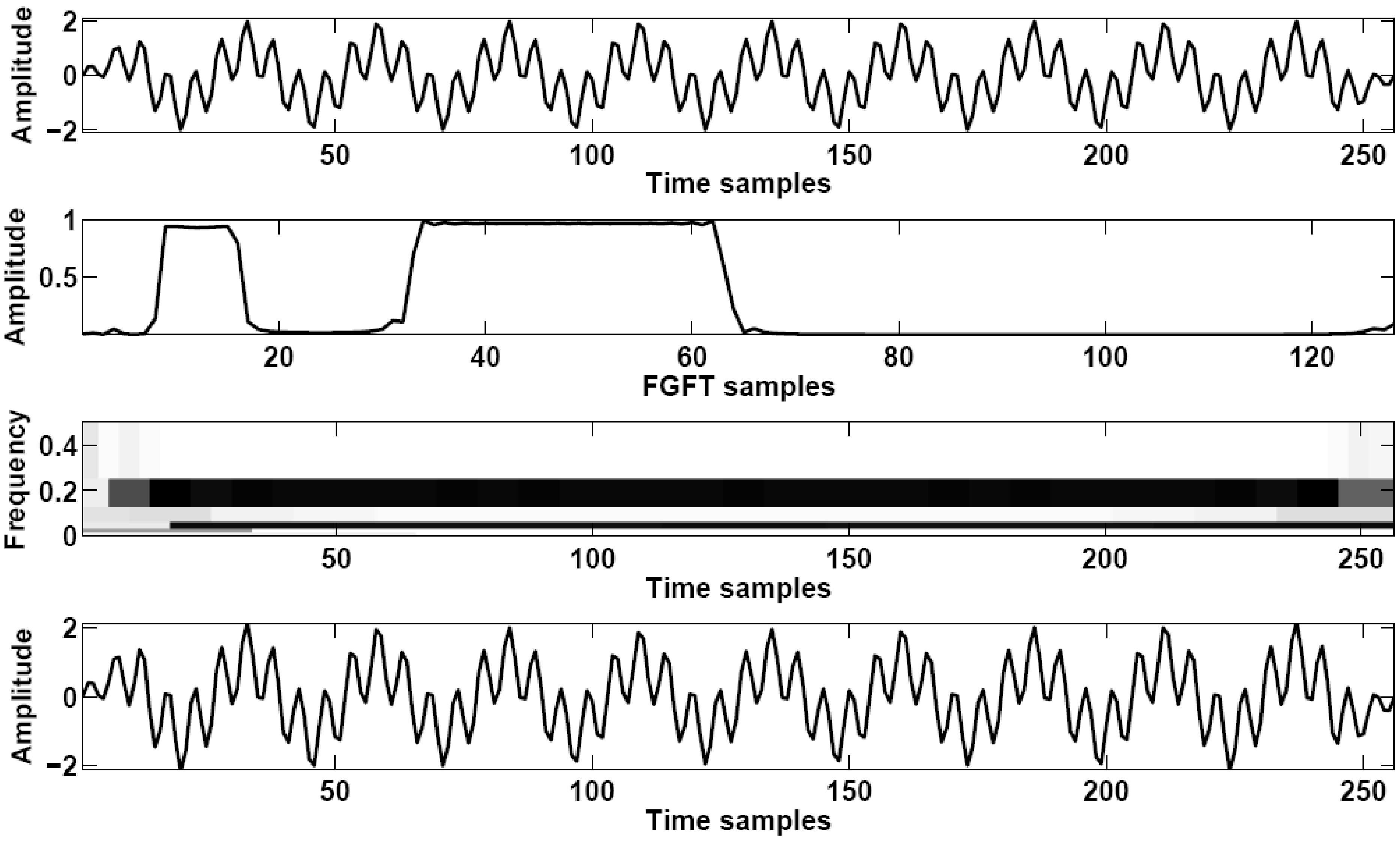
Chirp function



Another chirp function



Sine function



Fast Generalized Fourier Interpolation

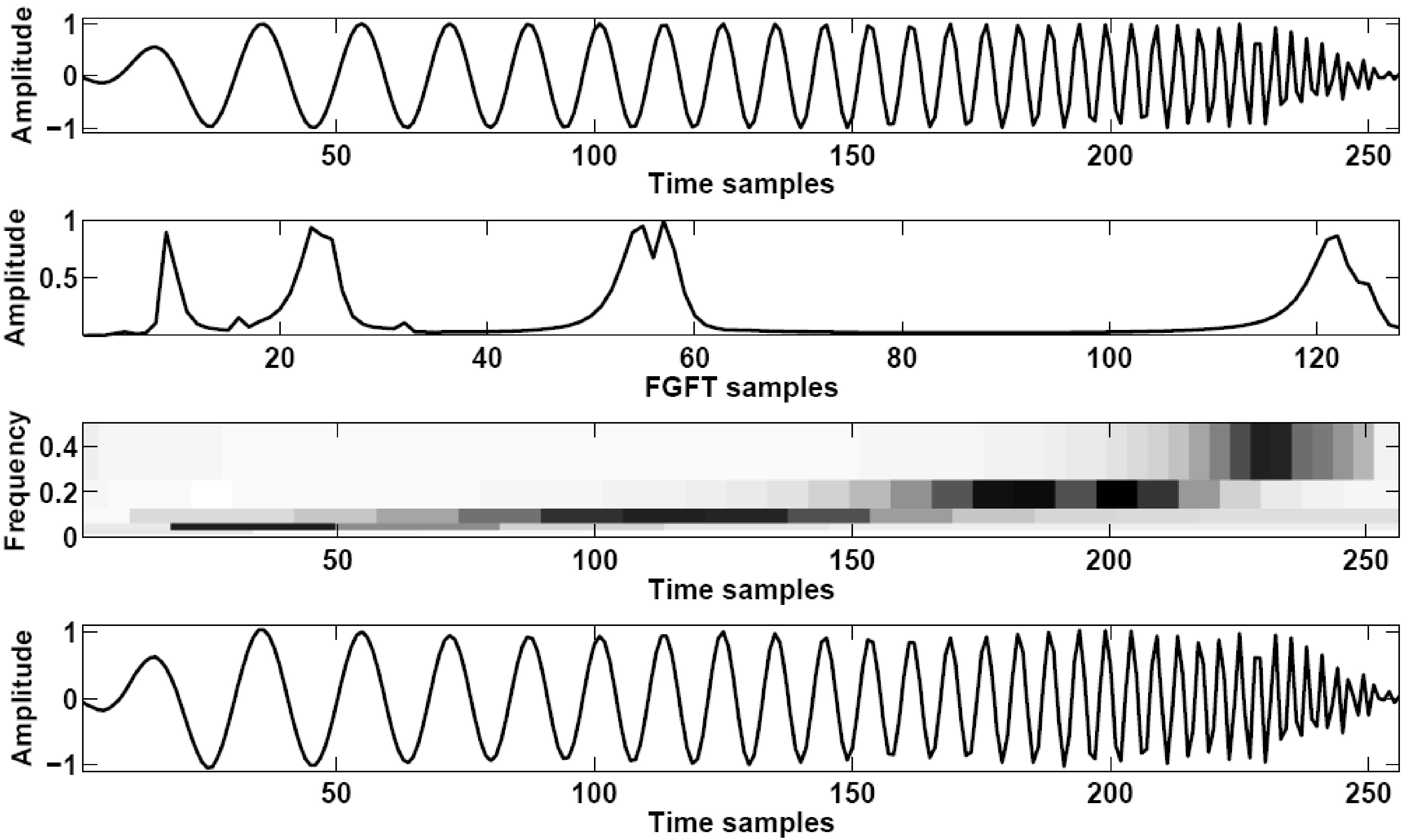
Fast Generalized Fourier Least-squares Interpolation

$$\texttt{Minimize} \quad \|\mathbf{m}\|_{\mathrm{W}}^2$$

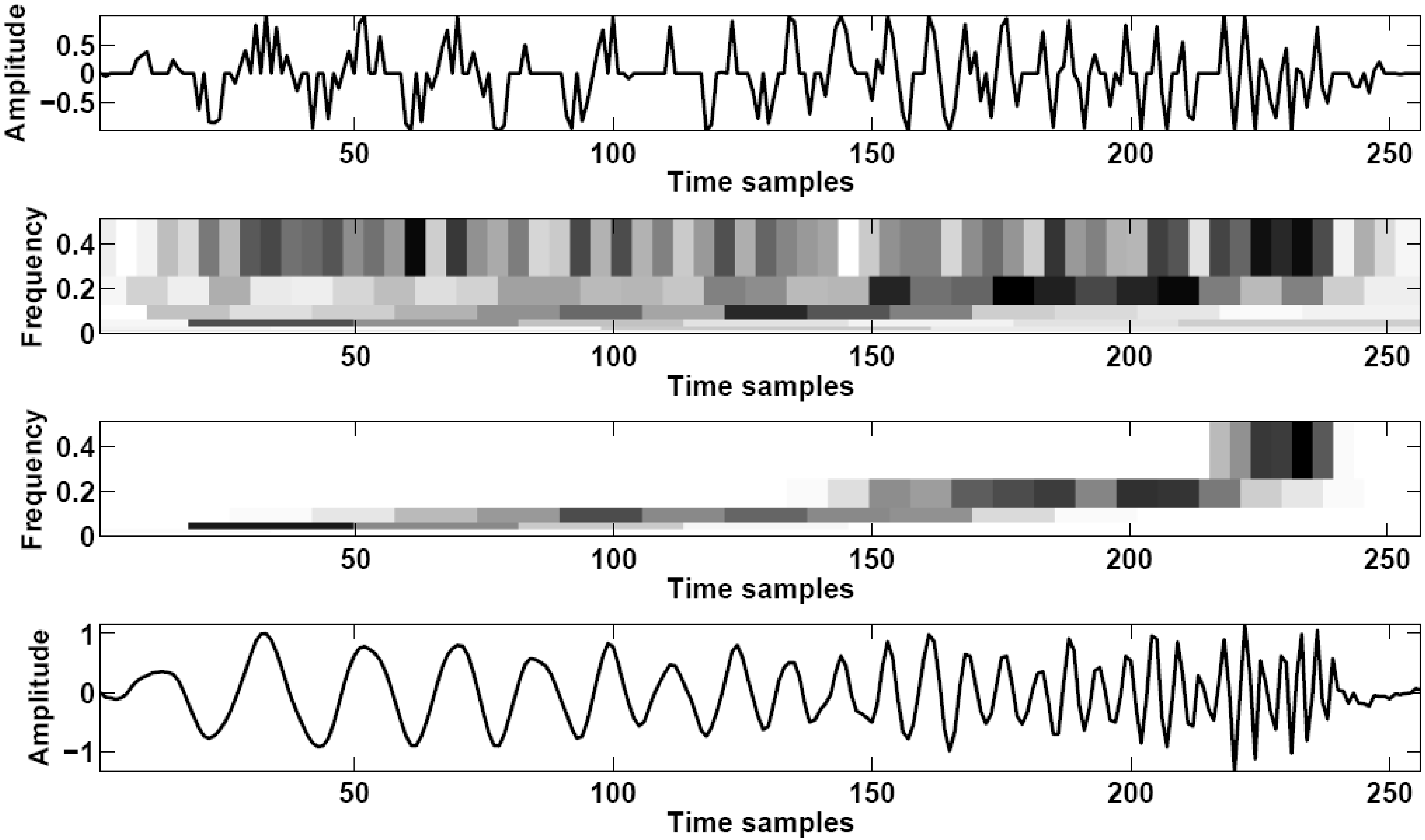
$$\texttt{Subject to} \quad \underbrace{\mathbf{T}}_{\text{Sampling matrix}}\underbrace{\mathbf{m}}_{\text{Desired interpolated data}} = \underbrace{\mathbf{d}}_{\text{Available data}}$$

$$\|\mathbf{m}\|_{\mathrm{W}}^2 = \mathbf{m}^H \underbrace{\mathbf{G}}_{\text{FGFT}}{}^H \underbrace{\boldsymbol{\Upsilon}}_{\text{Weight function}} \mathbf{G}\mathbf{m}$$

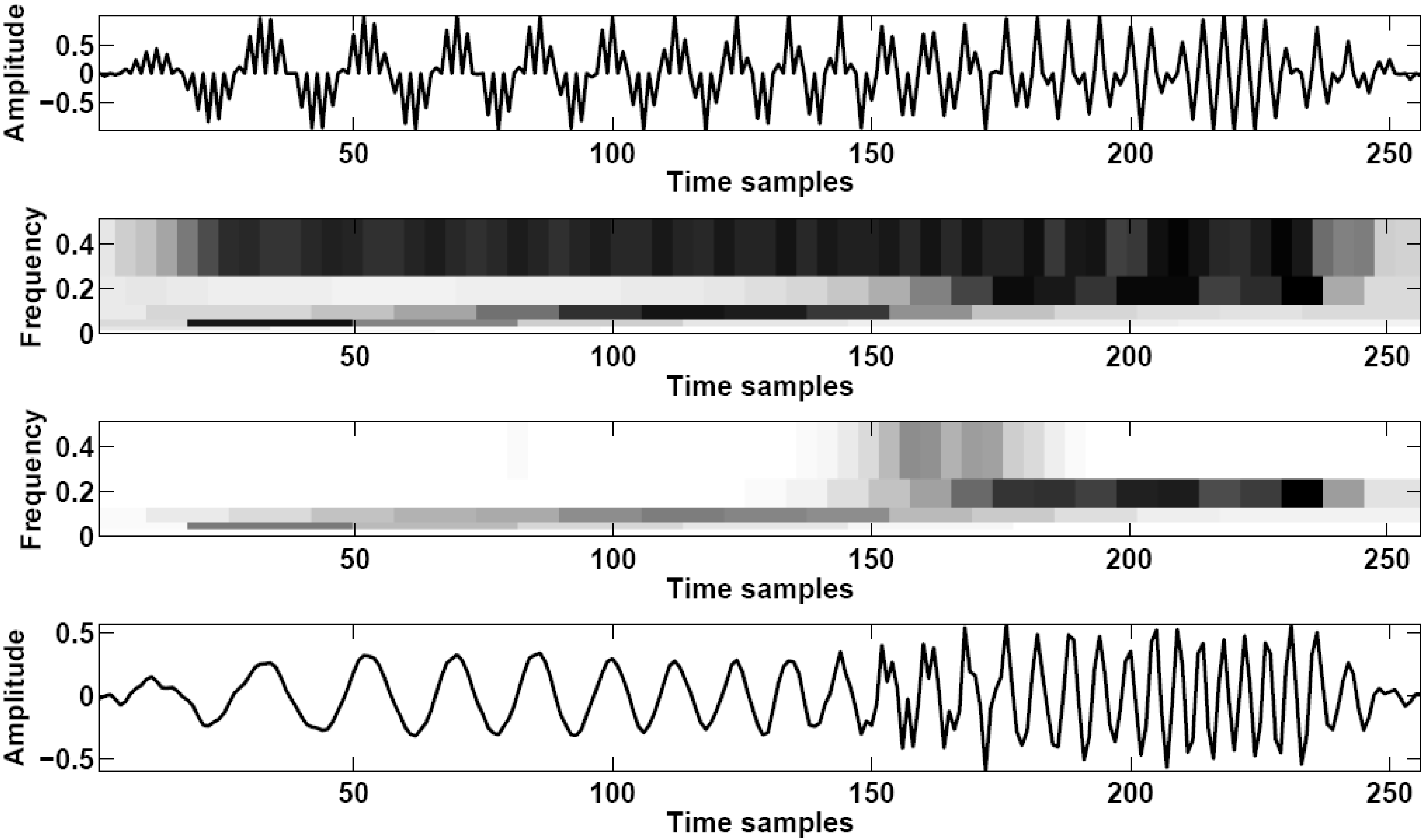
Chirp function



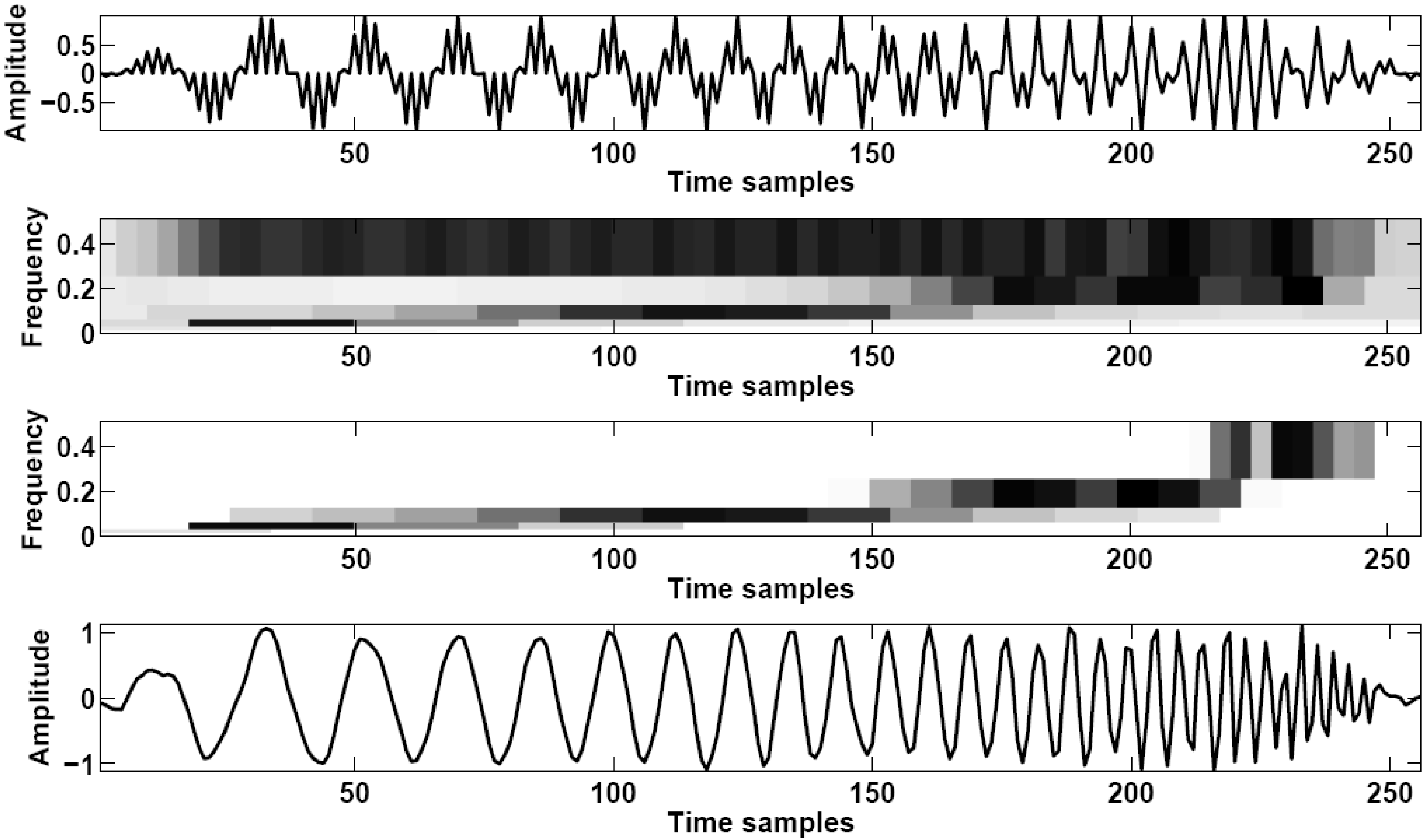
Reconstruction of irregularly sampled chirp



Reconstruction of regularly sampled chirp



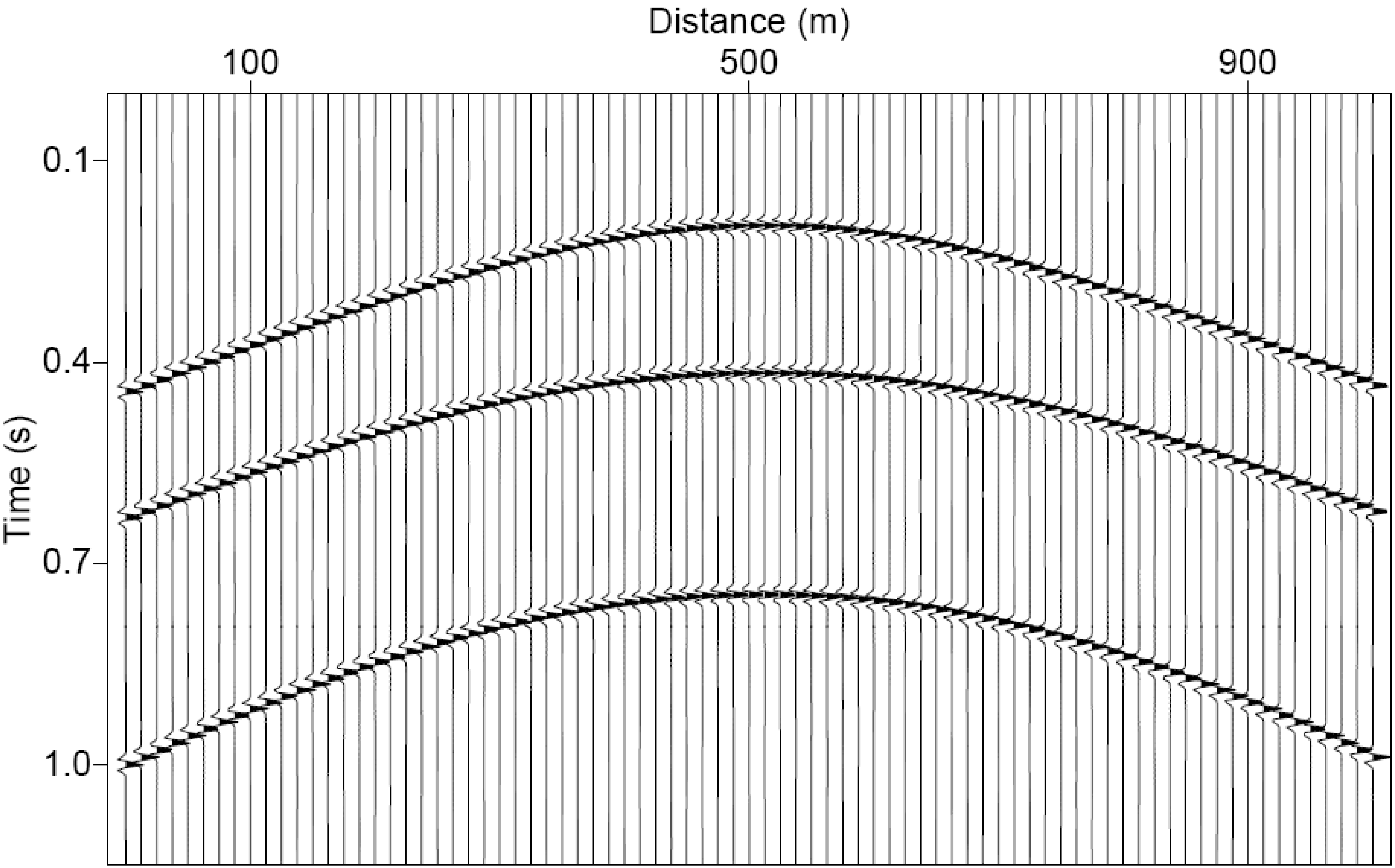
Reconstruction of regularly sampled chirp using band-limitation mask



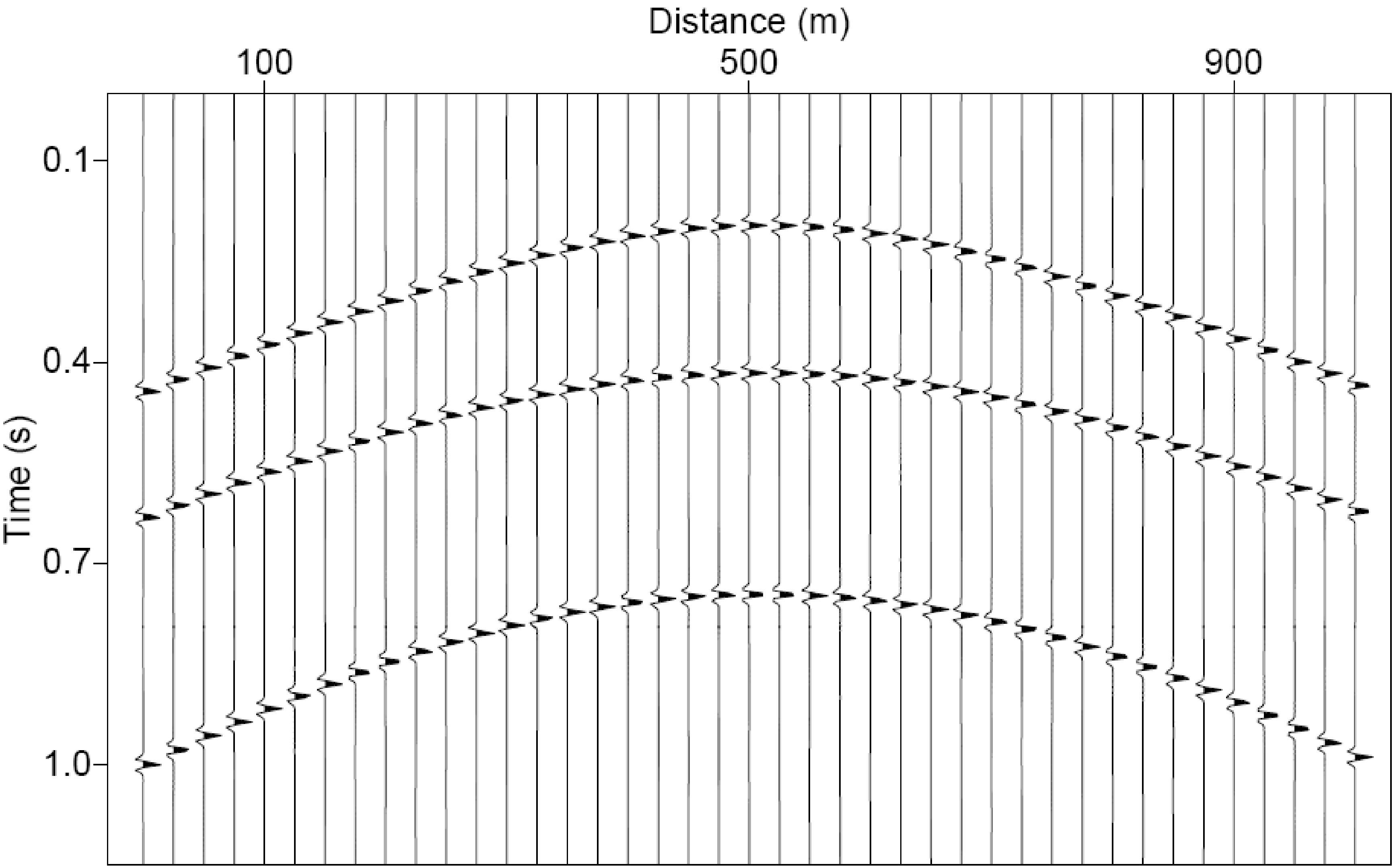
FGFI for regularly sampled seismic data

1. Transform the original data to f-x domain.
2. Compute FGFT of half frequencies.
3. Create a mask function by thresholding the FGFTs of step 2. The mask function will have values 1 for elements above threshold and 0 otherwise.
4. Interlace zero traces between each trace of step 1.
5. Apply Least-squares FGFI to each frequency of step 4 using the mask function from step 3.
6. Transform back the results of step 5 to t-x domain.

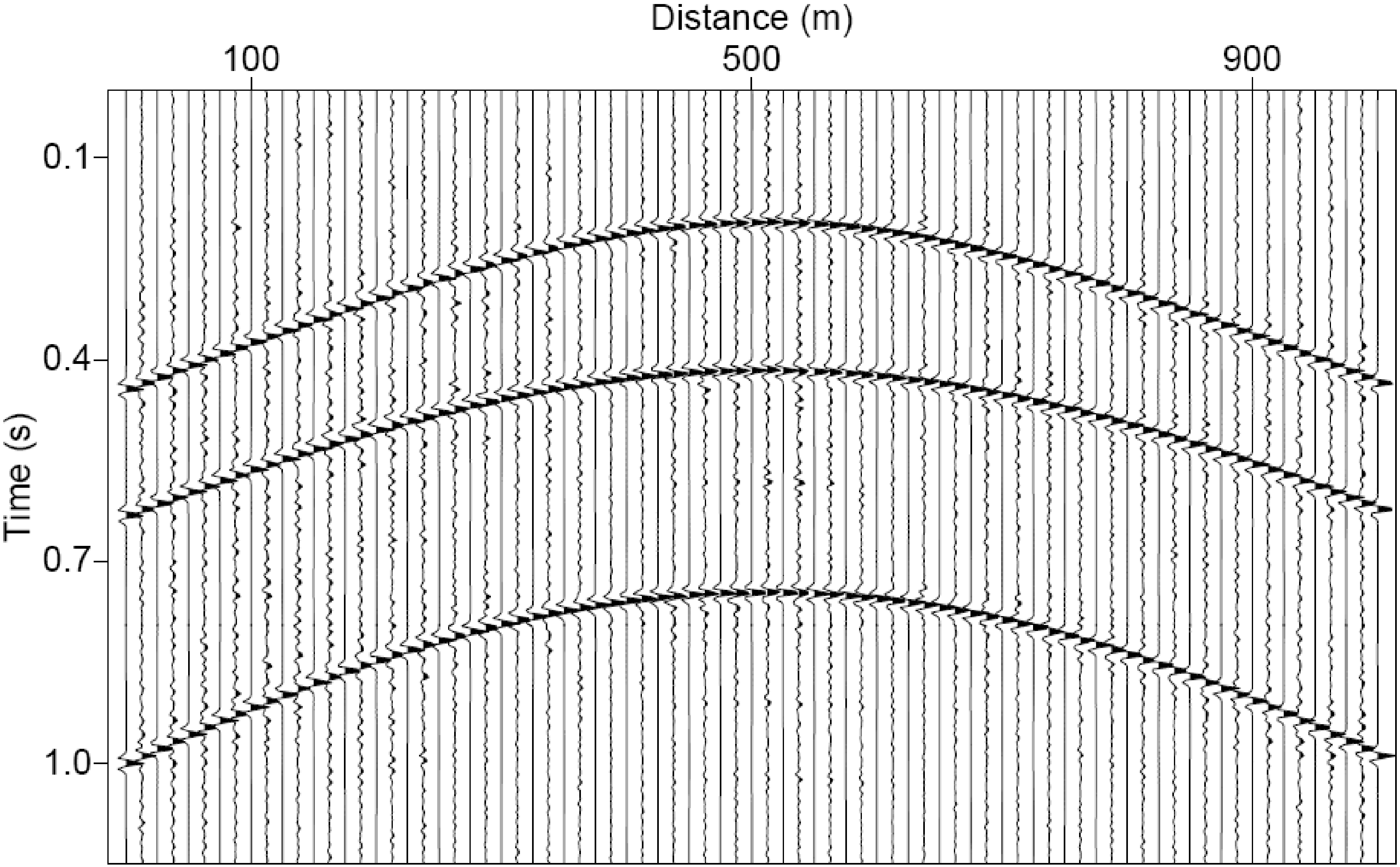
Original data with hyperbolic events



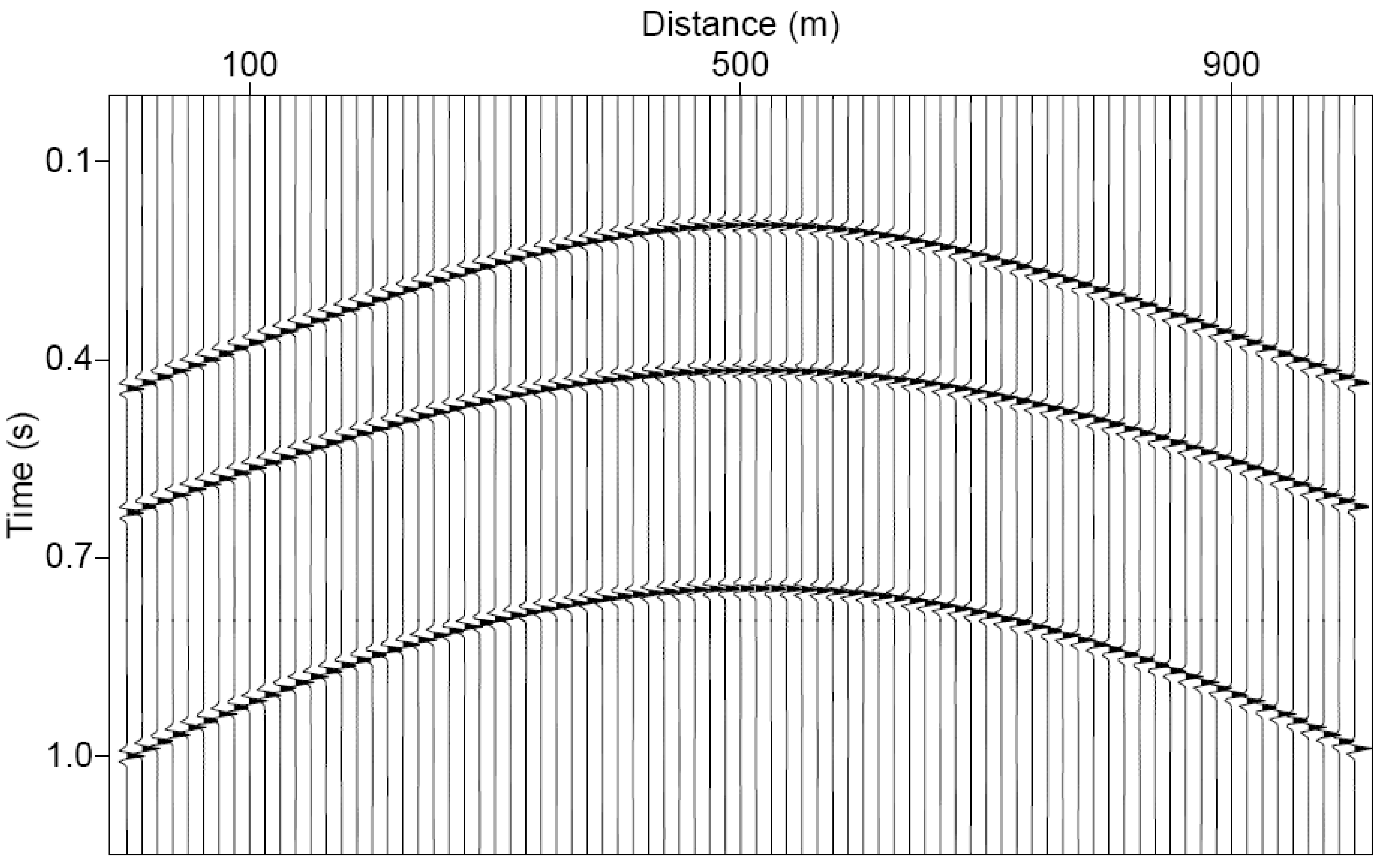
Decimated data



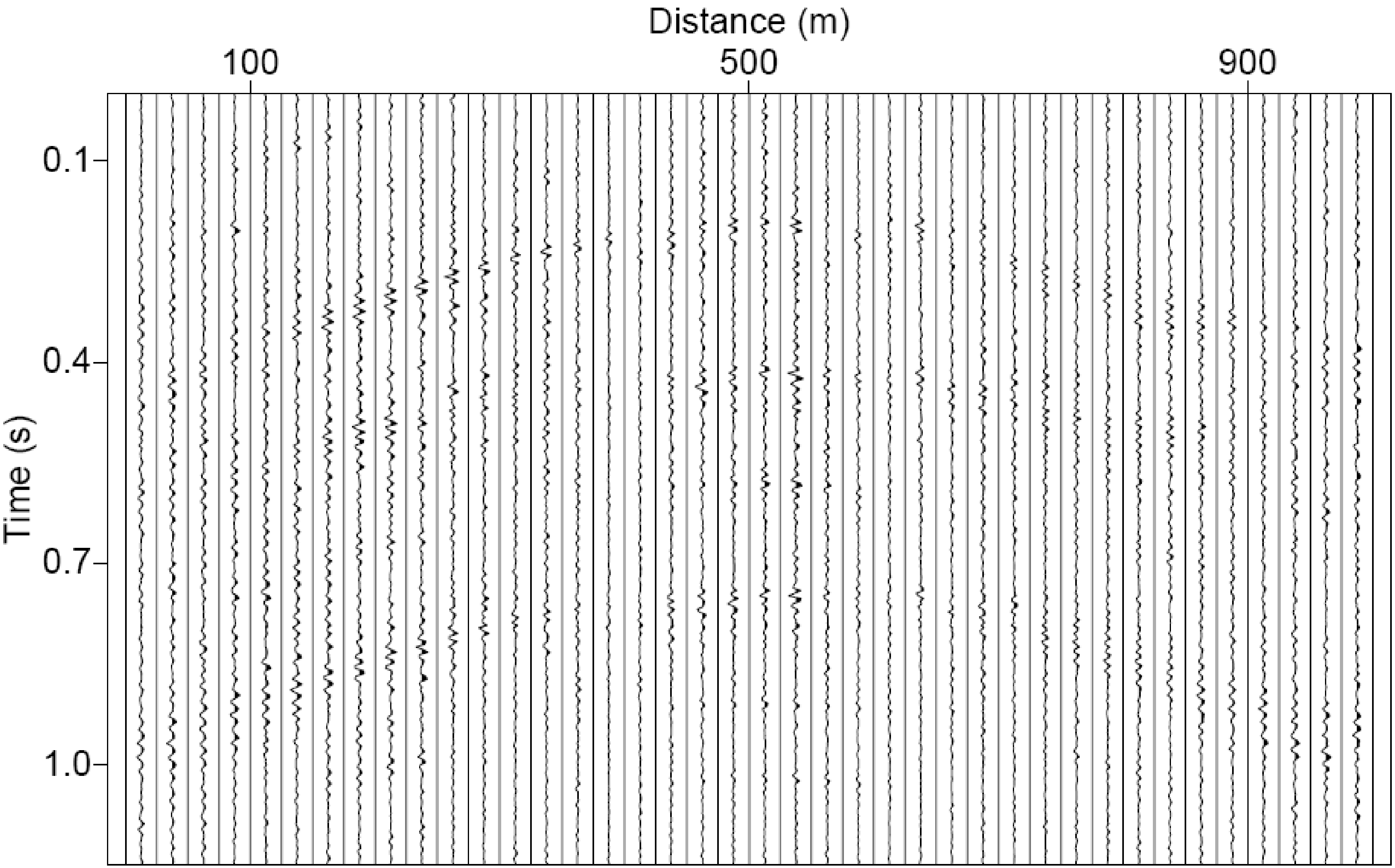
Reconstructed data



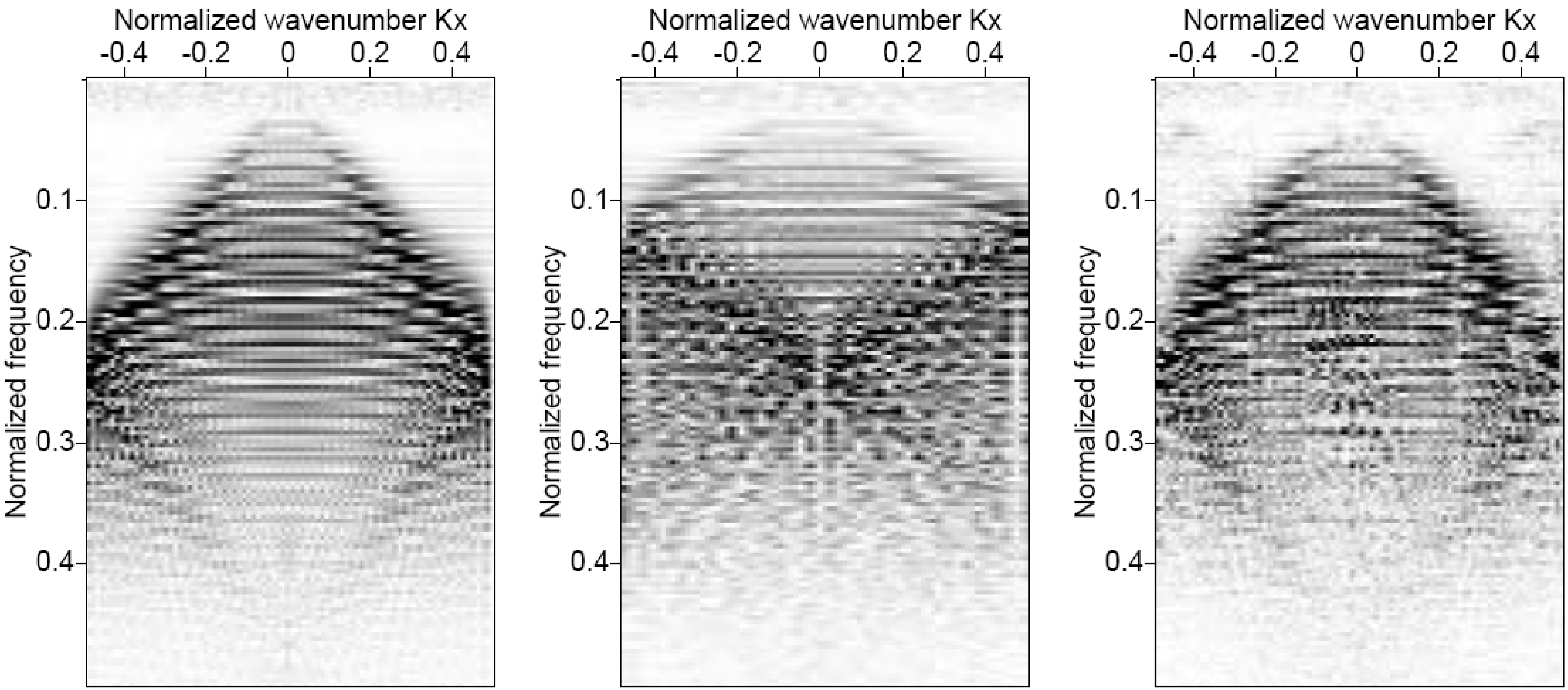
Original data



Difference section



F-K domain representation of data

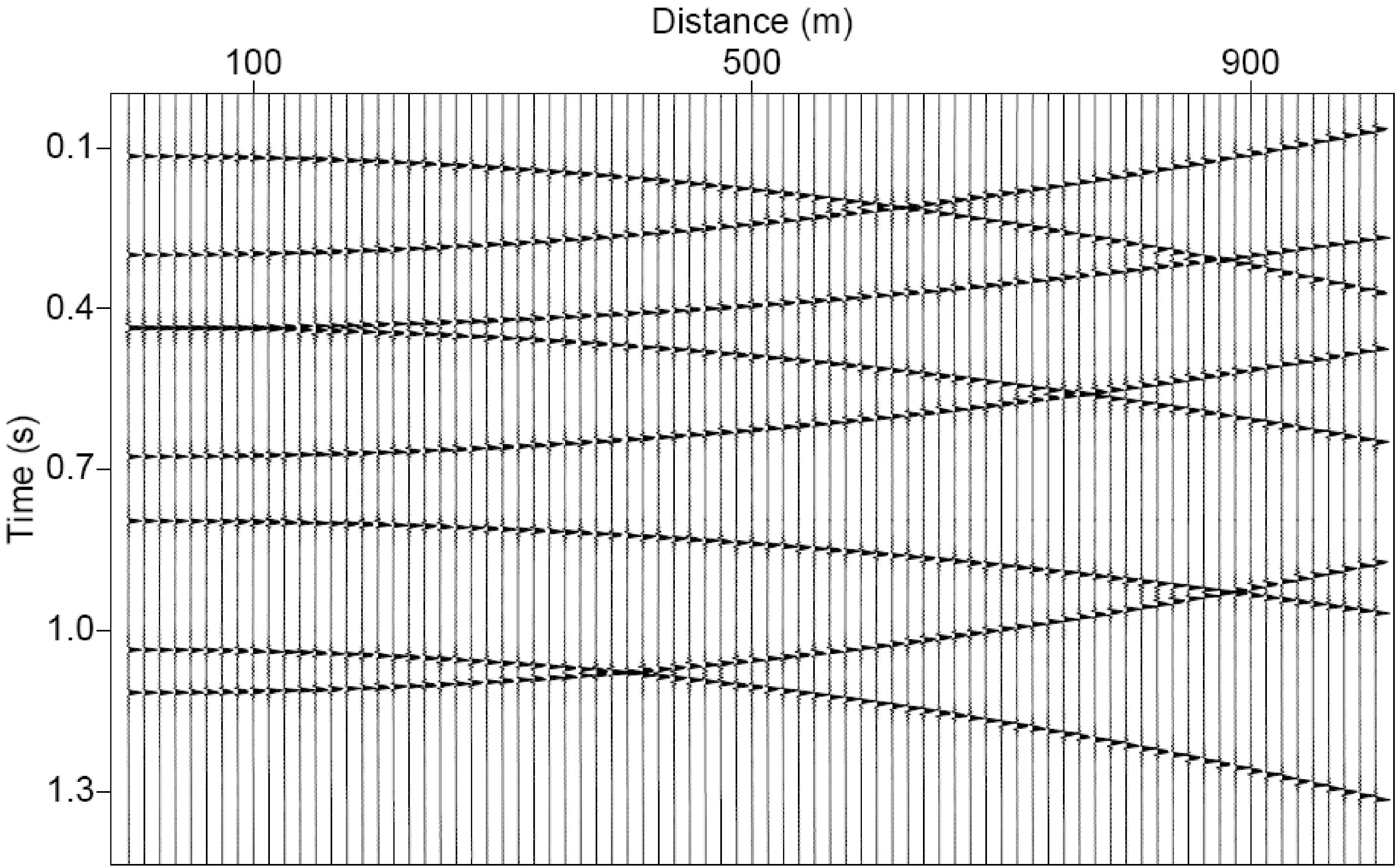


Original

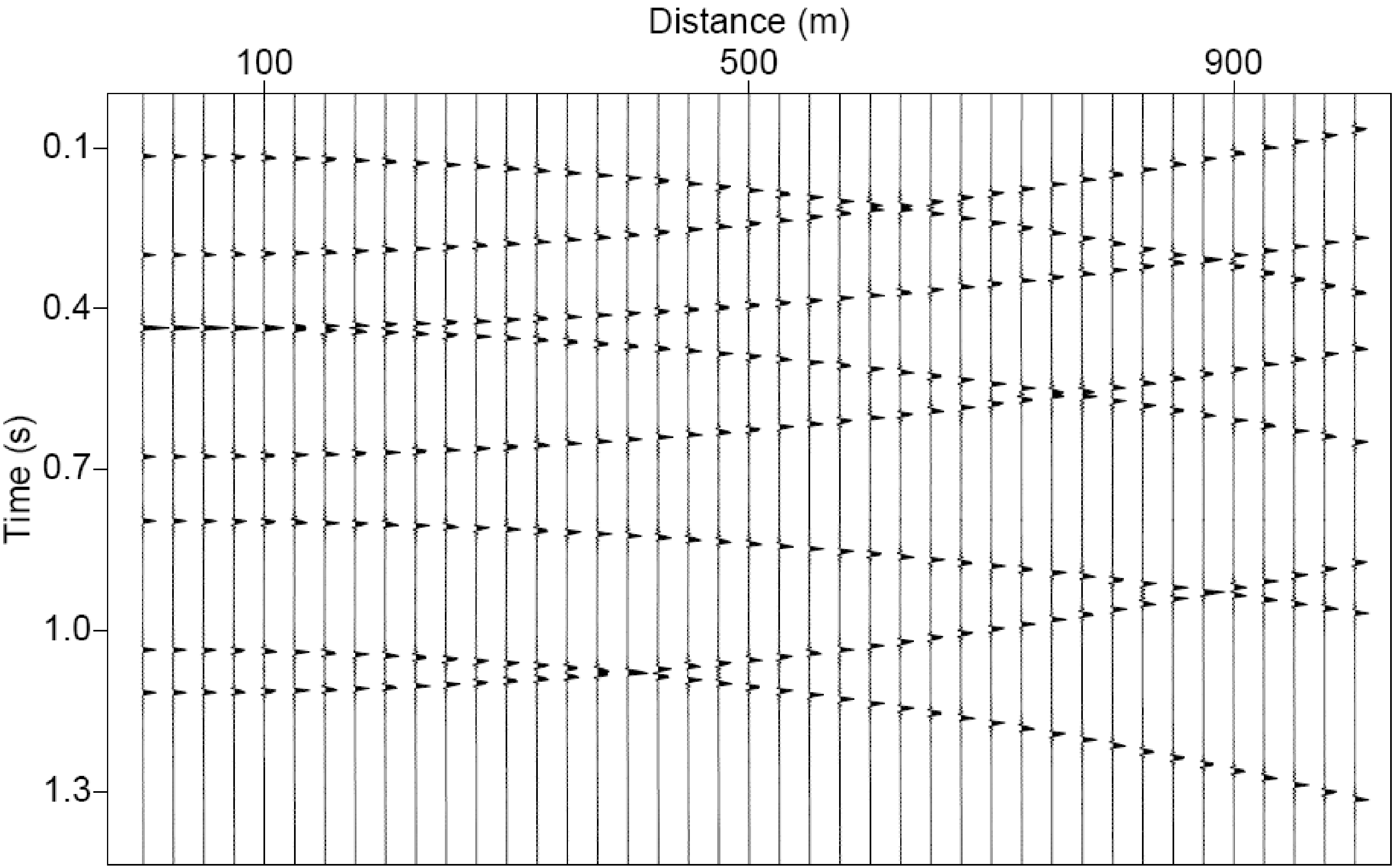
Decimated

Reconstructed

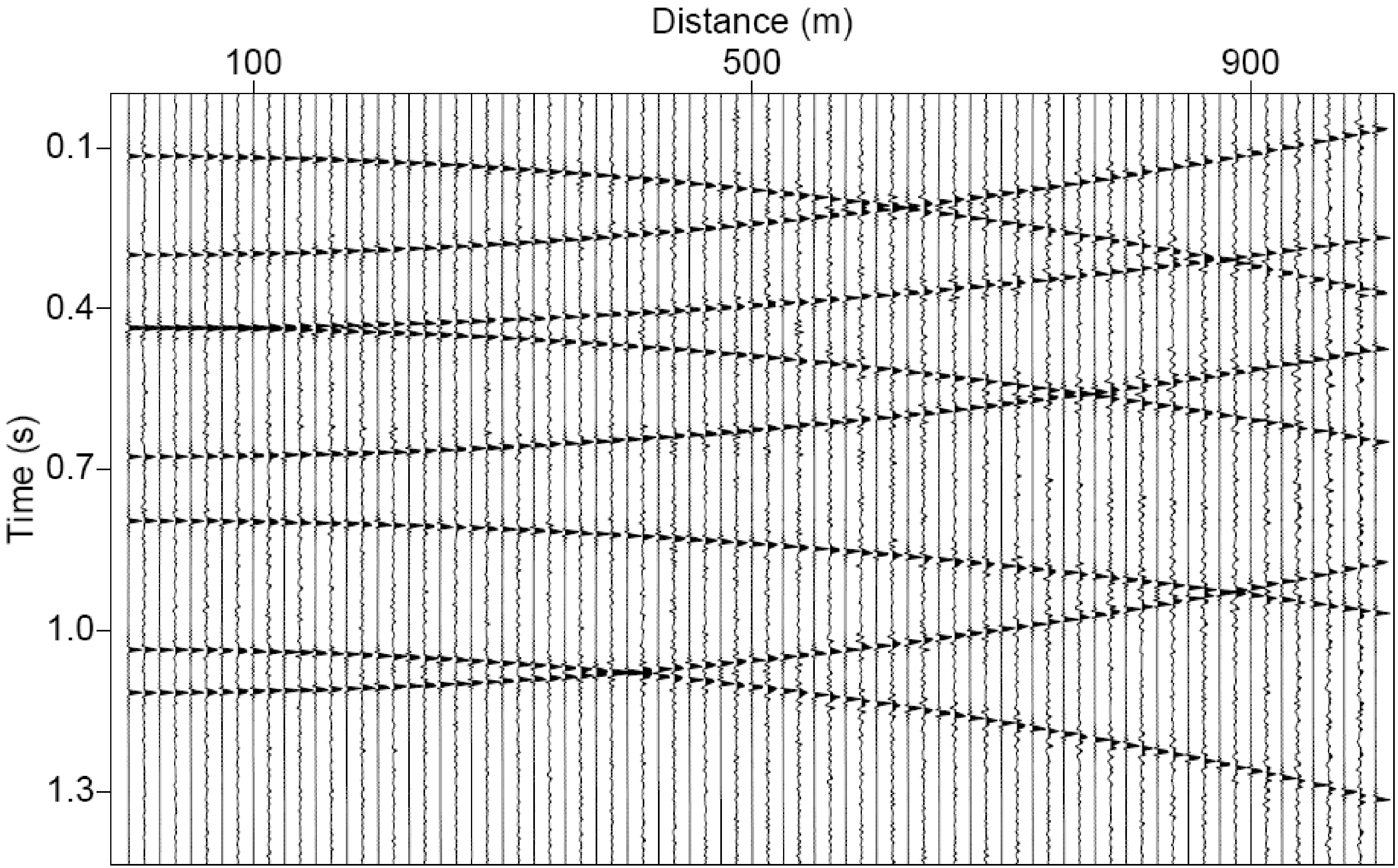
Original data with conflicting dip events



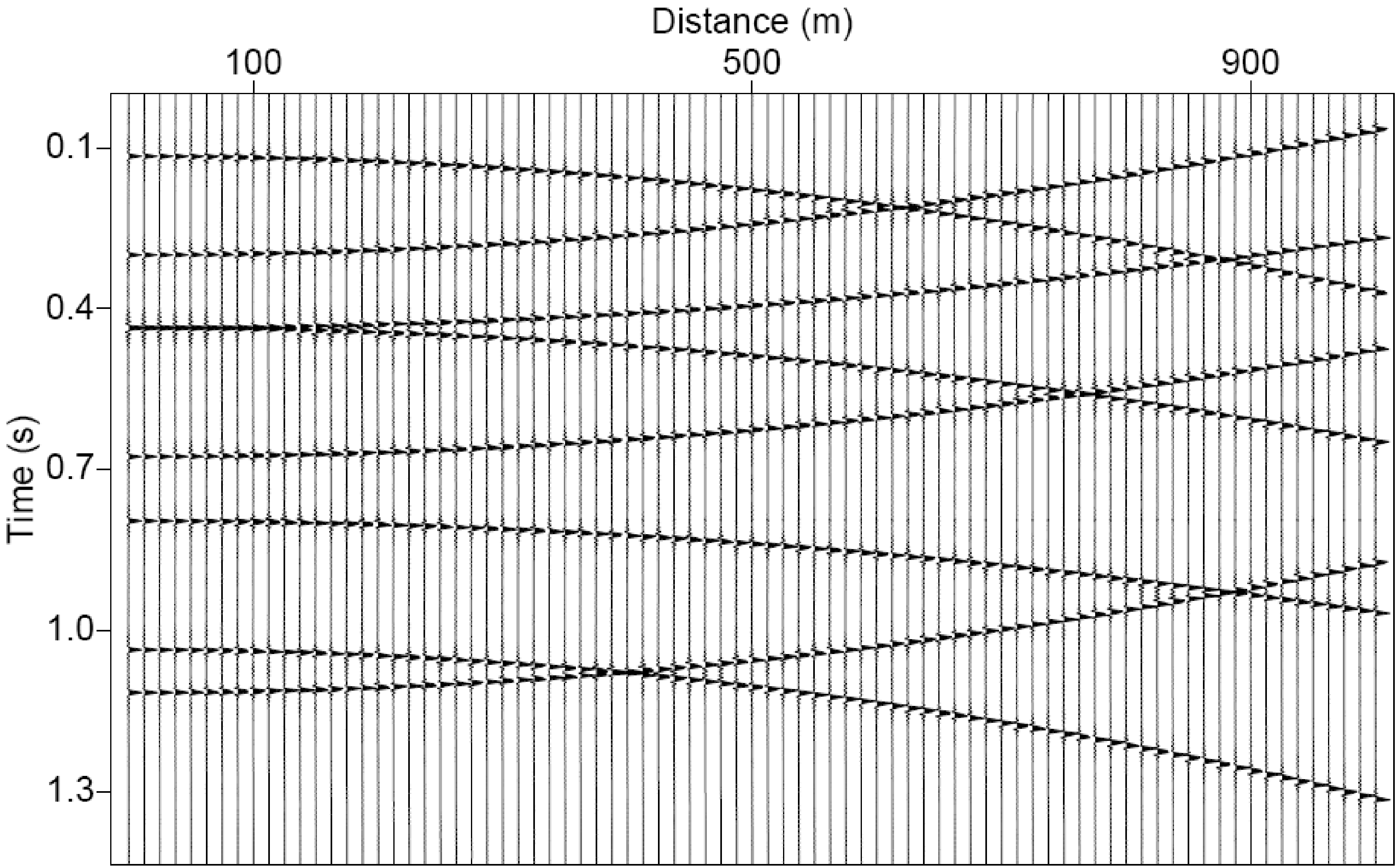
Decimated data



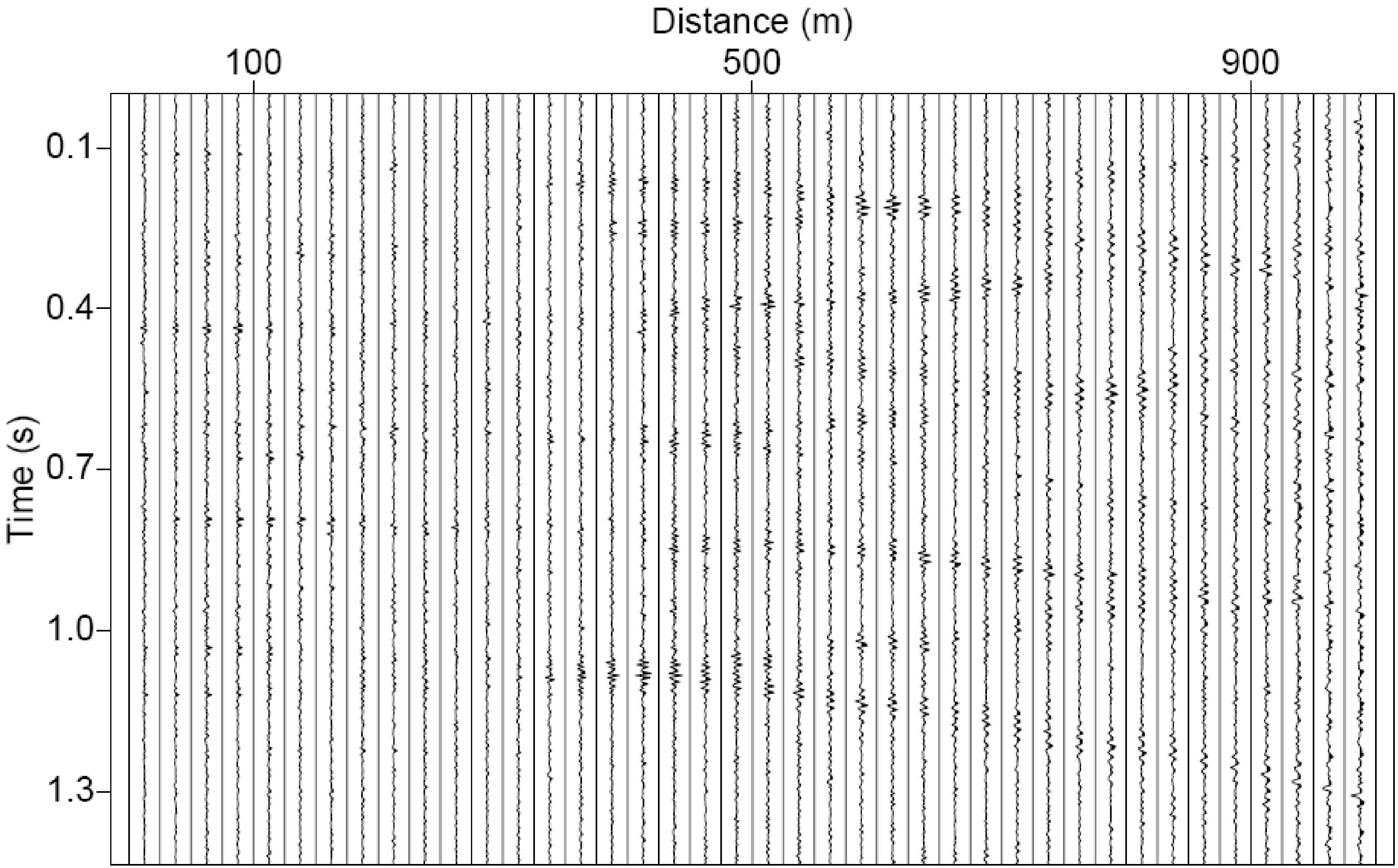
Reconstructed data



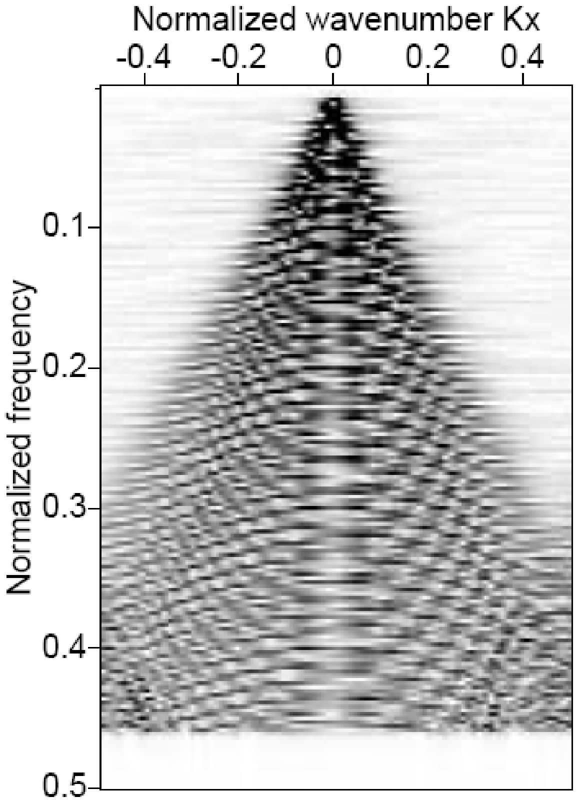
Original data



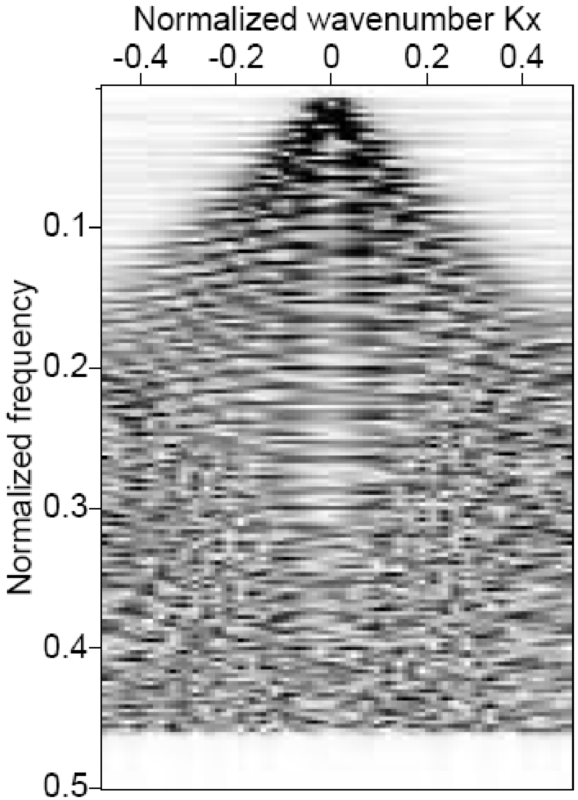
Difference section



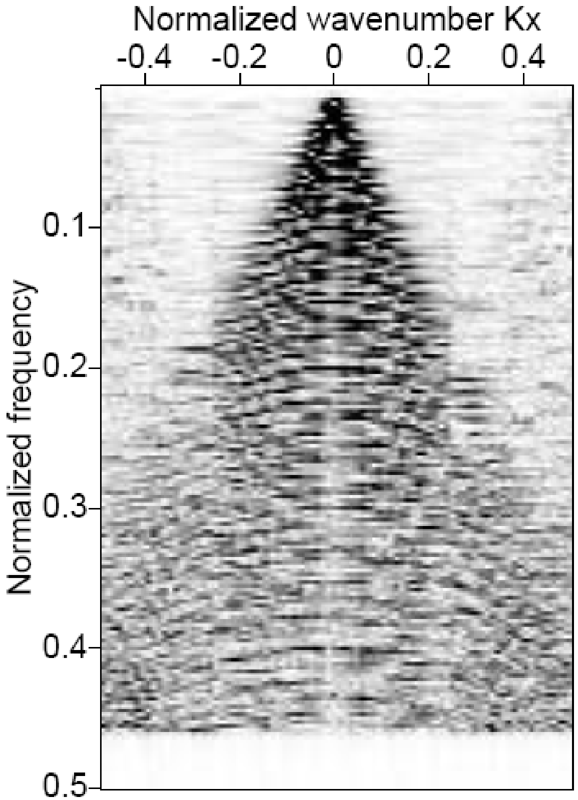
F-K domain representation of data



Original

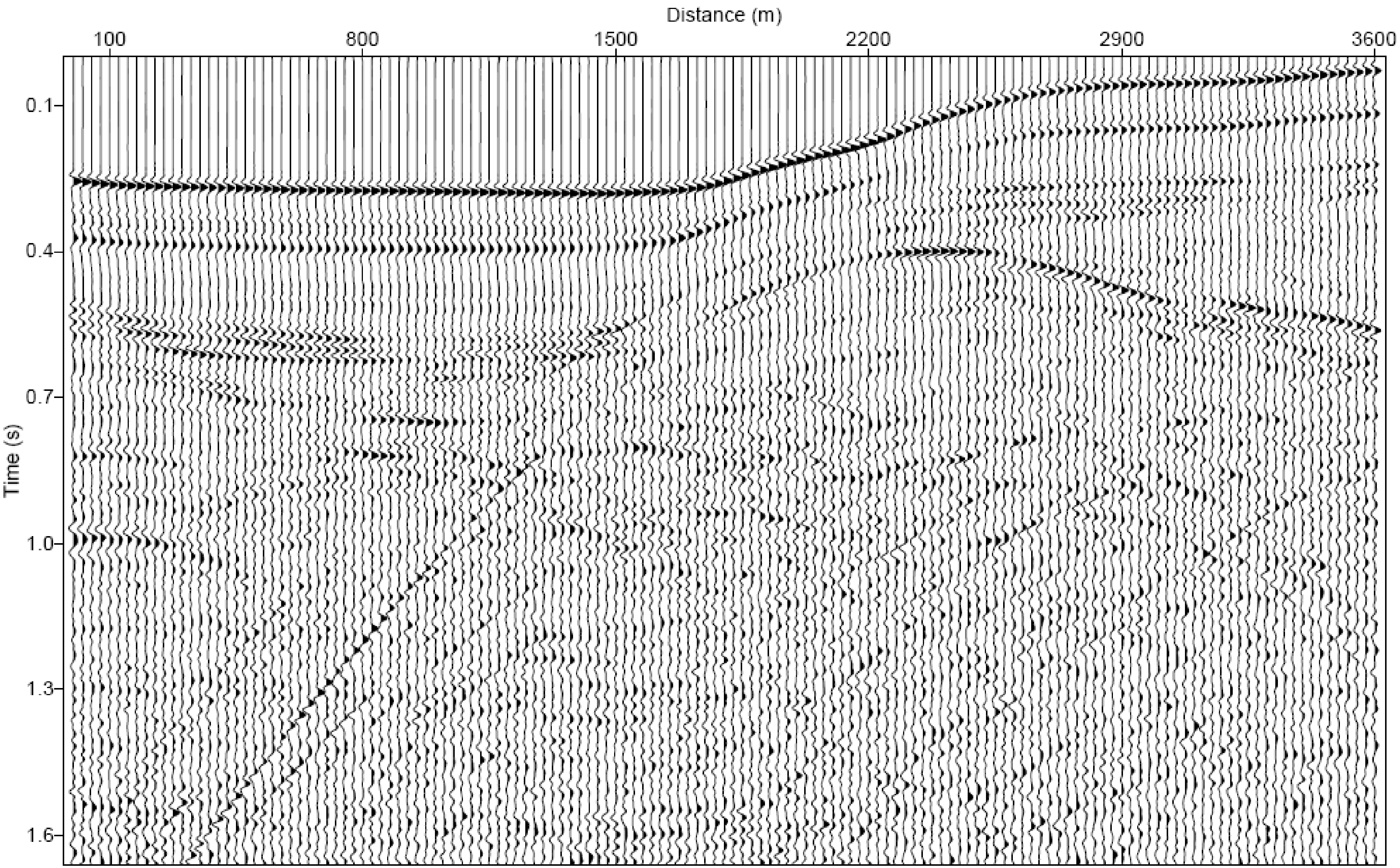


Decimated

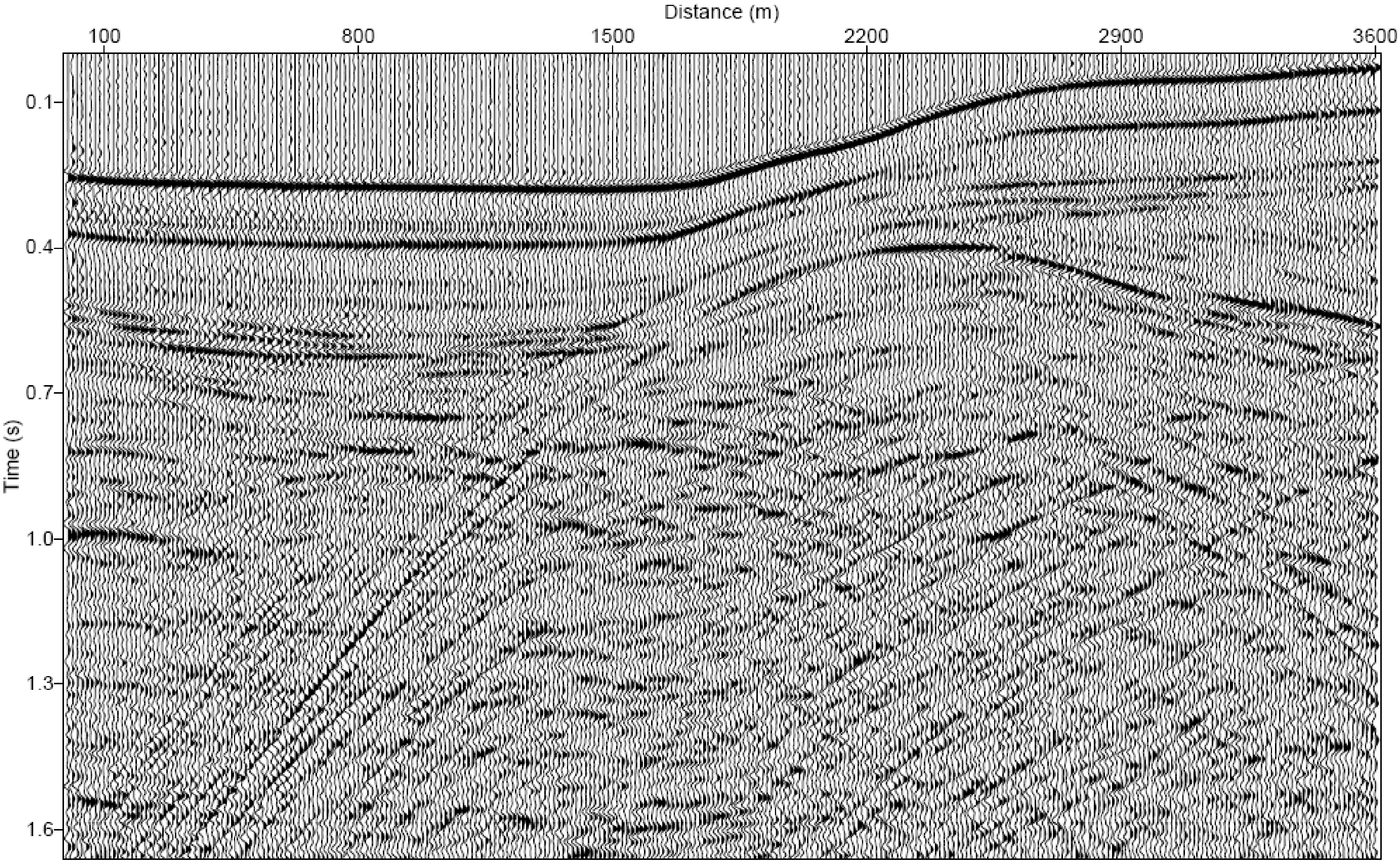


Reconstructed

A near-offset section from Gulf of Mexico



Interpolated data

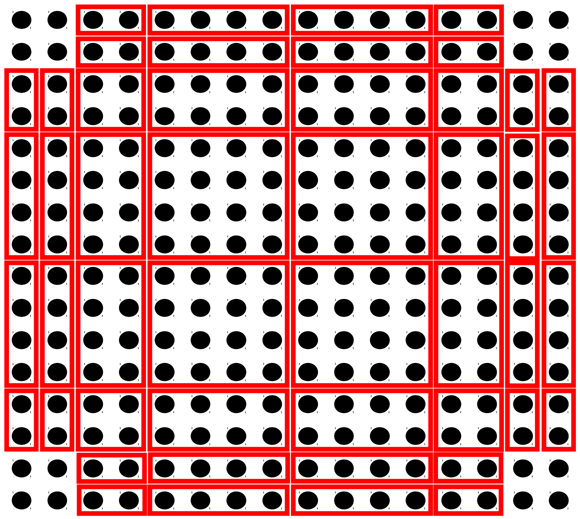


Conclusions:

- **Fast Generalized Fourier Transform (FGFT) is a fast and efficient way for analyzing nonstationry signals.**
- **FGFT does not increase the size of data (non-redundant) and is very simple to apply. In fact, it consists of FFTs inside an FFT.**
- **The fast and non-redundant properties of FGFT comes with loss of some precision which might make it unsuitable for some applications.**
- **Fast Generalized Fourier Interpolation (FGFT) can be used for:**
  - **Irregularly sampled data: Combination of a nonstationry signal and irregular sampling is hard to be reconstructed. FGFI shows somehow stable recovery in this situation.**
  - **Regular sampling: Regular sampling creates high amplitude artifact in Fourier domain. In FGFT, these artifact are only present at high frequencies. For successful FGFI a proper mask function is required for high frequencies.**

Conclusions (cont.):

- For regularly sampled aliased nonstationary seismic data one can make a mask function from half frequencies in the f-x domain of original data. The mask function can be used to guide FGFI.
- The extension of FGFT to multidimensional cases is straightforward.
- FGFT can be used in various geophysical applications which require time-frequency analysis. The next talk by Chris Bird is an example.



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- Dr. Robert A. Brown