Full Waveform Inversion (FWI) with wave-equation migration (WEM) and well control

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Banff, 3 Dec. 2010
Outline

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Initial impressions of FWI
Second impression of FWI
FWI Cycle

1) Forward model through $v_{k-1}$ to predict data $\psi_{r,k}$

2) Migrate “data residual” with $v_{k-1}$ and stack

3) “Calibrate” migration and deduce velocity perturbation $\delta v_k$

$$\delta v_k = \lambda M^{\dagger} [\delta \psi_{r,k}]$$

4) Update velocity model

$$v_k = v_{k-1} + \delta v_k$$

a) Initial velocity model $v_0$

b) Actual data $\psi_r$
Fundamental Theorem of FWI

Theorem (Tarantola, Lailly): Given real acoustic data and an approximate velocity model, then a linearized velocity model update, $dvel$, is given by

$$dvel(x, z) = \lambda \int \sum_s \sum_r \omega^2 \psi_s(x, z, \omega) \delta \psi^*_r(s)(x, z, \omega) d\omega$$

A prestack migration

$\lambda$ a scalar to be determined ("step length")

$\psi_s(x, z, \omega)$ monochromatic, forward propagated (downward continued), shot model

$\delta \psi_r(s)(x, z, \omega)$ monochromatic, reverse propagated (downward continued), data residual
Understanding the FTFWI

**Interpretation:** A prestack migration of the data residual is proportional to the desired update to the velocity model.

This is because the gradient of the data misfit function can be shown to be a type of prestack migration.

A cross correlation imaging condition arises naturally and there is a frequency squared factor. In the time domain this is a type of reverse –time migration (RTM).

\[
\int \sum_{s,r} \omega^2 \psi_s(x, z, \omega) \delta \psi^*_r(x, z, \omega) d\omega \Rightarrow - \int \sum_{s,r} \partial_t \psi_s(x, z, t) \partial_t \psi_r(x, z, T - t) dt
\]

*Frequency domain*  
*Time domain*
Understanding the FTFWI

Frequency dependence of $\lambda$:
• FTFWI assumes that the source wavelet is known.
• An unknown wavelet is equivalent to a complex-valued (i.e. amplitude and phase), frequency-dependent scalar.

So we write

$$dvel(x, z) = \int \lambda(\omega) \sum_s \sum_r \psi_s(x, z, \omega) \delta \psi^*_r(x, z, \omega) d\omega$$

where the factor $\omega^2$ has been absorbed into $\lambda$. 
What kind of migration?

• The direct interpretation of the FTFWI requires a prestack reverse-time migration (RTM) using time-differentiated wavefields.
• However, experience suggests that all depth migrations can produce comparable results.
• With $\lambda$ allowed to be complex and frequency-dependent, there seems no reason that a depth-stepping wave-equation migration (WEM) should not be used.

$$dvel(x, z) = \int \lambda(\omega) M^\dagger[\delta \psi_r(x, z, \omega)] d\omega$$

where $M^\dagger$ is a generalized migration operator and we expect that $\lambda$ depends on both the source wavelet and the type of migration.
Calibrating the migration

FTFWI: Find the scalar $\lambda$ such that

$$v_k(x, z) = v_{k-1} + \lambda M^\dagger[\delta \psi_k](x, z)$$

produces the best forward modelled data.

Standard practice finds $\lambda$ in a 1D search called a “line search”.
Calibrating the migration

This paper: Find the scalar $\lambda$ such that

$$\int (v_{true}(x_w, z) - v_k(x_w, z))^2 \, dz = \min$$

where $x_w$ is the well location at which $v_{true}$ is known.

Defining the velocity residual $\delta v_k(x_w, z) = v_{true}(x_w, z) - v_{k-1}(x_w, z)$

$$\int (\delta v_k(x_w, z) - \lambda(\omega) M^\dagger[\delta \psi_k](x, z, \omega))^2 \, dz = \min$$

So we will match the migrated data residual to the velocity residual at the well. This is a process very similar to standard impedance inversion.
Calibrating the migration

Our explicit calibration procedure is:

1. Convolve the migrated stack with a Gaussian smoother
2. Determine the best (least squares) scalar to match the smoothed migrated trace at the well to the residual velocity at the well.
3. Determine the best (least squares) constant-phase rotation to match the scaled, smoothed, migrated trace to the residual velocity at the well.
4. Apply the amplitude scalar and phase rotation to the entire smoothed stack to estimate $dvel$.

Many other, more sophisticated calibration methods are possible.
Numerical Experiment

Using data created from the Marmousi model, we implemented the four steps of FWI as:

1. Modelling using acoustic finite difference tools in Matlab.
2. Migration of data residual using PSPI in Matlab.
3. Calibration by matching to data residual to velocity residual at well using least-squares amplitude and constant-phase rotations.
4. Update using addition.
Marmousi Model
with shots (red), receivers (white), and well (black)

40 shots, split spread, far offset 2000 m.
Modelling

Spectrum of source wavelet (5 Hz dominant)

Spectrum of modelled data
Sample Shots

Shot 1

Shot 20

Shot 40

Seconds

Meters
Initial Velocity Model
Gaussian smoother 580m width
Shot 20

Original shot

Predicted by initial model
<table>
<thead>
<tr>
<th>Deconvolution Imaging condition</th>
<th>Migration of full data</th>
<th>Migration of data residual</th>
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<tbody>
<tr>
<td>Cross Correlation Imaging condition</td>
<td>Migration of full data</td>
<td>Migration of data residual</td>
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Calibration at the well
First iteration 0-5 Hz.

Blue is the exact velocity at the well. Red is the migration velocity.
After 1 Iteration 0-5 Hz

$v_{true}(x, z)$

$v_0(x, z) + \lambda M^\dagger[\delta \psi_0]$
Frequency Iteration

- Iteration 1, 1-4 Hz
- Iteration 2, 5-6 Hz
- Iteration 3, 5-10 Hz
- Iteration 5, 10-20 Hz
- Iteration 11, 25-35 Hz
- Iteration 22, 55-60 Hz
Smoother Iteration 1-40 Hz normal mute

Iteration 1, 1000m
Iteration 6, 200m
Iteration 7, 100m
Iteration 8, 50m
Iteration 9, 40m
Iteration 12, 10m
Data Residual L2 Norms

![Data Residual L2 Norms Graph](image)
Common Image Gather
Smoother Iteration 1-40 Hz wide mute

Iteration 1, 1000m
Iteration 6, 200m
Iteration 7, 100m
Iteration 8, 50m
Iteration 9, 40m
Iteration 12, 10m
Smoother Iteration 1-40 Hz harsh mute

Iteration 1, 1000m
Iteration 6, 200m
Iteration 7, 100m
Iteration 8, 50m
Iteration 9, 40m
Iteration 12, 10m
1-40 Hz Wide Mute, smoother fixed at 12 m
Data Residual L2 Norms

![Graph showing the data residual L2 norms over iterations. The norms start high, drop to a lower level, and remain constant thereafter.](image)
Shot 20

Original shot

Predicted on third iteration
Conclusions

The FTFWI suggests a generalized inversion scheme with many possible variations. FWI is an iterative modelling, migration, and calibration process. FWI can be done with migration algorithms other than RTM. Calibration generally involves a wavelet estimation and scaling, and is reminiscent of impedance inversion. The process demonstrated here is not claimed to be optimal, but is suggestive of many other variants.
Acknowledgements

We thank our industry sponsors for their generous support which makes our work possible. We thank Hussain Hammad for his thesis and discussions.
Calibrating the migration

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produces the best forward modelled data.

**This paper:** Find the scalar $\lambda$ such that

$$\int (v_{true}(x_w, z) - v_k(x_w, z))^2 \, dz = \min$$

where $x_w$ is the well location at which $v_{true}$ is known.
Calibrating the migration

Matching condition at the well

\[ \int (v_{true}(x_w, z) - v_k(x_w, z))^2 \, dz = \min \]

Substituting for \( v_k \):

\[ \int (v_{true}(x_w, z) - v_{k-1} - \lambda M^\dagger[\delta \psi_k](x, z))^2 \, dz = \min \]

Defining the velocity residual

\[ \int (\delta v_k(x_w, z) - \lambda M^\dagger[\delta \psi_k](x, z))^2 \, dz = \min \]

So we will match the migrated data residual to the velocity residual at the well. This is a process very similar to standard impedance inversion.